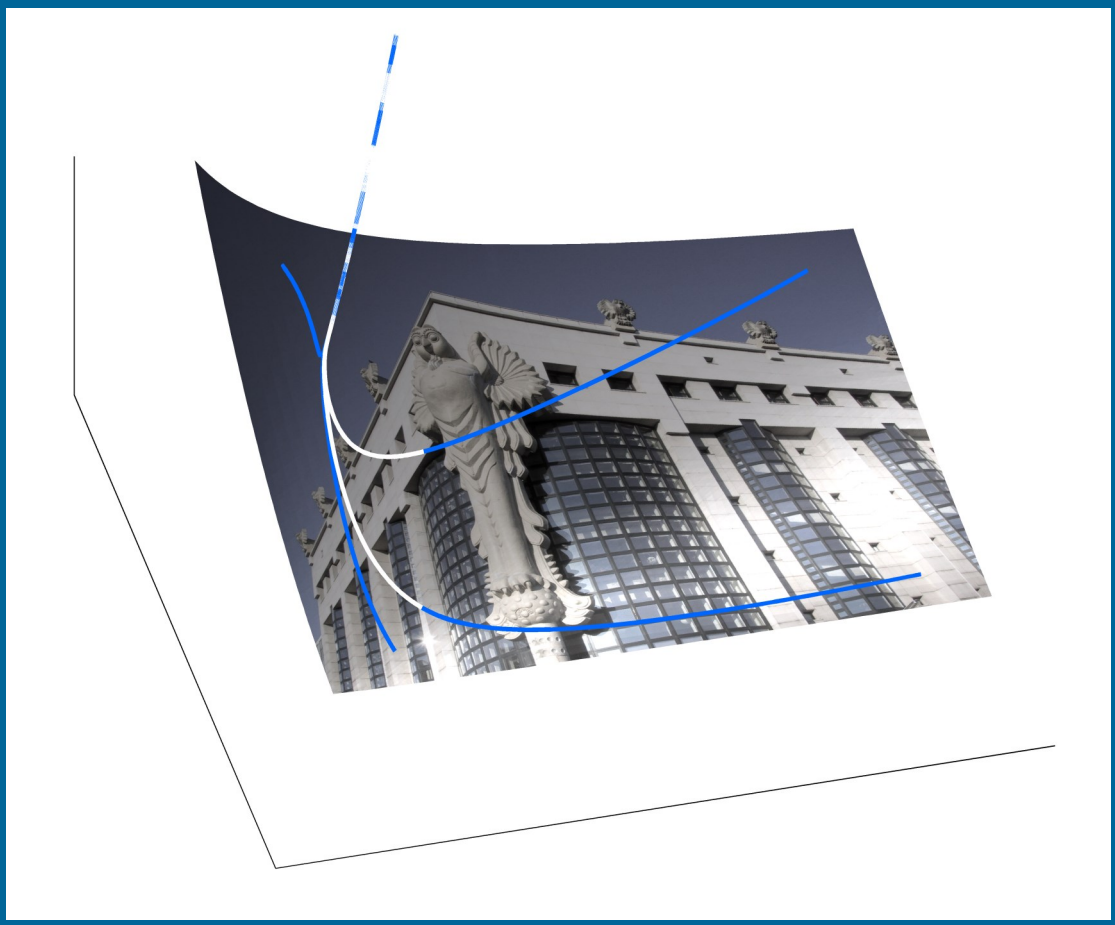
TECHNISCHE
UNIVERSITÄT
WIEN
Vienna University of Technology

15th Viennese Conference

Optimal Control and Dynamic Games



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Contents

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Karl Sigmund (AT)
Terry Rockafellar (US)
George Yin (US)

Semi-plenary Speakers

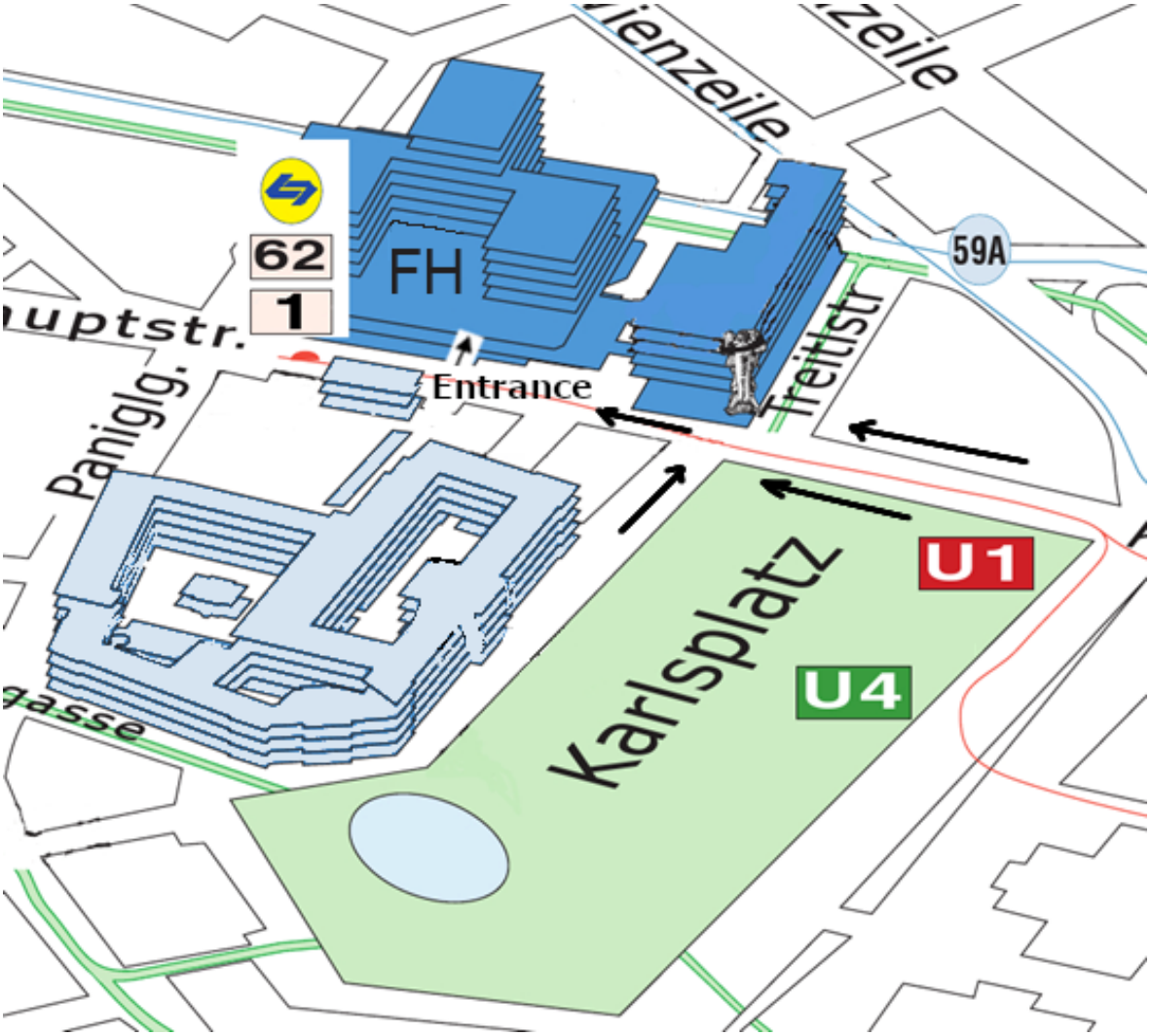
Eduardo Casas (ES)
Lars Grüne (DE)
Raouf Boucekkine (FR)
George Zaccour (CA)

I. Practical Information

1. Conference Venue

Vienna University of Technology (TU Wien)

Vienna University of Technology is located in the heart of Vienna. The conference takes place in the „**Freihaus**“ (building FH), Wiedner Hauptstrasse 8-10, 1040 Vienna. As indicated on the map, the TU Wien can be reached by the underground lines U1 and U4, station „Karlsplatz“, or tram lines 1 and 62 or „Badner Bahn“, station „Resselgasse“.



Lecture Rooms

All lecture rooms are in the **Freihaus** (D), red and yellow area at first and second floor. For an overview see the map on the next page.

FH HS 1: The main entrance of **FH Hörsaal 1** is on the first floor of the red area. It is also accessible from the second floor of the red area.

FH HS 5: is located on the second floor of the green area (accessible only from second floor).

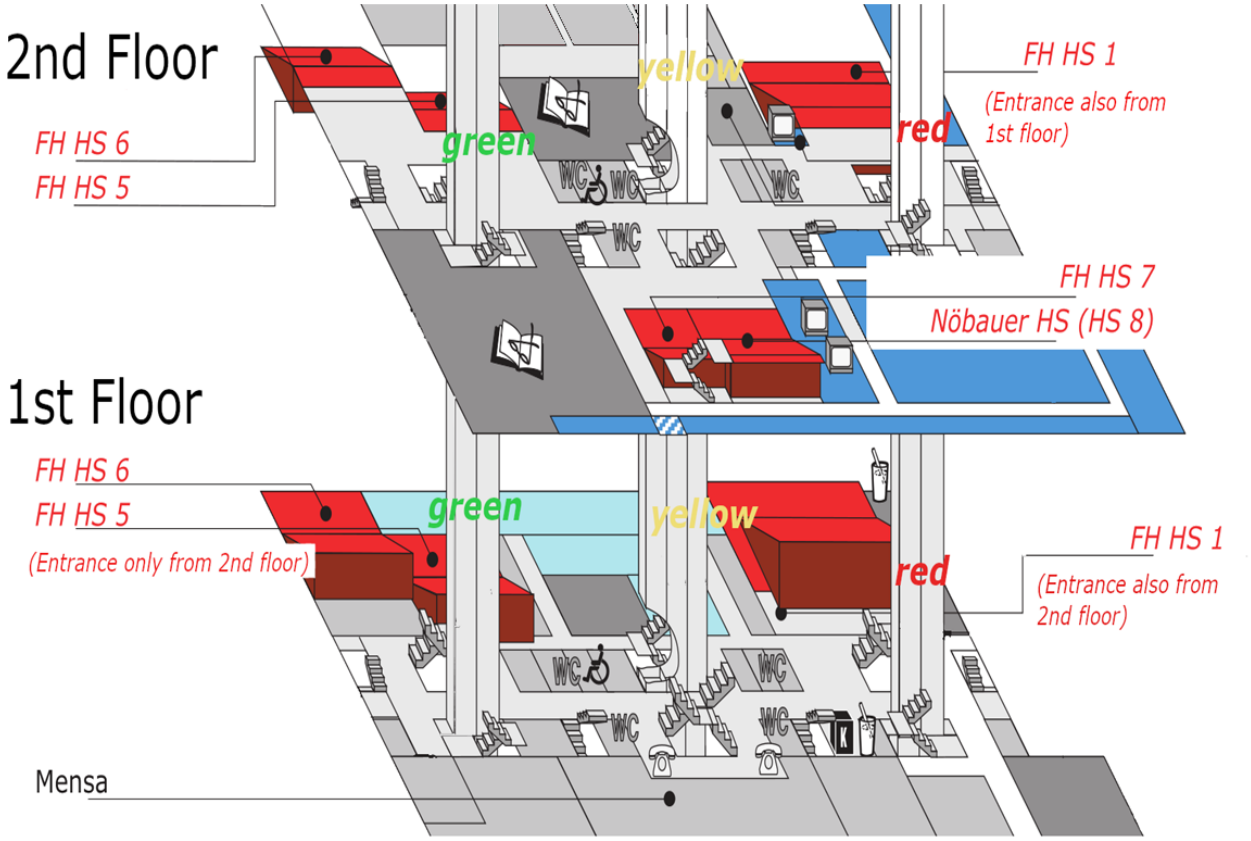
FH HS 6: is located on the second floor of the green area (accessible only from second floor).

FH HS 7: is located on the second floor of the yellow area.

FH HS 8: is located on the second floor of the yellow area.

Coffee Breaks will take place on the hallway of the yellow and red areas on the second floor.

The Mensa is located in the yellow area of the first floor.



2. Registration, Conference Desk and Welcome Reception

The **Registration Desk** will be open on Monday, July 11th, from 17:00 to 19:00 on the first floor in the Freihaus (building D) in front of the mensa (yellow area). The **Welcome Reception** will take place in the mensa and begins at 18:00. **Late registration** will be possible at the **Conference Desk** on the second floor, yellow area, on Tuesday July 12th, from 07:45 to 10:00 and during the coffee breaks on all following conference days.

Upon registration you receive your confirmation of payment, an information package including this booklet, vouchers for lunches in the university mensa and invitation cards for the social events.

3. Guidelines for Speakers and Session Organizers

All rooms are equipped with notebooks and projectors. Please prepare your presentation as pdf-file (recommended) or ppt-file on a USB memory-stick and copy it on the notebook **before** the session has started. Your presentation may last **25 minutes, including preparation and discussions**, thus 20 min. should be the actual duration of the talk.

The session organizers are supposed to chair their sessions or to take care of alternative chair persons. The chair persons are responsible for strictly keeping the schedule.

4. WLAN and Internet, Charging of devices

We encourage all participants to use their eduroam accounts, if provided by their university. A limited number of TU-based internet accounts is available at the conference desk.

It is possible to charge devices in Seminar room DB gelb 04 (4th floor of Freihaus, yellow area).

5. Lunch

Upon registration you receive vouchers which can be used for menu (including one drink from the machine) at the university mensa. The mensa is located at Freihaus, yellow area, first floor (see the map of Freihaus for details). It is self-service and will be open Tuesday to Friday 11:00-14:00.

Restaurants and grocery stores near the university are shown on the next page.


Karlsplatz
B1
Wieden
Kunsthalle
Wien Karlsplatz
Heuer
Kleines
Haus der
Kunst
Opernwurst
Stadl
Operngasse
Treitlstraße
Knockbox
Naschmarkt
Innere Stadt
Admiral
Porrhaus
Tolstoy
Hauptbibliothek
Eule
Wienzeile
Tipico
Resselgasse
Leckture
Hörsaal
FH1
Konsularabteilung
der Botschaft
der Republik
Kroatien
TU Wien,
Freihaus
INTU Paper
Wiedner Hauptstraße
kopie
mood's
hair
Freihausviertel
Schrödinger
Bahn mi
RE/MAX
Schaurhofergasse
Bank Austria
Paniglgasse
TTTech
Auto AG
Anker
Billa
Rilkeplatz
kangmei
massage
TeigWare
Trinity
Porto
Dr. Thomas
Strauch
HairConcept
Crupi Sizilianische

6. Transportation

Transportation from/to the Airport

Public Transportation

There are buses and trains going from and to the airport. The cheapest is the “S-Bahn” (see rates below).

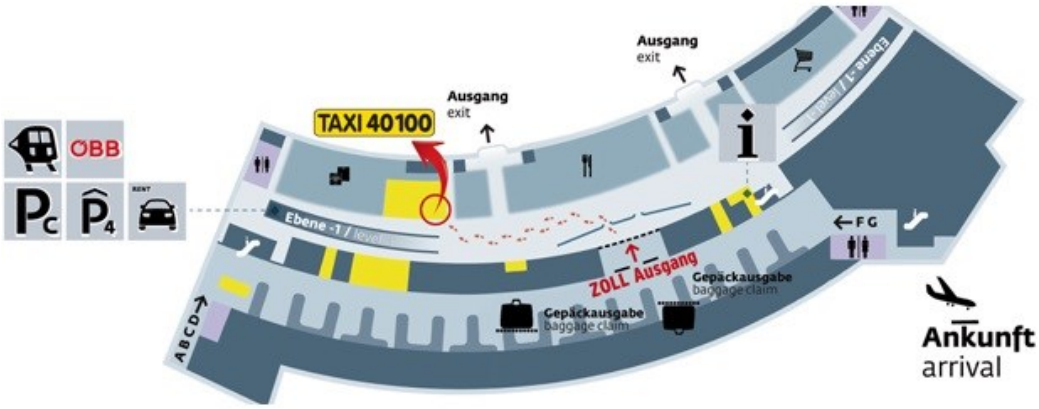
	Place of Departure	Interval (min)	Duration (min)	Price (€)
Railjet	Vienna Central Station (Wien Hauptbahnhof— Südtiroler Platz)	~30	15	4.30
City Airport Train	Wien Mitte	~30	16	11,-
Airport Bus 1187	Westbahnhof (Europaplatz)	~30	45	8,-
Airport Bus 1185	Schwedenplatz (Morzinplatz)	~30	20	8,-
S-Bahn	Wien Mitte	~30	25	4,30

Websites: www.cityairporttrain.com, **www.postbus.at**, **www.oebb.at**.

All timetables can also be found on **www.wienerlinien.at**.

Airport Taxi:

A taxi from/to the center costs about € 35.00, see www.taxi40100.at/flughafen (Tel: +43-(1) 40100).



Wiener Linien (Public Transport)

Map

See the city map of Vienna in your Welcome Package or **www.wienerlinien.at**.

Metro operation hours:

Mo-Thu, Su: 05:00 – approx. 00:20, then Nightlines (buses).

Fr-Sa: 05:00 - 00:20 in the usual intervals, then every 15 min for the whole night.

Tickets

Single Trip	24/48/72 Hours Ticket	The 8 Days Climate Ticket	One Week Ticket *
2,40 €	8,00/14.10/ 17.10 €	40.80 €	17.10 €

* valid from Monday 00:00 till Monday 09:00

Timetables, route planner and real time information

All information about tickets, time tables, nightlines, route planner, etc. can be found at **www.wienerlinien.at**. The app „WienMobil“ provides all information directly on your smartphone: www.wien.info/en/travel-info/transport/wienmobil

Taxi

The closest taxi stand in walking distance from the university is at Linke Wienzeile 4.

Some numbers and websites to order a taxi:

Phone 01/40100
www.taxi40100.at

Phone 01/31300
www.taxi31300.at

Transportation from/to Vienna's Railway Stations

- **Westbahnhof:** Metro U3 and U6, station "Westbahnhof"
- **Wien Meidling:** Metro U6, station "Philadelphiabrücke"
- **Wien Hauptbahnhof:** Metro U1, station "Hauptbahnhof(Südtiroler Platz)", Bus 13A station "Hauptbahnhof", Tram D station "Hauptbahnhof Ost"
- **Franz-Josefs-Bahnhof:** Metro U4, station "Friedensbrücke", Tram D station "Franz-Josefs-Bahnhof"

Homepage of the Austrian Federal Railways (ÖBB): www.oebb.at

City Bike

You can rent a bike from one of over 100 stations in Vienna and discover the city on a bike. The bike can be returned at any other station. Registration €1 by credit card, the first hour of every ride is free. Other rates and more information online: **www.citybikewien.at.**

7. Social Program

Welcome Reception

The Welcome Reception will take place Monday, July 11th, from 18:00-21:00 in front of the Mensa on the first floor, yellow area.

This event is covered by your registration fee.

Conference Dinner/Cocktail Reception

The Mayor and Governor of Vienna demands the pleasure of your company at a Cocktail Reception on Wednesday, July 13th, at the "Wiener Rathaus (town hall) - Wappensaal". The entrance time is 19:30.

Entrance: 1010 Vienna, Lichtenfelsgasse 2.

Public Transportation:

Tram 1 , D, U2Z to station Rathausplatz/Burgtheater

Walking distance:

about 2 km (25 min) from TU Wien.

This event is covered by your registration fee. Please bring the invitation you received upon registration.

Round Table Debate

A round table debate about connecting science with industry will take place as a luncheon in the canteen on Thursday, July 14th, 12:40 -13:40.

Achieving great scientific/research results is one of the main drivers in academia, while having the results applied in the respective areas in the industry makes the success double in a very organic way. Why then it is not easy to get it go hand in hand?

At VC2022 we would like to explore the opportunities for scientific conferences like this one, to support the bridging between science and its applicability. It is open invitation for all, interested in the topic. We want to hear from those of you, who have been successful with applying their research results as well from those, who have been experiencing barriers in it.

After opening by the chairman, V. Veliov, the debate will be moderated by the organizers of the round table –Virginia Peneva, IT architect, and Kathleen Jimenez-Mühlbach, from Funding Support and Industry Relations, TU Wien. No registration needed, however we kindly recommend e-mailing to viivpen@gmail.com, (Virginia Ivanova Peneva) so that we can ensure that your lunch&drinks are served before we begin the discussion. The table will be marked "reserved for the round table".

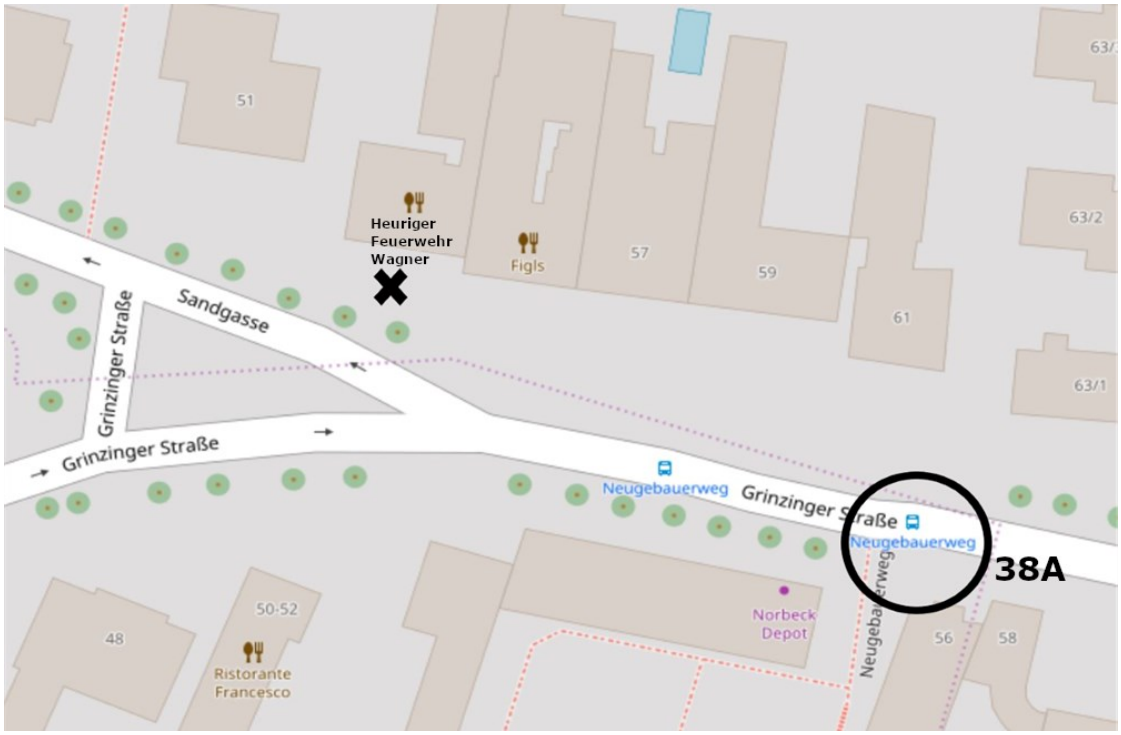
Any additional information in this regard, which may appear, will be emailed to the interested before July 14.

Discussion Dinner

A joint discussion dinner will take place on Thursday, July 14th, 19:00 at a "Heurigen" in Grinzing, "Feuerwehr Wagner", 1190 Vienna, Grinzinger Straße 53. Participants who paid for the dinner will receive invitations in their package.

Public Transportation:

Take Metro U4 to station Spittelau, from there use tram D to Grinzinger Straße and finally bus 38A to station Neugebauerweg.



8.Tourist Information

Tourist map

In the conference package a tourist map of the center of Vienna is included. The most important sights with opening hours are
described therein.

Tourist Information

At the corner Albertinaplatz/Maysedergasse, 1010 Vienna
www.wien.info or **www.wien.gv.at/tourismus**

Tickets for Concerts and Theater

WIEN-TICKET Pavillon in the State Opera House, 1010 Vienna (5 minutes from Vienna University of Technology)
www.wien-ticket.at/en

II. Scientific Program

List of Sessions

Session name (time-slot number)	*Organizers*
Plenaries and Semi-Plenaries (2,11,20,29,50)	Scientific committee
Set-valued approximation in control and optimization (21,25)	R. Baier, E. Farkhi
Optimal control and applications on biology and medicine (57,61)	P. Bettiol, V. Milisic, J.Rouot
Continuous optimization: theory and applications (46,51,55,67)	R. Bot, A. Daniilidis
Variational analysis in optimization and control (3,16,30,34)	R. Cibulka
Optimal control and Machine Learning (7,12)	M. Falcone, M. Palladino
Data-driven and learning-based control (35,39,43,52,56)	T. Faulwasser, K. Worthmann
Stochastic control models in economics and finance (6,10,37)	S. Federico, G. Ferrari
Optimal control in production, logistics and marketing (41,45)	G. Feichtinger, A. Seidl
Optimal control and calculus of variations on metric spaces (59,63)	H. Frankowska, M. Quincampoix
Infinite dimensional economic dynamics (15,19,24)	F. Gozzi, G. Fabbri, S. Faggian
Mathematical models for the human impact on the environment (40,44)	F. Gozzi, G. Fabbri, S. Faggian
Covid-19: optimal control approaches (23,65,69)	D. Grass, S. Wrzaczek
Recent advances in Model Predictive Control (13,17,22,26)	L. Grüne
Economic dynamics with regime switching (54,58)	J. Haunschmied, W. Semmler
Infinite horizon optimal control and applications (38,42)	N. Hayek, S. Pickenhain
Dynamic economic policy (62)	B. Heijdra, P. Heijnen
Dynamic games in industrial organization (45,49)	M. Kopel
Dynamics of firm (28,33,66,70)	P. Kort
Dynamic games in environmental and resource economics (14,18,53)	L. Lambertini
Numerical analysis for PDE constrained optimization (4,8,31,47,64,68)	J. Pfefferer, A. Rösch
Population ageing and inequality (27)	A. Prskawetz, M. Sanchez
Dynamic resource and environmental economics in spatial domains (5,9)	T. Upmann
Dynamic games and applications (32,36,48)	S. Wrzaczek
Mean field games (60)	Scientific committee
Round table “Industrial applications – opportunities and challenges” (see p. 13)	Organizing committee

Program Overview

Tue July 12

FH HS 1

08:30-9:10 **Session 1: Opening**

09:10-10:00 **Session 2: Plenary - Karl Sigmund**

Coffee Break at 1st/ 2nd floor, Freihaus

Freihaus	FH HS 8	FH HS 5	FH HS 6	FH HS 7
Slot/Session	3 Cibulka	4 Pfefferer, Roesch	5 Upmann	6 Federico, Ferrari
10:20-11:35	Cibulka, Radek	Chrysafinos, Konstantinos	Anita, Stefana-Lucia	Calvia, Alessandro
	Benko, Matúš	Fallahpour, Merlin	Fabbri, Giorgio	Dianetti, Jodi
	Fabian, Marian	Gedicke, Joscha	Faggian, Silvia	Ghilli, Daria
Slot/Session	7 Falcone, Palladino	8 Pfefferer, Roesch	9 Upmann	10 Federico, Ferrari
11:45-12:35	Aubin-Frankowski, Pierre-Cyril	Haubner, Christof	Gozzi, Fausto	Rodosthenous, Neofytos
	Wang, Chia-Li	Wollner, Winnifried	Singh, Bismark	Palczewski, Jan

Lunch at Canteen, 1st floor, Freihaus

14:00-14:50

FH HS 5	FH HS 6
Session 11: Semi Plenary - Lars Grüne	**Session 11: Semi Plenary - Georges Zaccour**

Coffee Break at 2nd floor, Freihaus

Slot/Session	12 Falcone, Palladino	13 Gruene	14 Lambertini	15 Gozzi, Fabbri, Faggian
15:10-16:25	Darbon, Jerome	Fleig, Arthur	El Ouardighi, Fouad	Biagini, Sara
	Falcone, Maurizio	Kögel, Markus	Feichtinger, Gustav	Fiaschi, Davide
	Pham, Huyên	Krügel, Lisa	Freiberger, Michael	Leocata, Marta
Slot/Session	16 Cibulka	17 Gruene	18 Lambertini	19 Gozzi, Fabbri, Faggian
16:40-17:55	Krastanov, Mikhail	Lehtomaa, Jere	Lambertini, Luca	Masiero, Federica
	Lajara, Sebastian	Müller, Matthias A.	Morales, José Rodolfo	Parenti, Angela
	Ribarska, Nadezhda	Püttschneider, Jens	Wrzaczek, Stefan	Ricci, Cristiano

Wed July 13

FH HS 8

08:30-09:20 **Session 20: Plenary - George Yin**

Coffee Break at 2nd floor, Freihaus

Freihaus	FH HS 8	FH HS 5	FH HS 6	FH HS 7
Slot/Session	21 Baier, Farkhi	22 Gruene	23 Grass, Wrzaczek	24 Gozzi, Fabbri, Faggian
09:40-10:55	Farkhi, Elza	Rakovic, Sasa V.	Neuvonen, Lauri	Behringer, Stefan
	Donchev, Tzanko	Rawlings, James	Neck, Reinhard	Brito, Paulo B.
	Baier, Robert	Sanfelice, Ricardo G.	Kim, Bowon	Hosoya, Yuhki
Slot/Session	25 Baier, Farkhi	26 Gruene	27 Prskawetz, Sanchez	28 Kort
11:10-12:50	Margaliot, Michael	Schaller, Manuel	Kindermann, Fabian	Brill, Maximilian
	Wolenski, Peter	Tanwani, Aneel	Siassi, Nawid	Stofferis, Berend Joost
	Steppich, Florian	Volkwein, Stefan	Trimborn, Timo	Gapeev, Pavel
	Mokhov, Alona	Worthmann, Karl	Fehr, Hans	Hagspiel, Verena

Lunch at Canteen, 1st floor, Freihaus

14:00-14:50

FH HS 5	FH HS 6
Session 29: Semi Plenary - Eduardo Casas	**Session 29: Semi Plenary - Raouf Boucekkine**

Coffee Break at 2nd floor, Freihaus

Slot/Session	30 Cibulka	31 Pfefferer, Roesch	32 Wrzaczek	33 Kort
15:10-16:25	Roubal, Tomáš	Neitzel, Ira	Parilina, Elena	Huisman, Kuno
	Zlateva, Nadia	Hoppe, Fabian	Wiszniewska-Matyszkiel, Agnieszka	Ketelaars, Martijn Willem
	Domínguez Corella, Alberto	Kahle, Christian	Zaccour, Georges	Thijssen, Jacco
Slot/Session	34 Cibulka	35 Faulwasser, Worthmann	36 Wrzaczek	37 Federico, Ferrari
16:40-17:55	Fusco, Giovanni	Maslovskaya, Sofya	Machowska, Dominika	Zanco, Giovanni
	Rampazzo, Franco	Grüne, Lars	Martin-Herran, Guiomar	Privileggi, Fabio
	Tam, Jonathan	Binfet, Philipp	Blueschke, Dmitri	

Thu July 14	Freihaus	FH HS 8	FH HS 5	FH HS 6	FH HS 7
Slot/Session	38	Hayek, Pickenhain	39 Faulwasser, Worthmann	40 Gozzi, Fabbri, Faggian	41 Feichtinger, Seidl
08:30-09:20		Crettez, Bertrand	Flaßkamp, Kathrin	Augeraud-Véron, Emmanuelle	Kogan, Konstantin
		Dikariev, Ilya	Bieker, Katharina	Boucekkine, Raouf	Muttoni, Maddalena
		Coffee Break at 2nd floor, Freihaus			
Slot/Session	42	Hayek, Pickenhain	43 Faulwasser, Worthmann	44 Gozzi, Fabbri, Faggian	45 Feichtinger, Seidl/Kopel
09:40-10:55		Hayek, Naila	Faulwasser, Timm	Dobson, Andy	Novak, Andreas
		Pickenhain, Sabine	Findeisen, Rolf	Loch Temzelides, Ted	Seidl, Andrea
		Shvartsman, Ilya	Heiland, Jan	Upmann, Thorsten	Dawid, Herbert
Slot/Session	46	Bot, Daniilidis:	47 Pfefferer, Roesch	48 Wrzaczek	49 Kopel
11:10-12:25		Contreras, Andrés	Tröltzsch, Fredi	Palokangas, Tapio Kalervo	Eigruber, Markus
		Geiersbach, Caroline	Leykekhman, Dmitriy	Bondarev, Anton A.	Cabo, Francisco
		Grapiglia, Geovani	König, Philipp	de Frutos, Javier	Tampieri, Alessandro
				Gromova, Ekaterina	
		Lunch at Canteen, 1st floor, Freihaus			
		Lunch and Round Table "Science and Industry" at marked tables in the Canteen, 12:40-13:40			
		FH HS 8			
14:00-14:50		**Session 50: Plenary - Terry Rockafellar**			
		Coffee Break at 2nd floor, Freihaus			
Slot/Session	51	Bot, Daniilidis:	52 Faulwasser, Worthmann	53 Lambertini	54 Haunschmied, Semmler
15:10-16:25		Giselsson, Pontus	Koltai, Péter	Goetz, Renan	Semmler, Willi
		Latafat, Puya	Nonhoff, Marko	Haastert, Simon Gero	Zou, Benteng
		Laude, Emanuel	Schaller, Manuel	Khmelnitsky, Eugene	Saglam, Cagri
Slot/Session	55	Bot, Daniilidis:	56 Faulwasser, Worthmann	57 Bettiol, Milisic, Rouot	58 Haunschmied, Semmler
16:40-17:55		Mishchenko, Konstantin	Sperl, Mario	Augier, Nicolas	Marin-Solano, Jesus
		Sedlmayer, Michael	Vasquez-Varas, Donato	Bettiol, Piernicola	Marzufero, Luciano
		Staudigl, Mathias	Voigt, Matthias	Comte, Éloïse	Xu, Wenji

Fri July 15	Freihaus	FH HS 8	FH HS 5	FH HS 6	FH HS 7
Slot/Session	59	Frankowska, Quincampoix	60 Scientific Committee	61 Bettiol, Milisic, Rouot	62 Heijdra, Heijnen
08:30-10:10		Bonnet-Weill, Benoit	Festa, Adriano	Djema, Walid	Heijdra, Ben
		Cannarsa, Piermarco	Murali, Divya	Oelz, Dietmar	Heijnen, Pim
		Cavagnari, Giulia	Neumann, Berenice Anne	Rouot, Jeremy	Van der Kwaak, Christiaan
		Frankowska, Helene	Wendt, Julian	Milisic, Vuk	Zwart, Gijsbert
		Coffee Break at 2nd floor, Freihaus			
Slot/Session	63	Frankowska, Quincampoix	64 Pfefferer, Roesch	65 Grass, Wrzaczek	66 Kort
10:30-11:45		Lorenz, Thomas	Meyer, Christian	Lykina, Valeria	Moreira, Afonso Moniz
		Marigonda, Antonio	Mateos, Mariano	Huberts, Nick F.D.	Nagy, Roel L.G.
		Quincampoix, Marc	Wachsmuth, Daniel	Kulikova, Yuliya	Faninam, Farzan
Slot/Session	67	Bot, Daniilidis:	68 Pfefferer, Roesch	69 Grass, Wrzaczek	70 Kort
11:55-13:10		Caldwell, Bethany	Winkler, Max	Nowakowski, Andrzej	Kort, Peter M.
		Malinovsky, Grigory	Jork, Nicolai Alexander	Lindström, Torsten Axel	Wen, Xingang
		Moghrabi, Issam		Angelov, Georgi Stoychev	Schünemann, Johannes

Detailed program

Tuesday, July 12th

7 Optimal control and Machine Learning **11:45 – 12:35**
***Chair*: M. Falcone, M. Palladino** **FH HS 8**

8 Numerical analysis for PDE constrained optimization **11:45 – 12:35**
***Chair*: J. Pfefferer, A. Rösch** **FH HS 5**

9 Dynamic resource and environmental economics in spatial domains **11:45 – 12:35**
***Chair*: T. Upmann** **FH HS 6**

10 Stochastic control models in economics and finance **11:45 – 12:35**
***Chair*: S. Federico, G. Ferrari** **FH HS 7**

Lunch Break **12:35 – 14:00**

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11 Semi Plenary **14:00 – 14:50**
***Chair*: R. Findeisen** **FH HS 5**

11 Semi Plenary **14:00 – 14:50**
***Chair*: L. Lambertini** **FH HS 6**

Coffee Break

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12 Optimal control and Machine Learning **15:10 – 16:25**
***Chair*: M. Falcone, M. Palladino** **FH HS 8**

13 Recent advances in Model Predictive Control **15:10 – 16:25**
***Chair*: L. Grüne** **FH HS 5**

14 Dynamic games in environmental and resource economics **15:10 – 16:25**
***Chair*: L. Lambertini** **FH HS 6**

15 Infinite dimensional economic dynamics **15:10 – 16:25**
***Chair*: F. Gozzi, G. Fabbri, S. Faggian** **FH HS 7**

Coffee Break

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16 Variational analysis in optimization and control **16:40 – 17:55**
***Chair*: R. Cibulka** **FH HS 8**

17 Recent advances in Model Predictive Control **16:40 – 17:55**
***Chair*: L. Grüne** **FH HS 5**

18 Dynamic games in environmental and resource economics **16:40 – 17:55**
***Chair*: L. Lambertini** **FH HS 6**

19 Infinite dimensional economic dynamics **16:40 – 17:55**
***Chair*: F. Gozzi, G. Fabbri, S. Faggian** **FH HS 7**

Wednesday, July 13^{th}

20 Plenary **08:30 – 09:20**
***Chair*: H. Frankowska** **FH HS 8**

Coffee Break

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21 Set-valued approximation in control and optimization **09:40 – 10:55**
***Chair*: R. Baier, E. Farkhi** **FH HS 8**

22 Recent advances in Model Predictive Control **09:40 – 10:55**
***Chair*: L. Grüne** **FH HS 5**

23 Covid-19: optimal control approaches **09:40 – 10:55**
***Chair*: D. Grass, S. Wrzaczek** **FH HS 6**

24 Infinite dimensional economic dynamics **09:40 – 10:55**
***Chair*: F. Gozzi, G. Fabbri, S. Faggian** **FH HS 7**

Break

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25 Set-valued approximation in control and optimization **11:10 – 12:50**
***Chair*: R. Baier, E. Farkhi** **FH HS 8**

26 Recent advances in Model Predictive Control **11:10 – 12:50**
***Chair*: L. Grüne** **FH HS 5**

27 Population ageing and inequality **11:10 – 12:50**
***Chair*: A. Prskawetz, M. Sanchez** **FH HS 6**

28 Dynamics of firm **11:10 – 12:50**
***Chair*: P.M. Kort** **FH HS 7**

Lunch Break **12:50 – 14:00**

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29 Semi Plenary **14:00 – 14:50**
***Chair*: A. Rösch** **FH HS 5**

29 Semi Plenary **14:00 – 14:50**
***Chair*: P.M. Kort** **FH HS 6**

Coffee Break

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30 Variational analysis in optimization and control **15:10 – 16:25**
***Chair*: R. Cibulka** **FH HS 8**

31 Numerical analysis for PDE constrained optimization **15:10 – 16:25**
***Chair*: J. Pfefferer, A. Rösch** **FH HS 5**

32 Dynamic games and applications **15:10 – 16:25**
***Chair*: S. Wrzaczek** **FH HS 6**

33 Dynamics of firm **15:10 – 16:25**
***Chair*: P.M. Kort** **FH HS 7**

Break

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34 Variational analysis in optimization and control **16:40 – 17:55**
***Chair*: R. Cibulka** **FH HS 8**

35 Data-driven and learning-based control **16:40 – 17:55**
***Chair*: T. Faulwasser, K. Worthmann** **FH HS 5**

36 Dynamic games and applications **16:40 – 17:55**
***Chair*: S. Wrzaczek** **FH HS 6**

37 Stochastic control models in economics and finance **16:40 – 17:55**

***Chair*: S. Federico, G. Ferrari** **FH HS 7**

Thursday, July 14th

38 Infinite horizon optimal control and applications **08:30 – 09:20**
***Chair*: N. Hayek, S. Pickenhain** **FH HS 8**

39 Data-driven and learning-based control **08:30 – 09:20**
***Chair*: T. Faulwasser, K. Worthmann** **FH HS 5**

40 Mathematical models for the human impact on the environment **08:30 – 09:20**
***Chair*: F. Gozzi, G. Fabbri, S. Faggian** **FH HS 6**

41 Optimal control in production, logistics and marketing **08:30 – 09:20**
***Chair*: G. Feichtinger, A. Seidl** **FH HS 7**

Coffee Break

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42 Infinite horizon optimal control and applications **09:40 – 10:55**
***Chair*: N. Hayek, S. Pickenhain** **FH HS 8**

43 Data-driven and learning-based control **09:40 – 10:55**
***Chair*: T. Faulwasser, K. Worthmann** **FH HS 5**

44 Mathematical models for the human impact on the environment **09:40 – 10:55**
***Chair*: F. Gozzi, G. Fabbri, S. Faggian** **FH HS 6**

45 Optimal control in production, logistics and marketing **09:40 – 10:55**
***Chair*: G. Feichtinger, M. Kopel, A. Seidl** **FH HS 7**

Break

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46 Continuous optimization: theory and applications **11:10 – 12:25**
***Chair*: R. Bot, A. Daniilidis** **FH HS 8**

47 Numerical analysis for PDE constrained optimization **11:10 – 12:25**
***Chair*: J. Pfefferer, A. Rösch** **FH HS 5**

48 Dynamic games and applications **11:10 – 12:25**
***Chair*: S. Wrzaczek** **FH HS 6**

49 Dynamic Games in Industrial Organization **11:10 – 12:25**
***Chair*: M. Kopel** **Saal**

Lunch Break **12:25 – 14:00**

Round Table "Science and Industry" (Canteen) **12:40 – 13:40**

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50 Plenary **14:00 – 14:50**
***Chair*: A. Daniilidis** **FH HS 8**

Coffee Break

51 Continuous optimization: theory and applications **15:10 – 16:25**
***Chair*: R. Bot, A. Daniilidis** **FH HS 8**

52 Data-driven and learning-based control **15:10 – 16:25**
***Chair*: T. Faulwasser, K. Worthmann** **FH HS 5**

53 Dynamic games in environmental and resource economics **15:10 – 16:25**
***Chair*: L. Lambertini** **FH HS 6**

54 Economic dynamics with regime switching **15:10 – 16:25**
***Chair*: J. Haunschmied, W. Semmler** **FH HS 7**

Coffee Break

55 Continuous optimization: theory and applications **16:40 – 17:55**
***Chair*: R. Bot, A. Daniilidis** **FH HS 8**

Friday, July 15^{th}

59 Optimal control and calculus of variations on metric spaces **08:30 – 10:10**
***Chair*: H. Frankowska, M. Quincampoix** **FH HS 8**

60 Mean field games **08:30 – 10:10**
***Chair*: Scientific Committee** **FH HS 5**

61 Optimal control and applications on biology and medicine **08:30 – 10:10**
***Chair*: P. Bettiol, V. Milisic, J.Rouot** **FH HS 6**

62 Dynamic economic policy **08:30 – 10:10**
***Chair*: B. Heijdra, P. Heijnen** **FH HS 7**

Coffee Break

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63 Optimal control and calculus of variations on metric spaces **10:30 – 11:45**
***Chair*: H. Frankowska, M. Quincampoix** **FH HS 8**

64 Numerical analysis for PDE constrained optimization **10:30 – 11:45**
***Chair*: J. Pfefferer, A. Rösch** **FH HS 5**

Abstracts

Tuesday, July 12^{th}

1. Opening — 08:30 – 09:10

FH HS 1

2. Plenary — 09:10 – 10:00

***Chair*: G. Feichtinger** — **FH HS 1**

Adaptive dynamics for signaling games

Karl Sigmund (1),

(1) Faculty for Mathematics, University of Vienna, Vienna, Austria;

Signaling games are used to investigate basic, nonverbal ways of conveying information. If the interests of senders and receivers are in conflict, this information is apt to be unreliable. We apply a single minimalistic framework to examples of frequently studied interactions, such as costly advertisements, ownership claims, and signals for help. We analyze their evolution within populations with the help of adaptive dynamics, a basic tool of evolutionary game theory describing a trial-and-error adjustment process. This new approach to signaling is particularly appropriate for investigating oscillations around partially revealing equilibria, communication failures, and paradoxical outcomes.

3. Variational analysis in optimization and control — 10:20 – 11:35

***Chair*: R. Cibulka** — **FH HS 8**

A quest for simple and unified proofs in regularity theory: perturbation stability

Radek Cibulka (1), Tomáš Roubal (1),

(1) NTIS - New Technologies for the Information Society and Department of Mathematics, Faculty of Applied Sciences, University of West Bohemia, Pilsen, Czech Republic;

Ioffe's criterion and various reformulations of it have become a standard tool in proving theorems guaranteeing metric regularity of a (set-valued) mapping. First, we demonstrate that one should always use directly the so-called general criterion which follows, for example, from Ekeland's variational principle, and that there is no need to make a detour through the slope-based consequences of this general statement. Second, we argue that when proving perturbation stability results, in the spirit of Lyusternik-Graves theorem, there is no need to employ the concept of a lower semi-continuous envelope even in the case of an incomplete target space. The gist is to use the "correct" function to which Ekeland's variational principle is applied; namely, the distance function to the graph of the set-valued mapping under consideration. This approach originates in the notion of graphical regularity introduced by L. Thibault, which is equivalent to the property of metric regularity. Our criteria cover also both metric subregularity and metric semiregularity, which are weaker properties obtained by fixing one of the points in the definition of metric regularity.

Second-order sufficient conditions are easy, in a way

Matúš Benko (1,2), Helmut Gfrerer (2), Patrick Mehlitz (3,4), Jane J. Ye (5), Jin Zhang (6), Jinchuan Zhou (7),

(1) Applied Mathematics and Optimization, University of Vienna, Vienna, Austria; (2) Institute of Computational Mathematics, Johannes Kepler University Linz, Linz, Austria; (3) Institute of Mathematics, Brandenburg University of Technology Cottbus–Senftenberg, Cottbus, Germany; (4) School of Business Informatics and Mathematics, University of Mannheim, Mannheim, Germany; (5) Department of Mathematics and Statistics, University of Victoria, Victoria, B.C., Canada; (6) Department of Mathematics, Southern University of Science and Technology, Shenzhen, P.R. China; (7) Department of Statistics, School of Mathematics and Statistics, Shandong University of Technology, Zibo, P.R. China;

In this talk, we discuss second-order sufficient optimality conditions for very general optimization problems, in which the curvature of the feasible set is captured via the second subderivative of the indicator function. We stress how how easy it it to derive these conditions even for very challenging classes of mathmatical programs, such as the bilevel programs, programs with constraints govern by quasi-variational inequalities, and the mathematical program with second-order cone complementarity constraints (SOC-MPCC). The key reason is that the calculus for second subderivatives is surprisingly favourable. Namely, we address two essential settings - the rules for compositions and for marginal functions. Both these results are needed to handle the aforementioned optimization problems. Interestingly, the desirable chain rule for compositions holds without any assumptions, which seems to be overlooked in standard as well as more recent literature. The rule for marginal functions is shown under the mild assumption of inner calmness*, which is a very recent notion.

Clarke Jacobians, Bouligand Jacobians, and compact connected sets of matrices

David Bartl (1), Marián Fabian (2), Jan Kolář (2),

(1) School of Business Administration in Karviná, Silesian University in Opava, Univerzitní náměstí 1934/3, 733 40 Karviná, Czech Republic; (2) Institute of Mathematics of Czech Academy of Sciences, Žitná 25, 115 67 Prague 1;

Recent results on Clarke Jacobians obtained by D. Bartl and M. Fabian in [1] and [2] are extended to Bouligand Jacobians (subdifferentials). In particular, we prove that every non-empty compact connected set of matrices can be expressed as the Bouligand Jacobian at the origin of a Lipschitz mapping which is moreover either countably piecewise affine or C^∞-smooth off the origin. Given proofs, coming from [3], are simpler and different from those in [1] and [2]. Finally, we discuss the existence of Lipschitz liftings (i.e. right inverses) for Lipschitz mappings from $\mathfrak{R}^n$ to $\mathfrak{R}^m$.

[1] D. Bartl, M. Fabian. Can Pourciau's open mapping theorem be derived from Clarke's inverse mapping theorem easily?, *J. Math. Anal. Appl.* 497 (2021) 124858.

[2] D. Bartl, M. Fabian, *Every compact convex subset of matrices is the Clarke Jacobian of some Lipschitzian mapping*, Proc. Amer. Math. Soc. 149 (2021), 4771–4779.

[3] D. Bartl, M. Fabian, J. Kolář. *Clarke Jacobians, Bouligand Jacobians, and compact connected sets of matrices*, submitted.

4. Numerical analysis for PDE constrained optimizatio 10:20 – 11:35

***Chair*: J. Pfefferer, A. Rösch** **FH HS 5**

Error estimates of fully-discrete approximations of an optimal control problem related to the Allen-Cahn equation

Konstantinos Chrysafinos (1), Dimitra Plaka (2),

(1) Department of Mathematics, National Technical University of Athens, Athens, Greece; (2) Department of Mathematics, National Technical University of Athens, Athens, Greece;

An optimal control problem related to the Allen-Cahn equation is considered. In particular, a tracking functional is minimized subject to the Allen-Cahn equation using distributed controls that satisfy pointwise control constraints. Several results regarding first and second order necessary and sufficient conditions are presented. The lowest order discontinuous Galerkin - in time - scheme is considered for the approximation of the control to state and adjoint state mappings. Under a suitable restriction on the maximum size of the temporal and spatial discretization parameters k, h respectively in terms of the parameter ϵ that describes the thickness of the interface layer, a-priori estimates are presented with constants depending polynomially upon $1/\epsilon$. Unlike to previous works for the uncontrolled Allen-Cahn problem our approach does not rely on a construction of an approximation of the spectral estimate, and as a consequence our estimates are valid under low regularity assumptions imposed by the optimal control setting. Finally, error estimates for the difference between local optimal controls and their discrete approximation as well as between the associated state and adjoint state variables and their discrete approximations are presented.

Shape optimization for composite materials in linear elasticity

Merlin Fallahpour (1), Helmut Harbrecht (1),

(1) Research Group of Computational Mathematics, University of Basel, Basel, Switzerland;

This talk is devoted to the optimal design of the microstructure in composite materials, which are governed by the equations of linear elasticity. To this end, we combine homogenization with shape optimization. In particular, we determine the sensitivity of the homogenized coefficients of the elasticity tensor with respect to the shape of the microstructure also in case of spatially varying material coefficients. We compute the respective Hadamard shape gradient and demonstrate the applicability and feasibility of our approach by numerical experiments for different problem settings.

P_1 finite element methods for an elliptic optimal control problem with pointwise state constraints

Susanne C. Brenner (1), Joscha Gedicke (2), Li-yeng Sung (1),

(1) Department of Mathematics and Center for Computation and Technology, Louisiana State University, Baton Rouge, USA; (2) Institute for Numerical Simulation, University of Bonn, Bonn, Germany;

We present theoretical and numerical results for two P1 finite element methods for an elliptic distributed optimal control problem on general polygonal/polyhedral domains with pointwise state constraints.

[1] S.C. Brenner, J. Gedicke, and L.-Y. Sung. P_1 finite element methods for an elliptic optimal control problem with pointwise state constraints. *IMA J. Numer. Anal.*, **40**(1):1–28, 2020.

5. Dynamic resource and environmental economics in spatial domains 10:20 – 11:35

***Chair*: T. Upmann** **FH HS 6**

Optimal control of a McKean-Vlasov equation via non-linear Fokker-Planck equation

Ştefana-Lucia Aniţa (1),

(1) 'Octav Mayer' Institute of Mathematics of the Romanian Academy, Iaşi, 700506, Romania;

We consider an optimal control problem for a McKean-Vlasov equation with feedback inputs. This stochastic problem is equivalent to a deterministic optimal control problem for a nonlinear Fokker-Planck equation with open-loop controllers. For this latest problem we prove the existence of an optimal control under appropriate hypotheses and we derive necessary optimality conditions. We use these results to gain a deeper insight into the initial stochastic optimal control problem. We apply the theoretical results to an example related to population dynamics and economics.

A dynamic theory of spatial externalities

Raouf Boucekkine (1), Giorgio Fabbri (2), Salvatore Federico (3), Fausto Gozzi (4),

(1) Rennes School of Business, 2 rue Robert d'Arbrissel, Rennes, France.; (2) Grenoble Alpes, CNRS, INRA, Grenoble INP, GAEL, 38000 Grenoble, France.; (3) Università degli Studi di Genova, Dipartimento di Economia. Via Vivaldi, 5 – Darsena – 16126, Italy.; (1) Dipartimento di Economia e Finanza, Libera Università degli Studi Sociali *Guido Carli*, Roma, Italy;

This work targets the class of spatiotemporal problems with free riding under natural (pollution, epidemics...etc) diffusion and spatial externalities. Such a class brings to study a family of differential games in continuous time and space.

In the fundamental pollution free riding problem we develop a strategy to solve completely the associated game contributing to the associated debate on environmental federalism.

We depart from the preexisting literature in several respects. First, instead of assuming *ad hoc* pollution diffusion schemes across space, we consider a realistic spatiotemporal law of motion for pollution (diffusion

and advection). Second, we tackle spatiotemporal non-cooperative (and cooperative) differential games instead of static games in the related literature. Precisely, we consider a circle partitioned into several states where a local authority decides autonomously about its investment, production and depollution strategies over time knowing that investment/production generates pollution, and pollution is transboundary. The time horizon is infinite. Third, we allow for a rich set of geographic heterogeneities across states while the literature assumes identical states.

We solve **analytically** the induced non-cooperative differential game under decentralization and fully characterize the resulting long-term spatial distributions. In particular, we prove that there exist a Perfect Markov Equilibrium, unique among the class of the affine feedbacks.

We further provide with full exploration of the free riding problem, reflected in the so-called *border effects*. Finally, we explore how geographic discrepancies (the most elementary being the asymmetry of players) affect the shape of the border effects. We check in particular that our model is consistent with the set of stylized facts put forward by the related empirical literature.

On competition for spatially distributed resources in networks

Giorgio Fabbri (1), Silvia Faggian (2), Giuseppe Freni (3),

(1) Univ. Grenoble Alpes, CNRS, INRA, Grenoble INP, GAEL, 38000 Grenoble, France; (2) Department of Economics, Ca' Foscari University of Venice, Italy;
(3) Department of Business and Economics, Parthenope University of Naples, Italy;

We study the dynamics of the exploitation of a natural resource distributed among and flowing between several nodes connected via a weighted, directed network. The network represents both the locations and the interactions of the resource nodes. A regulator decides to designate some of the nodes as natural reserves where no exploitation is allowed. The remaining nodes are assigned (one-to-one) to players, who will exploit the resource at the node. We show how the equilibrium exploitation and the resource stocks depend on the productivity of the resource sites, on the structure of the connections between the sites, and on the number and the preferences of the agents. The best locations to host nature reserves are identified according to the model's parameters, and we find that they correspond to the most central (in the sense of eigenvector centrality) nodes of a suitably redefined network that considers the nodes' productivity.

6. Stochastic control models in economics and finance 10:20 – 11:35

***Chair*: S. Federico, G. Ferrari** **FH HS 7**

Economic growth models in time-space on networks

Alessandro Calvia (1), Fausto Gozzi (1), Marta Leocata (1),

(1) Department of Economics and Finance, LUISS University, Rome, Italy;

In this talk, we will discuss a family of economic growth models in continuous-time, where the output of the economy is described by an AK production function and the dependence on space of the involved economic quantities is explicitly taken into account.

Considering a spatial dimension in these economic growth models allows to study phenomena such as capital diffusion across space and agglomeration; it provides also a ground to discuss policy issues where the space dimension is relevant.

We will consider the case where space is modeled as a network of locations (i.e., as a graph). This choice presents a clear advantage over models where space is described as continuum of locations (see, e.g., [1–3]), since our models can be more easily fitted to available data.

From a mathematical perspective, the resulting problem is an optimal control one with state constraints, where a planner aims at maximizing an intertemporal utilitarian social welfare function, by choosing an optimal consumption plan at each location in the network. Random shocks modeled by a Poisson random measure may affect the evolution of the state variable, i.e., capital.

We aim at characterizing the value function of this optimal control problem both as the unique fixed point of a suitable contraction mapping and as the unique constrained viscosity solution of the corresponding *Hamilton-Jacobi-Bellman* equation. Classical results on state constrained problems cannot be applied (see, e.g., [4]) and a novel analysis is required.

[1] R. Boucekkine, C. Camacho, and G. Fabbri. Spatial dynamics and convergence: The spatial AK model, *J. Econom. Theory*, **148**:2719–2736, 2013.

[2] R. Boucekkine, G. Fabbri, S. Federico, and F. Gozzi. Growth and agglomeration in the heterogeneous space: A generalized AK approach. *J. Economic Geography*, **19**:1287–1318, 2019.

[3] A. Calvia, S. Federico, and F. Gozzi. State constrained control problems in Banach lattices and applications. *SIAM J. Control. Optim.*, **59**:4481–4510, 2021.

[4] H. M. Soner. Optimal control with state-space constraint. I. *SIAM J. Control Optim.*, **24**:552–561, 1986.

Stationary discounted and ergodic mean field games of singular control

Haoyang Cao (1), Jodi Dianetti (2), Giorgio Ferrari (2),

(1) Centre de Mathématiques Appliquées, École Polytechnique, Paris France; (2) Center for Mathematical Economics, Bielefeld University, Bielefeld, Germany;

We study stationary mean field games with singular controls in which the representative player interacts with a long-time weighted average of the population through a discounted and an ergodic performance criterion. This class of games finds natural applications in the context of optimal productivity expansion in dynamic oligopolies. We prove existence and uniqueness of the mean field equilibria, which are completely characterized through nonlinear equations. Furthermore, we relate the mean field equilibria for the discounted and the ergodic games by showing the validity of an Abelian limit. The latter allows also to approximate Nash equilibria of - so far unexplored - symmetric N-player ergodic singular control games through the mean field equilibrium of the discounted game.

A Mean Field Game model in economics with spatial spillovers on the human capital

Daria Ghilli (1), Cristiano Ricci (2), Giovanni Zanco (3),

(1) Department of Economics and Management, University of Pavia, Pavia, Italy; (2) Department of Economics and Management, University of Pisa, Pisa, Italy; (3) Department of Economics and Finance, LUISS University of Rome, Rome, Italy;

We study an economic model where each agent chooses its position in space and its level of human capital maximizing its own utility and interacts through the human capital. The main peculiarity of the model we consider consists in the presence of a spatial interaction term in the dynamic of the human capital, i.e. spatial spillovers on the accumulation of human capital and in the utility, i.e. spatial spillovers on the consumption. We adopt a Mean Field Game (MFG) approach and study a system of two partial differential equations, the MFG system, which arises as the continuum macroscopic description of the discrete multi-agents system emerging, heuristically, as the limit as the number of agents tends to infinity. The MFG system we study is expected to approximate Nash equilibria for the discrete multi-agents system when the number of players tends to infinity.

7. Optimal control and Machine Learning 11:45 – 12:35

***Chair*: M. Falcone, M. Palladino** **FH HS 8**

State-constrained Linear-Quadratic Optimal Control is a shape-constrained kernel regression

Pierre-Cyril Aubin-Frankowski (1),

(1) SIERRA, INRIA, Paris, France;

We show in this talk that linearly controlled trajectory spaces are vector-valued reproducing Hilbert kernel spaces when equipped with the scalar product corresponding to the quadratic cost. The associated LQ kernel is related to the inverse of the Riccati matrix [1] and to the controllability Gramian [2]. This kernel allows to deal by a simple "representation theorem" with state constraints for linear-quadratic (LQ) optimal control problems with varying time [2]. By the Pontryagine Maximum Principle (PMP), one has in the unconstrained case a closed feedback loop, however the optimal trajectory must be obtained by numerical approximations of the dynamics. Conversely, the LQ kernel is computed by integration of a matrix-valued Hamiltonian system and parsimoniously encodes the optimal trajectory. Numerically, this enables an exact continuous-time solution of path planning problems, e.g. constrained to "avoid ramming into walls".

[1] P.-C. Aubin-Frankowski, *Interpreting the dual Riccati equation through the LQ reproducing kernel*, Comptes Rendus. Mathématique, **359(2)**, 199–204, 2021.

[2] P.-C. Aubin-Frankowski, *Linearly-constrained Linear Quadratic Regulator from the viewpoint of kernel methods*, SIAM Journal on Control and Optimization,**59(4)**, 2693–2716, 2021.

Optimal intensity control of point processes

Jeongsim Kim (1), Bara Kim (2), Chia-Li Wang (3),

(1) Department of Mathematics Education, Chungbuk National University, Cheongju, Korea; (2) Department of Mathematics, Korea University, Seoul, Korea; (3) Department of Applied Mathematics, National Dong Hwa University, Hualien, Taiwan;

Consider the intensity control of a point process to maximize the expectation of a function of the time when a predetermined count of events occurs. We show that if the objective function is unimodal, then the optimal control is of the bang-bang type. Moreover, if the objective function is log-concave, so is the value function. These results resolve the generalized intensity control problem that were raised by [1] and [2] as well as the two conjectures proposed by [2].

[1] P. Brémaud. Bang-bang controls of point processes. *Advances in Applied Probability*, **8**, 385-394, 1976.

[2] B. Defourny. Optimality of an affine policy for maximizing the probability of an arrival count in point-process intensity control. *Operations Research Letters*, **46**, 51-55, 2018.

8. Numerical analysis for PDE constrained optimization 11:45 – 12:35

***Chair*: J. Pfefferer, A. Rösch** **FH HS 5**

Optimal control and regularization of a simplified Signorini problem

Christof Haubner (1),

(1) IMCS, Universität der Bundeswehr München, Munich, Germany;

In the context of optimal control we consider a simplified Signorini problem, an elliptic variational inequality of first kind with unilateral constraints on the boundary. The state is discretized using linear finite elements while a variational discretization is applied to the control. We derive a priori error estimates for control and state based on strong stationarity and a quadratic growth condition. The convergence rates depend on $H1$ and $L2$ error estimates of the simplified Signorini problem.

We verify the theoretical findings with numerical tests, which are done by considering a regularized problem. The corresponding regularization error is also discussed.

Coupling stochastic gradient methods with mesh refinement for PDE constrained optimization under uncertainty

Caroline Geiersbach (1), Winnifried Wollner (2),

(1) Weierstraß-Institut (WIAS), Berlin, Germany; (2) Fachbereich Mathematik, TU Darmstadt, Darmstadt, Germany;

In this talk, we consider the optimization of a convex and smooth functional subject to a PDE constraint with uncertain coefficients. For the solution a stochastic gradient method is utilized to avoid detailed sampling of the random coefficients in every iteration. We will interpret the inexact PDE-solution, and thus gradient evaluation, as a bias term in the stochastic gradient method. We can then provide an

a-priori coupling between iteration number and discretization accuracy to obtain the convergence rates known for the stochastic gradient method with exact, unbiased, gradient evaluation. Numerical examples will complement the theoretical investigations.

9. Dynamic resource and environmental economics in spatial domains 11:45 – 12:35

***Chair*: T. Upmann** **FH HS 6**

Time-space evolution of economic activities: from deterministic to stochastic models

Fausto Gozzi (1),

(1) Luiss University, Roma, Italia;

In this talk we present some recent results on the time-space evolution of economic activities. In particular we focus on recent stochastic models and on models where the space is a network. This is based on recent and partly ongoing papers with various authors: R. Boucekkine, A. Calvia, G. Fabbri, S. Federico, M.Leocata.

Balancing preferential access and fairness with an application to waste management in Bavaria

Christian Schmitt (1), Bismark Singh (2),

(1) Department of Mathematics, Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany;
(2) Department of Mathematics, Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany;

Typically, within facility location problems, fairness is defined in terms of accessibility of users. However, for facilities perceived as undesirable by communities hosting them, fairness between the usage of facilities becomes especially important. Limited research exists on this notion of fairness. To close this gap, we develop an optimization framework for the allocation of populations of users to facilities such that access for users is balanced with a fair utilization of facilities. The optimality conditions of the underlying models provide a precise tradeoff between accessibility and fairness.

Our work is motivated by pervasive ecological challenges faced by the waste management community as policymakers in Bavaria seek to reduce the number of recycling centers in the last few years. Applying our models on data for the state of Bavaria, we find that even after the closure of a moderate number of recycling centers, large degrees of access can be ensured provided the closures are conducted optimally. Fairness, however, is impacted more, with facilities in rural regions shouldering larger loads of visiting populations than those in urban regions. Such informed decisions can help determine and guide future policy decisions.

10. Stochastic control models in economics and finance 11:45 – 12:35

***Chair*: S. Federico, G. Ferrari** **FH HS 7**

Two-sided singular control of an inventory with unknown demand trend

Salvatore Federico (1), Giorgio Ferrari (2), Neofytos Rodosthenous (3),

(1) Università di Genova, Italy; (2) Bielefeld University, Germany; (3) University College London, UK;

We study the problem of optimally managing an inventory with unknown trend. Our formulation leads to a stochastic control problem under partial observation, in which a Brownian motion with non-observable drift can be singularly controlled in both an upward and downward direction. After first deriving the equivalent Markovian problem, we aim at solving this completely. We show substantial regularity of its value function, we construct an optimal control rule, and we show that the free boundaries delineating action and inaction regions are Lipschitz continuous. Our approach uses the transition amongst three different but equivalent problem formulations and a link between two-dimensional bounded-variation stochastic control problems and games of optimal stopping. In order to show that the value function of the control problem possesses the sufficient regularity needed to perform a verification theorem, we develop a probabilistic method in combination with refined viscosity theory arguments.

Non-zero sum game of exit from a stochastic market

H. Dharma Kwon (1), Jan Palczewski (2),

(1) Gies College of Business, University of Illinois at Urbana-Champaign, Champaign, United States; (2) School of Mathematics, University of Leeds, Leeds, United Kingdom;

The timing of strategic exit is one of the most important but difficult business decisions, especially under competition and uncertainty about future profits. We examine a game of exit in a stochastic market when firms, additionally, do not know their competitor's exit barrier. Because a competitor's exit improves a firm's profit, the exit decision is beset with a free-rider problem.

Mathematically, the problem is described by a non-zero sum stopping game with asymmetric information. The market uncertainty, observed by both players, is represented by a general one-dimensional diffusion. Under the condition that a single player exit problem has a solution of a threshold type, we construct a symmetric equilibrium in randomised strategies. This equilibrium is further shown to be unique in a certain class of symmetric strategies.

11. Semi Plenary — 14:00 – 14:50

***Chair*: R. Findeisen** — **FH HS 5**

The turnpike property — a classic revisited

Lars Grüne (1),

(1) Chair of Applied Mathematics, University of Bayreuth, Germany;

The turnpike property is a classical property in optimal control. It describes the fact that optimal or near-optimal trajectories stay close to a particular solution most of the time. This particular solution—the so-called turnpike—is often a constant, i.e., equilibrium or steady state solution, but may also have a more complicated form.

The turnpike property was first described by Ramsey and von Neumann in the 1920s and 1930s and its name was coined by Dorfman, Samuelson and Solow in the 1950s. It was extensively analysed in mathematical economy in the 1970 and 1980s and also appears occasionally (though not always under this name) in the control engineering literature.

In the last couple of years the turnpike property received a lot of renewed interest, fueled by two complementary developments. On the one hand, the turnpike property turned out to be pivotal to analyse model predictive control schemes. It provides a redundancy property for optimal control problems on long or infinite time horizons, which allows to use model predictive control as a complexity reduction method in time. On the other hand, the turnpike property was shown to exist for various classes of optimal control problems governed by partial differential equations, where it can be seen as a particular instant of a whole class of sensitivity properties with respect to perturbations in the data, which can for instance be exploited to design efficient numerical solution algorithms.

This semi-plenary talk gives a survey on these recent developments.

11. Semi Plenary — 14:00 – 14:50

***Chair*: L. Lambertini** — **FH HS 6**

Space debris: another tragedy of the commons?

Georges Zaccour (1),

(1) GERAD & Department of Decision Sciences, HEC Montréal, Montréal, Canada;

Since the successful launch of Sputnik 1 in 1957, the exploitation of space has increased rapidly. Satellites are providing crucial services, e.g., weather forecast, navigation systems, managing disasters, and it would be hard to imagine our daily life without them. Two notable changes are worth mentioning in this area. First, private companies jumped in, that is, launching satellites is no more a state/government business. Second, there are active plans for launching tens of thousands of satellites during the next few years. Inevitably, this will create space congestion, and the multiplication of space debris (objects with no function), especially in low orbits, which could ultimately render space non exploitable physically and/or economically.

In this talk, I first discuss some of the main issues related to space debris and remediation strategies. Next, I introduce a dynamic game model that considers the (future) presence of few mega constellations

(e.g., Starlink, Kuiper, OneWeb), and derive the conditions for a sustainable exploitation of space, and therefore avoid another tragedy of the commons.

(Presentation based on joint work with Pierre Bernhard and Marc Deschamps)

12. Optimal control and Machine Learning 15:10 – 16:25

***Chair*: M. Falcone, M. Palladino** **FH HS 8**

On neural network architectures for solving high dimensional Hamilton-Jacobi equations arising in optimal control

Jérôme Darbon (1), Peter M. Dower (2), Tingwei Meng (1),

(1) Division of Applied Mathematics, Brown University, Providence, RI, USA; (2) Department of Electrical and Electronic Engineering, The University of Melbourne, Australia;

We propose new mathematical connections between Hamilton-Jacobi (HJ) partial differential equations (PDEs) with initial data and neural network architectures. Specifically, we prove that some classes of neural networks correspond to representation formulas of HJ PDE solutions whose Hamiltonians and initial data are obtained from the parameters or the activation functions of the neural networks. These results do not require any learning stage. In addition these results do not rely on universal approximation properties of neural networks; rather, our results show that some classes of neural network architectures naturally encode the physics contained in some HJ PDEs. Our results naturally yield efficient neural network-based methods for evaluating solutions of some HJ PDEs in high dimension without using grids or numerical approximations.

An optimal control approach to Reinforcement Learning and its algorithms

Maurizio Falcone (1), Agnese Pacifico (2), Michele Palladino (2), Andrea Pesare (1),

(1) Università di Roma "La Sapienza", Rome, Italy; (2) Università di L'Aquila, L'Aquila, Italy;

We consider an optimal control problem where the exact dynamics is unknown and only a probability distribution on the vector field is given. The associated optimal control problem tries to minimize an average cost, where the average is made over the ensemble given by π. Our goal is to determine if the value function and the optimal control of this average cost problem converge respectively to the value function and the optimal control of the classical optimal control problem with underlying dynamics f, when the distribution π gets closer and closer to the Dirac delta on the vector field f [2,3]. Finally, we show how these results can give a better understanding of some Reinforcement Learning algorithms and give some hints on their implementation [1].

[1] A. Pacifico, A. Pesare, M. Falcone *A New Algorithm for the LQR Problem with Partially Unknown Dynamics*, Lecture Notes in Computer Science, 13127 LNCS, 2022, 322-330

[2] A. Pesare, M. Palladino, M. Falcone, *Convergence results for an averaged LQR problem with applications to reinforcement learning*, Mathematics of Control, Signals, and Systems, (33) 2021, 379-411

[3] A. Pesare, *An optimal control approach to Reinforcement Learning*, PhD Thesis, Dipartimento di Matematica, Università di Roma "La Sapienza", 2022

Differential learning methods for solving fully nonlinear Bellman PDEs

William Lefebvre (1), Grégoire Loeper (2), Huyên Pham (1),

(1) LPSM, Université Paris Cité, France; (2) Monash university and BNP-PAR;

We propose machine learning methods for solving fully nonlinear partial differential equations (PDEs) with convex Hamiltonian. Our algorithms are conducted in two steps. First the PDE is rewritten in its dual stochastic control representation form, and the corresponding optimal feedback control is estimated using a neural network. Next, three different methods are presented to approximate the associated value function, i.e., the solution of the initial PDE, on the entire space-time domain of interest. The proposed deep learning algorithms rely on various loss functions obtained either from regression or pathwise versions of the martingale representation and its differential relation, and compute simultaneously the solution and its derivatives. Compared to existing methods, the addition of a differential loss function associated to the gradient, yields a better estimation of the PDE's solution derivatives, in particular of the second derivative, which is usually difficult to approximate. Furthermore, we leverage our methods to design algorithms for solving families of PDEs when varying terminal condition (e.g. option payoff in the context of mathematical finance) by means of the class of DeepOnet neural networks aiming to approximate functional operators. Numerical tests illustrate the accuracy of our methods on the resolution of a fully nonlinear PDE associated to the pricing of options with linear market impact, and on the Merton portfolio selection problem.

13. Recent advances in Model Predictive Control 15:10 – 16:25

***Chair*: L. Grüne** **FH HS 5**

Simulation-based Human-Computer Interaction

Miroslav Bachinski (1), Florian Fischer (1), Arthur Fleig (1), Markus Klar (1), Jörg Müller (1),

(1) University of Bayreuth, Germany;

Simulation-based methods have been used extensively for decades in industry and in various fields of academia and are evolving rapidly, the latest trend being data-driven or learning-based techniques. However, the field of Human-Computer Interaction (HCI) has been comparatively slow in this regard – in research and in practice – which contrasts the huge increase in the design space of interaction techniques, e.g., by smartphones and augmented or virtual reality (VR) headsets.

In this talk we give a short overview about recent simulation-based approaches in HCI to simulate human movement during interaction, such as mid-air pointing in VR. We start by looking at feedback control methods such as the Linear-Quadratic (Gaussian) Regulator for simple, linear underlying models. We show the ability of Model Predictive Control to generate human-like movement trajectories during mid-air interaction using a state-of-the-art biomechanical model of the human upper extremity. Finally, we compare it to a Reinforcement Learning-based approach, for the same biomechanical model.

Multiple tubes for robust model predictive control: basic theory and application

Markus Kögel (1), Rolf Findeisen (2),

(1) IFAT, Otto-von-Guericke-University Magdeburg, Magdeburg, Germany; (2) Control and Cyber-physical Systems Laboratory, TU Darmstadt, Darmstadt, Germany;

Model predictive control (MPC) calculates an input based on a finite-horizon optimal control problem at each sampling instant. However, an exact prediction of the future is usually not possible due to uncertainties in the system model or the presence of external disturbances. One way to consider and counteract disturbances in the prediction is to utilize a virtual fixed linear feedback gain. The feedback allows determining bounds on the difference between the predictions and the actual system evolution in the form of sets called tubes. However, if poorly chosen, the feedback/the resulting tube can limit the domain of attraction and the control performance significantly, requiring to compromise objectives. To overcome this challenge, we present methods to combine online multiple tubes/gains to reduce the conservatism and improve the performance, see [1,2]. We discuss the resulting optimization problem and the properties of the closed-loop. Examples illustrate the approach and extensions.

[1] M. Kögel and R. Findeisen. Robust MPC with reduced conservatism by blending multiples tubes. In *Proc. American Control Conference*, 1949-1954, 2020.

[2] M. Kögel and R. Findeisen. Fusing multiple time varying tubes for robust MPC. In *Proc. IFAC World Congress*, pages 7137–7144, 2020.

Multiobjective MPC and its application

Gabriele Eichfelder (1), Lars Grüne (2), Lisa Krügel (2), Jonas Schießl (2),

(1) Institute of Mathematics, Technische Universität Ilmenau, Ilmenau, Germany; (2) Chair of Applied Mathematics, Mathematical Institute, Universität Bayreuth, Bayreuth, Germany;

In model predictive control (MPC), it is a natural idea that not only one but multiple objectives have to be optimized. This leads to the formulation of a multiobjective optimal control problem (MO OCP). In this talk, we consider a nonlinear model predictive control scheme with multiple competing cost functions. In each step of the scheme, a multiobjective optimal control problem with nonlinear system dynamics and terminal conditions is solved. We introduce a simplified version of the algorithm presented in [2], which yields system theoretic properties and, due to the optimization-based nature of the method, performance results. More precisely, by assuming strict dissipativity and the existence of a compatible terminal cost for one of the competing objective functions only, we are able to obtain averaged and non-averaged performance estimates for all considered objective functions. Moreover, we give conditions that ensure asymptotic stability of the closed-loop solution. As the proposed algorithm requires the selection of an efficient solution in each iteration, we also examine numerically how different selection rules impact the results. Application examples and numerical simulations will illustrate our findings.

This talk is based on the results in [1].

[1] G. Eichfelder, L. Grüne, L. Krügel, J. Schießl. Relaxed assumptions and a simplified algorithm for multiobjective MPC. Preprint, arXiv:2202.08560, 2022.

[2] L. Grüne and M. Stieler. Multiobjective model predictive control for stabilizing cost criteria. *Discrete & Continuous Dynamical Systems — B* 24, pp. 3905–3928, 2019.

14. Dynamic games in environmental and resource economics 15:10 – 16:25

***Chair*: L. Lambertini** **FH HS 6**

How effective is a cooperative agreement between producers and deforesters for forests' restoration?

Fouad El Ouardighi (1), Giorgio Gnecco (2), Marcello Sanguineti (3),

(1) ESSEC Business School, France; (2) IMT - School for Advanced Studies, Lucca, Italy; (3) University of Genova, Genova, Italy;

This paper suggests a differential game model with the two countries: a forest-owner that draws revenues from deforestation (deforester) and a forest-non-owner that draws revenues from production (producer). While both deforestation and production generate polluting emissions that accumulate over time, the former also diminishes the environmental absorption efficiency due to forests. Both the deforester and the producer incur a cost from the negative externalities engendered by the accumulated pollution. Two questions are addressed in a decentralized setting with the cooperative solution as a benchmark, that is, i) if the players are unable to cooperate, should the restoration of forests be achieved by the deforester or the producer?, and ii) how beneficial is it to implement a cooperative support program and who should have the leadership for the restoration of forests? To answer these questions, the deforester and the producer are successively involved in a non-cooperative game where the effort to restore the environmental absorption efficiency of forests is performed alternatively by the deforester or the producer, and then in a cooperative game with a support program where the restoration of the environmental absorption efficiency due to forests is jointly performed by the deforester and the producer, with the former and the latter alternatively undertaking the Stackelberg leadership.

Resource extraction and global warming: Cournot vs Bertrand

Gustav Feichtinger (1), Luca Lambertini (2), George Leitmann (3), Stefan Wrzaczek (4),

(1) ORCOS, Vienna University of Technology, Vienna, Austria; (2) Department of Economics, University of Bologna, Bologna, Iyaly; (3) College of Engineering, University of California at Berkeley, Berkeley, CA, USA; (4) International Institute for Applied Systems Analysis (IIASA), Laxenburg, Austria;

We build upon Lambertini and Leitmann (2019) and Feichtinger et al. (2022) to investigate a differential oligopoly game in which firms exploit a renewable resource, and GHG emissions caused by either production or consumption contribute to global warming. A public authority may regulate access to the common pool and tax emissions, to stimulate investments in abatement technologies, and the impact of such measures is comparatively assessed under price and quantity competition. Regulating entry to induce the industry to harvest at the maximum sustainable yield (MSY) allows us to show that the optimal number of firms at the MSY may be lower in Bertrand that in Cournot, depending on the intensity of the Bertrand-Nash pricing behaviour, which we model as in Dastidar (1995). We also perform analogous evaluations for welfare and the environmental damage, to find that they become insensitive to the nature of market competition once access to the commons is being regulated.

[1] K.G. Dastidar. On the existence of pure strategy Bertrand equilibrium. *Economic Theory*, **5**: 9-32, 1995.

[2] G. Feichtinger, L. Lambertini, G. Leitmann and S. Wrzaczek. Managing the tragedy of commons and polluting emissions: A unified view. *European Journal of Operational Research*, forthcoming, 2022.

[3] L. Lambertini, and G. Leitmann. On the attainment of the maximum sustainable yield in the Verhulst–Lotka–Volterra model. *Automatica*, **110**, article 108555: 1-5, 2019.

Modelling behavioral drivers of disaster risks

Michael Freiberger (1), Roman Hoffmann (2), Alexia Prskawetz (3),

(1) Economic Frontiers, International Institute for Applied Systems Analysis (IIASA), Laxenburg, Austria; (2) Population and Just Societies, IIASA, Laxenburg, Austria; (3) ECON, Vienna University of Technology, Vienna, Austria;

In the last decades, many parts of the world faced an increase in the number of extreme weather events and worsening climatic conditions with negative impacts for local populations and their livelihoods. While various empirical studies have identified driving forces of disaster preparedness and vulnerability, we still lack an conceptual understanding of how different policy interventions can support disaster prevention efforts. This study develops a dynamic household model, in which households face stochastic environmental hazards, which can lead to a loss of their wealth. To respond to the risk, households can either relocate to a safer area or undertake preventive measures to protect their physical assets. Both actions require material and immaterial resources, which constrain the household's decision. Households are assumed to be heterogeneous with respect to income, risk awareness, time preference and their access to preventive measures.

Theoretical insights on the optimal household strategies are derived from the FOC in the HJB-equations. Furthermore a calibrated numerical approximation of the optimal policy functions is presented using data from Thailand and Vietnam. Using Monte-Carlo-Simulations the corresponding stationary distributions are derived and compared to their empirical counterparts and the impacts of different household characteristics are systematically assessed.

15. Infinite dimensional economic dynamics — 15:10 – 16:25

***Chair*: F. Gozzi, G. Fabbri, S. Faggian** — **FH HS 7**

Robust portfolio choice with sticky wages

Sara Biagini (1), Fausto Gozzi (1), Margherita Zanella (2),

(1) LUISS, Rome, Italy; (2) Politecnico di Milano, Milan, Italy;

We present a robust version of the life-cycle optimal portfolio choice problem in the presence of labor income, as introduced in Biffis, Gozzi and Prosdocimi [1] and Dybvig and Liu [2]. In particular, in [1] the influence of past wages on the future ones is modelled linearly in the evolution equation of labor income, through a given weight function. The optimisation relies on the resolution of an infinite dimensional HJB equation.

We improve the state of art in three ways. First, we allow the weight to be a Radon measure. This accommodates for more realistic weighting of the sticky wages, like, e.g., on a discrete temporal grid according to some periodic income. Second, there is a general correlation structure between labor income and stocks market. This naturally affects the optimal hedging demand, which may increase or decrease according to the correlation sign. Third, we allow the weight to change with time, possibly lacking perfect identification. The uncertainty is specified by a given set of Radon measures K, in which the weight process

takes values. This renders the inevitable uncertainty on how the past affects the future, and includes the standard case of error bounds on a specific estimate for the weight. Under uncertainty averse preferences, the decision maker takes a maxmin approach to the problem. Our analysis confirms the intuition: in the infinite dimensional setting, the optimal policy remains the best investment strategy under the worst case weight.

[1] E. Biffis, F. Gozzi and C. Prosdocimi. Optimal portfolio choice with path dependent labor income: the infinite horizon case. *SIAM Journal on Control and Optimization,* **58***(4): 1906-1938, 2020.*

[2] P.H. Dybvig and H. Liu. Lifetime consumption and investment: retirement and constrained borrowing. Journal of Economic Theory, **145**: 885-907, 2010.

The geography of income distribution dynamics: theory

Davide Fiaschi (1), Cristiano Ricci (1),

(1) University of Pisa, Pisa, Italy;

We propose a microfoundation of a spatial model of growth which encompasses amenities, local factor accumulation, spatial spillovers, and factors and technological flows among locations driven by differential factors returns in an explicit geographical space. Factors reallocate through space by searching for their highest returns, while individuals move toward locations where their utility is higher. We show how the model can be used to investigate the actual geography of income distribution dynamics characterized by spatial agglomeration of economic activity and population, differential locations of production and housing, metastable out-of-equilibrium dynamics, and path dependence in the equilibrium spatial distribution.

The time-space evolution of economic activities: a focus on the stochastic setting

Fausto Gozzi (1), Marta Leocata (1),

(1) Deparment of Economics and Finance, Luiss University, Roma, Italy.;

Aim of the talk is to introduce some models for the evolution in time and space of some economical variables, when heterogeneity in space is considered. After an introduction on the deterministic setting, see [1], we will focus on the case where uncertainty is taken into account, see [2]. Specifically, we consider the case where there is one planner which maximizes utility across space with heterogeneous productivity. The problem is formalized as an infinite dimensional stochastic control problem, where the state equation is a SPDE driven by a cylindrical Wiener process. We will present the optimal strategy for this control problem and also the asymptotic behavior of the optimal trajectories.

[1] R. Boucekkine, G. Fabbri, S. Federico, F. Gozzi. Growth and agglomeration in the heterogeneous space: a generalized AK approach.*Journal of Economic Geography* 19.6 (2019): 1287-1318.

[2] F. Gozzi, M. Leocata. A Stochastic Model of Economic Growth in Time-Space. *SIAM Journal on Control and Optimization* (2022), 60(2), 620-651.

16. Variational analysis in optimization and control 16:40 – 17:55

***Chair*: R. Cibulka** **FH HS 8**

On the problem of calculus of variations with pure state constraints

Mikhail Krastanov (1,2), Nadezhda Ribarska (1,2),

(1) Faculty of Mathematics and Informatics, Sofia University, Sofia, Bulgaria; (2) Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Sofia, Bulgaria;

The basic problem of calculus of variations with non-smooth criterion is considered in the presence of pure state constraints of equality type. A necessary optimality condition is obtained under suitable assumptions. An example illustrating the applicability of the obtained results is presented.

Pseudo-Jacobians and invertibility of nonsmooth maps

Jesus A. Jaramillo (1), Sebastian Lajara (2), Oscar Madiedo (3),

(1) Department of Mathematical Analysis and Applied Mathematics, Complutense University, Madrid, Spain; (2) Department of Mathematics, University of Castilla-La Mancha, Albacete, Spain; (3) Department of Financial and Actuarial Economics and Statistics, Madrid, Spain;

We study an analogue of the concept of pseudo-Jacobian matrix introduced by Jeyakumar and Luc for continuous maps between infinite-dimensional Banach spaces. Using that notion, we obtain several results concerning local and global invertibility in this context.

A necessary optimality condition involving measures of noncompactness

Mikhail Krastanov (1,2), Nadezhda Ribarska (1,2),

(1) Faculty of Mathematics and Informatics, Sofia University, Sofia, Bulgaria; (2) Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Sofia, Bulgaria;

A sufficient condition for tangential transversality involving measures of non-compactness as well as a Lagrange multiplier theorem for the infinite-dimensional optimization problem are obtained. The relation of the obtained results to the basic problem of calculus of variations is discussed. Also, an application to the problem of calculus of variations in the presence of pure state constraints of inequality type is presented.

17. Recent advances in Model Predictive Control 16:40 – 17:55

***Chair*: L. Grüne** **FH HS 5**

Learning-based model predictive control for risk-aware climate policy assessment

Jere Lehtomaa (1),

(1) Department of Management, Technology, and Economics, ETH, Zurich, Switzerland;

We develop a framework for solving stochastic climate-economy integrated assessment models (IAMs) using learning-based model predictive control (MPC). MPC approximates infinite-horizon optimal policies by repeatedly solving a finite surrogate problem over a receding planning window, bringing significant computational savings compared to the conventional dynamic programming algorithms. However, truncating the optimization horizon can lead to myopic actions in tasks that require long-term planning. To reduce shortsightedness, we infer an approximate value function from previously collected data using recursive Bayesian learning, which guides the myopic controller to high-value intermediate goals. Then, to enforce risk-aware behaviour, we formulate the IAM as a chance-constrained problem where the risk of exceeding a catastrophic temperature threshold must remain below a predefined probability level. Our experiments show that the approach is accurate, computationally tractable, and allows an intuitive treatment of risk appetite in stochastic economic decision-making problems by tuning the tolerated constraint violation probabilities.

Data-based model predictive control with stability and robustness guarantees

Julian Berberich (1), Christian Klöppelt (2), Johannes Köhler (3), Frank Allgöwer (1), Matthias A. Müller (2),

(1) University of Stuttgart, Institute for Systems Theory and Automatic Control, Stuttgart, Germany; (2) Leibniz University Hannover, Institute of Automatic Control, Hannover, Germany; (3) ETH Zürich, Institute for Dynamic Systems and Control, Zurich, Switzerland;

Model predictive control (MPC) has become one of the most successful modern control concepts, mainly thanks to its ability to directly incorporate hard state and input constraints as well as some performance criterion into the controller design. In the last decades, various MPC schemes for linear and nonlinear systems have been proposed which allow for closed-loop stability, robustness, and performance guarantees. To this end, a reasonably well identified model of the considered system is needed. However, in some applications, obtaining such a model by (classical) system identification can be difficult or the physical modeling process might be expensive. In such cases, MPC schemes are of high interest which suitably employ collected data for predictions. In this talk, we discuss a data-based MPC framework for which rigorous closed-loop stability and robustness guarantees can be given. In particular, a trajectory-based system representation is used for prediction which allows to express all input/output trajectories of a linear time-invariant (LTI) system in terms of a single, sufficiently exciting, input/output trajectory. Using this representation, we design data-based MPC schemes and derive closed-loop stability and robustness guarantees for LTI systems, even if the measured data are affected by noise. Our theoretical analysis reveals various connections between design parameters, data properties and the resulting closed-loop behavior. Finally, we discuss how these results can be extended to control unknown nonlinear systems based on input-output data with closed-loop stability guarantees.

[1] J. Berberich, J. Köhler, M. A. Müller, F. Allgöwer, "Data-driven Model Predictive Control with Stability and Robustness Guarantees", IEEE Transactions on Automatic Control, vol. 66, no. 4, pp. 1702 - 1717, 2021.

[2] J. Berberich, J. Köhler, M. A. Müller, F. Allgöwer, "Linear tracking MPC for nonlinear systems Part II: The data-driven case", arXiv:2105.08567, 2021.

[3] C. Klöppelt, J. Berberich, F. Allgöwer, M. A. Müller, "A novel constraint tightening approach for robust data-driven predictive control ", arXiv:2203.07055, 2022.

Turnpikes in Deep Learning

Jens Püttschneider (1), Timm Faulwasser (1),

(1) Institue for Energy Systems, Energy Efficiency and Energy Economics, Department of Electrical Engineering and Information Technology, TU Dortmund University, Dortmund, Germany;

This talk investigates the turnpike phenomena in optimal control approaches to deep learning, i.e. the training of neural network [1,2]. To this end, we formulate the training of residual neural networks as an optimal control problem and investigate its dissipaitivity properties [3] when introducing a stage cost based on the loss function. This was already done for quadratic losses [2,4] and in this talk we consider the cross-entropy, the classical loss used in classification tasks. The considered turnpikes are verified for the MNIST benchmark problem and they are shown to generalize to test data. Based on this, those layer that constitute the turnpike can be pruned since they do not contribute to the transformation learned. This results in an network with the minimum number of layers required for learning without the need for extensive hyperparameter tuning. Moreover, the training with and without a stage cost is compared.

[1] Q. Li, L. Chen, C. Tai, and E. Weinan, Maximum principle based algorithms for deep learning. In *The Journal of Machine Learning Research*, vol. 18, 2017

[2] C. Esteve Yagüe, B. Geshkovski, D. Pighin, and E. Zuazua Large-time asymptotics in deep learning. *arXiv preprint arXiv:2008.02491*, 2020.

[3] D. Angeli, R. Amrit, and J. Rawlings, On Average Performance and Stability of Economic Model Predictive Control. In *EEE Transactions on Automatic Control*, vol. 57, 2012

[4] T. Faulwasser, A.-J. Hempel, S. Streif On the Turnpike to Design of Deep Neural Nets: Explicit Depth Bounds. *arXiv preprint:2101.03000*, 2021.

18. Dynamic games in environmental and resource economics 16:40 – 17:55

***Chair*: L. Lambertini** **FH HS 6**

On nonlinear feedback strategies in differential games with multiple state variables

Luca Lambertini (1),

(1) Department of Economics, University of Bologna, Bologna, Italy;

Relying on a method used in Wirl (2010) and Colombo and Labrecciosa (2013), I illustrate a linear-quadratic linear state game with multiple states, which lends itself to be solved for nonlinear feedback strategies as if each player were facing a single state equation. Then, I also show that in general this solution does not coincide with that delivered by the method based on undetermined coefficients. The illustration relies upon a linear-quadratic game which may describe either the extraction of a renewable resource or GHG emissions in a dynamic Cournot oligopoly.

[1] L. Colombo and P. Labrecciosa. Oligopoly exploitation of a private property productive asset. *Journal of Economic Dynamics and Control*, **37**: 838-53, 2013.

[2] F. Wirl. Dynamic demand and noncompetitive intertemporal output adjustments. *International Journal of Industrial Organization*, **28**: 220-29, 2010.

Transboundary pollution and New Economic Geography in a dynamic game framework

Guiomar Martín-Herrán (1), María Pilar Martínez-García (2), José Rodolfo Morales (1),

(1) IMUVA, Universidad de Valladolid, Spain; (2) Universidad de Murcia, Spain;

This paper formulates and analyzes a dynamic game between two trading regions that face a transboundary pollution problem. We study how the distribution of firms and trade costs affect the optimal emission decision of governments and how the resulting environmental policy would alter the allocation of the industry between both countries. The underlying microeconomic behaviour of consumers and firms is framed within the New Economic Geography literature, in particular within the Footloose Capital Model. The macroeconomic model that arises is a transboundary pollution linear-quadratic dynamic game. For the case of fully symmetric regions, we find that higher transport costs lead to higher industrial production and higher stock of pollution at the steady state. However, while domestic consumption increases, exports could shrink depending on how the representative consumer weighs the homogeneous goods and the pollution damage in his welfare. For the case of regions that differ in their shares of industrial activity, we find some preliminary results. Governments modify their production with respect to changes in the pollution stock in a similar way to the symmetric case, although the optimal emissions are different in both cases. In a second stage we observe that the production decisions of the governments can generate core-periphery structures, even in the fully symmetric region case, which is new in this type of linear models.

Opening up the DICE black box

Michael Freiberger (1), Michael Kuhn (1), Stefan Wrzaczek (1),

(1) International Institute for Applied Systems Analysis (IIASA), Economic Frontiers, Laxenburg, Austria;

The Dynamic Integrated Climate-Economy (DICE) model is one of the most prominent Integrated Assessment Model (IAM) to assess the development of the climate and economy. However, the results form the freely available DICE solver (including extensions of it) as well as the social cost of carbon (SCC) are treated like the outcome of a black box. We are formulating a continuous version of the DICE model and (i) derive the SCC explicitly (separating different effects), (ii) show its relation to the socially optimal marginal abatement costs (SMAC) and (iii) prove that a tax in a decentralized economy has to be equal to the SCC.

19. Infinite dimensional economic dynamics 16:40 – 17:55

***Chair*: F. Gozzi, G. Fabbri, S. Faggian** **FH HS 7**

Stochastic maximum principle for problems with delay with dependence on the past through general measures

Giuseppina Guatteri (1), Federica Masiero (2),

(1) Politecnico di Milano, Milano, Italy; (2) Università di Milano-Bicocca, Milano, Italy;

In this talk we present stochastic optimal control problems with delay both in the state and in the control, motivated in particular by a stochastic dynamic model in marketing for problems of optimal advertising
These problems are studied by means of the Pontryagin stochastic maximum principle, and we notice that besides delay in the state and in the control in the state equation, we aim to consider an associated cost functional with dependence on the past trajectory also in the final cost. Following the standard steps in the variational approach for control problems, we formulate the maximum principle by means of an adjoint equation. The novelty is that the adjoint equation turns out to be an anticipated backward stochastic differential equation of a new form. This is due to the fact that we allow dependence on the past trajectory also in the final cost.
Besides problems of optimal advertising, we apply the results to an optimal portfolio problem with execution delay.

The geography of income distribution dynamics: empirics

Angela Parenti (1), Davide Fiaschi (1), Cristiano Ricci (1),

(1) University of Pisa, Pisa, Italy;

We propose a methodology to estimate a spatial income model in continuos space which encompasses amenities, local factor accumulation, spatial spillovers, and factors and technological flows among locations driven by differential factors returns in an explicit geographical space. The application to the proposed methodology to the sample of Italian municipalities over the period 2008-2019 points out to the presence of conditional convergence in income per square kilometre, but also of i) spatial agglomeration, which we

trace back to positive spatial spillovers, and to ii) tendency of income to spread toward poorer locations, which we trace back to the reallocation of factors toward more productive locations.

Numerical methods for a Mean Field Game model for human capital.

Cristiano Ricci (1), Daria Ghilli (2), Giovanni Zanco (3),

(1) Department of Economics, University of Pisa, Pisa, Italy; (2) Department of Economics, University of Pavia, Pavia, Italy; (2) LUISS Guido Carli, Roma, Italy;

In this talk I will present a mean field game model with spatial interaction, to describe the time evolution of a class of agent both in the space variable and in their level of human capital. One of the key feature of the model is the presence of spatial spillover effect, in the form a reinforcement mechanism between agents that are in neighboring locations. A particular focus will be devoted on the numerical simulation of the model, both in and out of equilibrium.

Wednesday, July 13th

20. Plenary — 08:30 – 09:20

***Chair*: H. Frankowska** — **FH HS 8**

Stochastic Kolmogorov Systems and Applications: Some Recent Progress

George Yin (1),

(1) University of Connecticut, Storrs, CT, USA;

In this talk, we report some of our recent work on stochastic Kolmogorov systems. The motivation stems from dealing with important issues of ecological and biological systems. Focusing on environmental noise, we aim to address such fundamental questions: "what are the minimal conditions for long-term persistence of a population, or long-term coexistence of interacting species". Some optimal control problems are also examined.

[1] N.T. Dieu, D.H. Nguyen, N.H. Du, and G. Yin, Classification of asymptotic behavior in a stochastic SIR model, *SIAM J. Appl. Dynamic Sys.*, **15** (2016), 1062-1084.

[2] D. Nguyen, N. Nguyen, and G. Yin, General Nonlinear Stochastic Systems Motivated by Chemostat Models: Complete Characterization of Long-Time Behavior, Optimal Controls, and Applications to Wastewater Treatment, *Stochastic Process. Appl.*, **130** (2020) 4608-4642.

[3] D. Nguyen, N. Nguyen, and G. Yin, Stochastic functional Kolmogorov equations I: Persistence, *Stochastic Process Appl.* **142** (2021), 319-364.

[4] D. Nguyen, N. Nguyen, and G. Yin, Stochastic functional Kolmogorov equations II: Extinction, *J. Differential Eqs.*, **294** (2021), 1-39.

21. Set-valued approximation in control and optimization 09:40 – 10:55

***Chair*: R. Baier, E. Farkhi** **FH HS 8**

Filippov-Type theorems for the Non-Lipschitzian case

Robert Baier (1), Elza Farkhi (2),

(1) Chair of Applied Mathematics, University of Bayreuth, Bayreuth, Germany ; (2) School of Math. Sciences, Tel-Aviv University, Tel Aviv, Israel;

We consider differential inclusions with strengthened one-sided Lipschitz (SOSL) right-hand sides. The class of SOSL set-valued maps is wider than the class of Lipschitz ones and contained in the family of one-sided Lipschitz maps. We prove a Filippov stability theorem for the solutions of such differential inclusions with perturbations in the right-hand side, both in the state and in the set. The obtained estimate extends the known Filippov estimate for Lipschitz maps to SOSL ones and is better than the estimate known for one-sided Lipschitz right-hand sides. Some applications to perturbation stability estimates for some semi-discrete approximations of differential inclusions and the Euler approximation of differential inclusions with SOSL right-hand sides are made.

Properties of the solution set of nonlocal evolution inclusions

Tzanko Donchev (1),

(1) Department of Mathematics, University of Architecture and Civil Engineering, Sofia, Bulgaria;

In this talk we study the following problem with nonlocal initial condition

$$\begin{cases} \dot{x}(t) \in Ax(t) + f_x(t), \ t \in (0,T), \\ f_x(t) \in F(t, x(t))), \\ x(0) = g(x(\cdot)). \end{cases} \tag{1}$$

Here A is m-dissipative (linear or nonlinear) operator.

Two different approaches:

I. F and g satisfy compactness type conditions w.r.t. some measure of non-compactness.

II. Dissipative type of conditions are used, i.e. G is Lipschitz and F is quasi-dissipative (one-sided Lipschitz) or quasi-accretive (opposite-sided Lipschitz).

Examples to illustrate the applicability of our results are given.

Acknowledgements. The work was supported by the Bulgarian National Science Fund under Project KP-06-N32/7.

An adaptive subdivision scheme approximating reachable sets involving Filippov's estimates

Robert Baier (1), Matthias Gerdts (2), Wolfgang Riedl (3),

(1) Chair of Applied Mathematics, University of Bayreuth, Bayreuth, Germany; (2) Engineering Mathematics, University of Federal Armed Forces, Neubiberg/Munich, Germany; (3) Witron Co., Parkstein, Germany;

The talk presents an adaptive technique to approximate reachable sets of nonlinear control problems that is based on an subdivision framework by M. Dellnitz, A. Hohmann and O. Junge to calculate attractors and invariant measures. For this framework, reachable sets are regarded as feasible sets of nonlinear control problems or of their discretization, i.e., nonlinear optimization problems. As objective function the distance function of some grid point of a selected box to the corresponding feasible set is considered. To avoid the high computational costs for solving many optimal control problems, we iteratively refine boxes of the computed collection with decreased diameter. Only these refined boxes are further selected for which at least one test point of the box is mapped to a feasible point inside the same box, others can be dropped.

By projection arguments from the subdivision framework and by convergence results of set-valued Runge-Kutta methods, the computed union of boxes in the collection will shrink to the reachable set. 2d and 6d test examples will illustrate the concept and show how analytical and numerical estimates based on Filippov theorems for Lipschitz and one-sided Lipschitz set-valued maps help to choose a suitable bounding box.

22. Recent advances in Model Predictive Control 09:40 – 10:55

***Chair*: L. Grüne** **FH HS 5**

Model Predictive Control: Nonconvex constraints and nonquadratic costs

Saša V. Raković (1),

(1) Beijing Institute of Technology, Beijing, China;

Contemporary human–engineered, intelligent, and autonomous systems, as encountered, for instance in autonomous vehicles, aerospace engineering, and smart grids, lead to nontraditional constraints and performance measures in order to capture the corresponding safety, intelligence, privacy, and resilience related design specifications. It is an evident fact that the computational aspects, and hence real–life implementation, of model predictive control under nonconvex constraints and nonquadratic costs are critically less developed. This is a serious shortcoming of model predictive control, especially as many of modern design specifications result naturally in either nonconvex constraints or nonquadratic costs. For example, the obstacle and collision avoidance control problems, which are of prime importance for autonomous and space vehicles, are inherently nonconvex optimal and model predictive control problems.

This contribution revisits model predictive control in order to facilitate its computationally efficient and realistically implementable utilization under nonconvex constraints and nonquadratic costs. In particular, the contribution provides an overview of recently developed techniques for locally convexified model predictive control under obstacle and/or collision avoidance constraints, which does not require mixed integer programming formulation and which unlike many of existing approaches is implementable in most basic form by solving one or two strictly convex quadratic programming problems throughout the actual control process and in an advanced form by solving a sequence of such strictly convex quadratic programming problems at each step of an actual control process. The talk also provides a novel stabilizing condition

for model predictive control with Minkowski cost functions. This class of functions is considerably more flexible and topologically richer than quadratic cost functions; Thus, it enables a plethora of modern design specifications to be more naturally transcribed within the formalism of optimal and model predictive control.

Nonlinear optimization-based state estimation; robustness analysis by Q functions

James B. Rawlings (1), Douglas A. Allan (2), Titus Quah (1),

(1) Department of Chemical Engineering, University of California, Santa Barbara, CA, USA; (2) KeyLogic, 3168 Collins Ferry Road, Morgantown, WV, USA;

State estimation can be posed as an optimal control/tracking problem. From this perspective, the stability and robustness properties of the estimator should be derivable from the properties of the optimization problem, as is commonly done in the analysis of Model Predictive Control. To achieve this goal in state estimation, we introduce a Lyapunov-like function, termed a Q function, and show that for general nonlinear systems satisfying a nonlinear detectability assumption and a nonlinear, incremental stabilizability assumption, the optimal full information state estimate is robustly asymptotically stable in the presence of bounded process and measurement disturbances. We also show that the state estimate converges to zero for asymptotically convergent disturbances.

Stabilizability is in general not required for robust asymptotic stability. To explore this issue further, we first review the linear time invariant case for unstabilizable systems. Then we offer some conjectures about the utility of the stabilizability assumption for general nonlinear systems.

These general theoretical results are illustrated by application to some numerical examples using the freely available software CasADi/MPCTools for solving the optimal control problems. Implications of these full information results for moving horizon estimation are also discussed.

Hybrid Model Predictive Control: Framework, basic properties, and open problems

Ricardo G. Sanfelice (1),

(1) Hybrid Systems Laboratory, University of California at Santa Cruz, California, USA;

Hybrid systems model the behavior of dynamical systems in which the states can evolve continuously and, at isolate time instances, exhibit instantaneous jumps. Such systems arise when control algorithms that involve digital devices are applied to continuous-time systems, or when the intrinsic dynamics of the system itself has such hybrid behavior, for example, in mechanical systems with impacts, switching electrical circuits, spiking neurons, atc. Hybrid control may be used for improved performance and robustness properties compared to conventional control, and hybrid dynamics may be unavoidable due to the interplay between digital and analog components in a cyber-physical system. In this talk, we will introduce analysis and design tools for model predictive control (MPC) schemes for hybrid systems. We will present recently developed results on asymptotically stabilizing MPC for hybrid systems based on control Lyapunov functions. After a short overview of the state of the art on hybrid MPC, and a brief introduction to a powerful hybrid systems framework, we will present key concepts and analysis tools. After that, we will lay out the theoretical foundations of a general MPC framework for hybrid systems, with guaranteed stability and feasibility. In particular, we will characterize invariance properties of the feasible set and the terminal constraint sets, continuity of the value function, and use these results to establish asymptotic stability of the hybrid closed-loop system. To conclude, we will illustrate the framework in several applications and summarize some of the open problems, in particular, those related to computational issues.

23. Covid-19: optimal control approaches 09:40 – 10:55

***Chair*: D. Grass, S. Wrzaczek** **FH HS 6**

Supporting strategy selection in a complex multiobjective decision problem: A COVID-19 case study

Lauri Neuvonen (1), Matthias Wildemeersch (2), Eeva Vilkkumaa (1),

(1) Department of Information and Service Management, School of Business, Aalto University, Espoo, Finland; (2) International Institute for Applied Systems Analysis, Vienna, Austria;

Decision makers are often faced with complex decision-making problems that require making trade-offs between multiple, possibly conflicting objectives under various uncertainties. The task is even more difficult when considering dynamic, non-linear processes and when the decisions themselves are complex, for instance in case of selecting trajectories for multiple decision variables. One way to approach these kinds of problems is through (i) building a model to capture the dynamics of the underlying processes, and (ii) using multi-objective optimization (MOO) to find solutions that optimize the values of relevant outcomes of this model. A problem with MOO is that the number of Pareto optimal solutions can be very large, whereby the selection of a single preferred solution is difficult. Moreover, preference between solutions may not be determined only by their objective function values, but also in terms of how robust or implementable these solutions are. In this paper, we present a methodological framework to support the identification of a small but diverse set of robust Pareto optimal solutions, applied to the COVID-19 epidemic. We use a 12-state compartmental model to simulate epidemic dynamics, and identify control strategies that help minimize deaths and economic impact with only a low risk of exceeding intensive care capacity. The methodological framework enables a visual inspection of the remaining solutions to compare them in terms of practical implementability. In particular, we eliminate non-robust solutions from the Pareto front, and cluster the remaining solutions based on their similarity in the decision variable space.

How to deal with the Covid-19 pandemic in Slovenia? Simulations with a macroeconometric model

Klaus Weyerstrass (1), Dmitri Blueschke (2), Reinhard Neck (2), Miroslav Verbič (3,4),

(1) Macroeconomics and Business Cycles Group, Institute for Advanced Studies, Vienna, Austria;
(2) Department of Economics, University of Klagenfurt, Klagenfurt, Austria; (3) Faculty of Economics, University of Ljubljana, Ljubljana, Slovenia; (4) Institute for Economic Research, Ljubljana, Slovenia;

In this paper we analyze the effectiveness of fiscal policies during the Covid-19 pandemic in the small open economy of Slovenia. Simulating the SLOPOL11 model, an econometric model of the Slovenian economy, we analyze the effectiveness of various categories of public spending and taxes during the period 2020 to 2030, counterfactually assuming that no crisis occurs, and compare the simulation results with those obtained when considering the effects of the pandemic, which are modelled as a combined demand and supply shock. Our simulations show that those public spending measures that entail both demand- and supply-side effects are more effective at stimulating real GDP and increasing employment than pure demand-side measures. Successful stabilization policies should thus contain a supply-side component in addition to a demand-side component. We also provide a first simulation of optimal fiscal policies to combat the effects of the pandemic and to stabilize the economy under three scenarios for the future course of developments under Covid-19.

The relative impact of international travel restrictions during the COVID-19 pandemic

Fouad El Ouardighi (1), Bowon Kim (2), Christian Van Delft (3),

(1) ESSEC Business School, France; (2) Korea Advanced Institute of Science and Technology Business School, Seoul, Korea; (3) HEC, France;

In this paper, we develop a differential game of the current pandemic's dynamics to determine how preventive and therapeutic measures curb its spreading in the context of two neighboring countries. Given the R&D competition among the countries to develop their treatment capability, the exogenous impact of boundaries' control is assessed along with those of preparedness and R&D spillovers. Using the cooperative solution as a benchmark, we investigate non-cooperative (i.e., closed-loop Nash equilibrium) strategies and show how they can be detrimental in various contexts, both in terms of effectiveness and efficiency in preventing infections and saving lives.

24. Infinite dimensional economic dynamics

09:40 – 10:55

***Chair*: F. Gozzi, G. Fabbri, S. Faggian**

FH HS 7

The value of information in circular settings

Stefan Behringer (1), Roman Belavkin (2),

(1) SciencesPo, France; (2) Middlesex University London, UK;

This paper investigates two seemingly different concepts of the Value of Information, the classical Hartley information that has many uses in economics and decision theory and the Shannon/Stratonovich approach [1,2] that is inspired by results in thermodynamics for models that are set on the boundary of a circle. Among other things we generalize the original circle example in [3] to n possible realizations and investigate the limit case when the discrete distribution of the random variable approaches a density and its implications for the two Value of Information concepts.

[1] C. Shannon. A Mathematical Theory of Communication *Bell System Technical Journal, 27, p.379-423*, 1948.

[2] R.L. Stratonovich. *Theory of Information and its Value*, R.V. Belavkin, P.M. Pardalos, J.C. Principle (eds.) 2020, original 1975 (in Russian)

[3] R.L. Stratonovich. On the Value of Information (in Russian) *Izv. USSR Acad. Sci. Tech. Cybern. 5, 2-12*, 1965.

Inequality and growth: endogeneous dynamics of heterogeneity

Paulo Brito (1),

(1) ISEG, Universidade de Lisboa, Lisbon, Portugal;

This paper presents a distributional version of the Uzawa-Lucas model of endogenous growth and based upon the joint distributional dynamics of the human and physical capital. Assuming a Bergson-Samuelson social planner, the optimal distribution dynamics is driven by a coupled system of Hamilton-Jacobi and Fokker-Plank- Kolmogorov equations. Using benchmark preferences and technologies we find close form solutions to the optimal control of PDEs problem. We find that the distributional endogenous growth is non-ergodic, featuring asymptoticaly unbounded first and second moments for the endogenous income distribution. Our model clarifies a puzzling characteristic present in the data, featuring both an upward trend in the standard deviation and close to constant inequality measures.

Violation of the Hamilton-Jacobi-Bellman equation in economic dynamics

Yuhki Hosoya (1),

(1) Chuo University, Hachioji, Japan;

We consider an extension of the classical capital accumulation model, and present an example in which the Hamilton-Jacobi-Bellman (HJB) equation is neither necessary nor sufficient for a function to be the value function. Next, we present assumptions under which the HJB equation becomes a necessary and sufficient condition for a function to be the value function, and using this result, we propose a new method for solving the original problem using the solution of the HJB equation. Our assumptions are so mild that many macroeconomic growth models satisfy them. Therefore, our results ensure that the solution of the HJB equation is rigorously the value function in many macroeconomic models, and present a new solving method for these models.

25. Set-valued approximation in control and optimization 11:10 – 12:50

***Chair*: R. Baier, E. Farkhi** **FH HS 8**

Compound matrices in systems and control theory

Michael Margaliot (1),

(1) School of Electrical Engineering, Tel Aviv University, Israel;

The multiplicative and additive compounds of a matrix play an important role in several fields of mathematics including geometry, multi-linear algebra, combinatorics, and the analysis of nonlinear time-varying dynamical systems. There is a surge of interest in the theory and applications of these compounds, and their generalizations, in systems and control theory, see e.g., [1][2][3]. We provide a gentle and self-contained introduction to these topics with an emphasis on the geometric interpretation of the compounds, and describe some recent applications including several non-trivial generalizations of positive systems and cooperative systems [1][2], contracting systems [3], and more.

[1] E. Weiss and M. Margaliot. A generalization of linear positive systems with applications to nonlinear systems: invariant sets and the Poincaré–Bendixson property. *Automatica*, **123**:109358, 2021.

[2] M. Margliot and E. D. Sontag. Revisiting totally positive differential systems: A tutorial and new results. *Automatica*, **101**:1-14, 2019.

[3] C. Wu , I. Kanevskiy and M. Margaliot. k-contraction: Theory and applications. *Automatica*, **136**:110048, 2021.

Minimum-time problems with piecewise constant dynamics

Peter Wolenski (1),

(1) Louisiana State University, Baton Rouge, Louisiana, USA;

We first consider a control problem defined on two disjoint open half-spaces $\mathbf{M}_0$ and $\mathbf{M}_1$ with a common interface $\mathbf{\Sigma} := \mathrm{cl}(\mathbf{M}_0) \bigcap \mathrm{cl}(\mathbf{M}_1)$. Associated to each $\mathbf{M}_i$ is a fixed nonempty velocity set $\mathbf{F}_i$ $(i = 0, 1)$ that is closed, convex, bounded, and containing $\mathbf{0}$ in its interior. For $\mathbf{x}_0 \in \mathbf{M}_0$ and $\mathbf{x}_1 \in \mathbf{M}_1$, we consider the control problem (with discontinuous dynamics) of a trajectory $\mathbf{x}(\cdot)$ going from $\mathbf{x}_0$ to $\mathbf{x}_1$ in least time, meaning that $\dot{\mathbf{x}}(t) \in \mathbf{F}_i$ when $\mathbf{x}(t) \in \mathbf{M}_i$ $(i = 0, 1)$. We provide necessary and sufficient conditions for optimality which is a generalization of the classical Snell's Law. A characterization of the boundary of the reachable set from $\mathbf{x}_0$ is given. Finally, we present results where $\mathbf{\Sigma}$ can act like a highway with its own velocity set and situations where there are more than two mediums.

Comparison of reachable sets and polynomial chaos expansions for obstacle avoidance under uncertainties

Florian Steppich (1), Matthias Gerdts (1),

(1) Institut für Mathematik und Rechneranwendung (LRT), Universität der Bundeswehr, Neubiberg, Germany;

Considering moving obstacles is one aspect of path planning used with automatic and autonomous vehicles. Naive approaches treat dynamically moving obstacles as static and add another security perimeter around them to account for movements. In some cases, this leads to situations that prevent path planning algorithms from finding a feasible solution.

In this talk, I will explore two methods to calculate the possible positions of a dynamically moving obstacle. The first one calculates the reachable set based on adaptive use of solvers for optimal control problems as presented in [1]. Reachable sets are worst-case estimates of the possible position of an obstacle and tend to be too restrictive on the path planning problem. The second method propagates the given uncertainty using (generalized) polynomial chaos described in [2,3]. This method overlays the reachable set with a probability distribution. Based on this distribution, we can reduce the size of the restriction to allow the path planning algorithm to find a feasible solution in more cases. Despite this positive result, this method also incurs a residual risk of a collision.

[1] Riedl, W. et al. *Optimization-based subdivision algorithm for reachable sets*, Journal of Computational Dynamics, 8(1), doi:10.3934/jcd.2021005, 2021.

[2] Smith, R. *Uncertainty Quantification: Theory, Implementation, and Applications*, Society for Industrial and Applied Mathematics, 2014

[3] Xiu, D. *Numerical Methods for Stochastic Computations: A Spectral Method Approach*, Princeton University Press, 2010

High order approximation of set-valued functions

Nira Dyn (1), Elza Farkhi (1), Alona Mokhov (2,1),

(1) Tel-Aviv University, Israel; (2) Afeka, Tel-Aviv Academic College of Engineering, Dept. of Mathematics, Israel;

The goal of this work is to obtain high order approximation of set-valued functions (SVFs, multifunctions) with general compact images in $\mathbb{R}^d$.

High order approximation of real-valued functions is usually obtained from the smoothness of the approximated function or, in case of interpolation, from the boundedness of its divided differences of high order. For SVFs, notions of smoothness are defined only in some special cases, in particular when the multifunction is represented by a family of "uniformly smooth" single-valued functions.

In the absence of a general notion of higher order regularity of multifunctions, we introduce the metric divided differences of SVFs. This notion extends the classical divided difference for real-valued functions. We show that the boundedness of such high order divided differences leads to high order approximation of SVFs by metric piecewise-polynomial interpolants of high degree. These results are also used to obtain high order approximation by local sample-based linear approximation operators reproducing high degree polynomials.

26. Recent advances in Model Predictive Control 11:10 – 12:50

***Chair*: L. Grüne** **FH HS 5**

Adaptive space-time discretization for MPC of PDEs

Lars Grüne (1), Manuel Schaller (2), Anton Schiela (1),

(1) University of Bayreuth, Germany; (2) Technische Universität Ilmenau, Germany;

In every step of a Model Predictive Controller an optimal control problem on a possibly long horizon is solved and only an initial part of the optimal control is used as a feedback for the system under control. In this talk, we will present a specialized a posteriori space-time grid refinement technique to numerically exploit the fact that only an initial part of the control enters the MPC-feedback. Further, we will show by means of sensitivity analysis that discretization errors that occur in the far future only have a negligible effect on the MPC-feedback, i.e., the control on an initial part. All considerations will be accompanied by numerical examples of linear and nonlinear parabolic PDEs with distributed or boundary control.

Stochastic Model Predictive Control with randomly sampled measurements

Aneel Tanwani (1),

(1) CNRS, LAAS, Université de Toulouse, Toulouse, France;

For dynamical systems arising in engineering and economics, we are often interested in addressing optimal control problems and whether our computational methods can provide reasonable solutions to this problem. Model Predictive Control (MPC) presents a technique for addressing an optimal control problem over a large, or infinite, time horizon by solving the problem on a smaller finite horizon, which is computationally tractable. When we implement the control actions obtained by solving the finite horizon

problems, the resulting cost over the infinite horizon is larger than the minimal cost. An important question is to analyze how much the performance resulting from the computationally tractable algorithms has degraded compared to the theoretically optimal performance. One way to do so is by quantifying the increase in the value of the cost functional obtained by implementing MPC-based policies. Such questions are addressed in [1, 2]. Building on such works, and based on the work reported in [3], the problem of analyzing the performance of model predictive controllers in minimizing infinite-horizon cost functionals associated with stochastic dynamical systems when the measurements received by the controller are randomly sampled in time. In contrast to the standard model predictive control algorithms which rely on availability of the state measurements at all times, we compute control policies which minimize cost functionals over a (finite) rolling-horizon conditioned upon the information that arrives at random time instants; although a hard upper bound equal to the length of the optimization horizon is imposed on consecutive sampling instants. Sufficient conditions are provided on the system dynamics, the cost functionals, and the statistics of the sampling process, such that the proposed policies result in computable upper bounds on the infinite-horizon average cost. The case of linear time-varying system with quadratic cost functionals is studied for the illustration of our results.

[1] D. Chatterjee and J. Lygeros. On stability and performance of stochastic predictive control techniques. *IEEE Transactions on Automatic Control*, 60(2):509–514, 2015.

[2] L. Grüne. Approximation properties of receding horizon optimal control. *Jahresbericht der Deutschen Mathematiker Vereinigung*, 118:3–37, 2016.

[3] A. Tanwani, D. Chatterjee, and L. Grüne. Performance bounds for stochastic receding horizon control with randomly sampled measurements. In *Proc. 58th IEEE Conf. Decision & Control*, pages 2330–2335, 2019.

Data-driven modeling and control of complex dynamical systems

Sabrina Casper (1), Doris Fuertinger (1), Peter Kotanko (2), Luca Mechelli (3), Jan Rohleff (3), Stefan Volkwein (3),

(1) Fresenius Medical Care Deutschland GmbH, Bad Homburg, Germany.; (2) Renal Research Institute New York, New York, USA.; (3) Department of Mathematics and Statistics, University of Konstanz, Konstanz, Germany.;

In this talk control strategies are discussed for dynamical systems; cf. [1]. By data-driven strategies we derive surrogate models for the complex dynamics. The proposed strategies are illustrated by numerical experiments for mathematical model of erythropoiesis for anemia [2, 3] and for heat-convection phenomena [4]. The data-driven modeling extended dynamic mode decomposition (EDMD) and empirical grammians are utilized. This allows for efficient and reliable strategies based on a combination of EDMD and model predictive control (MPC), which produces results comparable with the one obtained in [5] for the original model. Moreover, multiobjective problems are considered as well and numerical tests show that advantages of empirical gramian approach compared to standard POD surrogate modeling.

[1] S. Casper, D.H. Fuertinger, P. Kotanko, L. Mechelli, J. Rohleff and S. Volkwein. Data-driven modeling and control of complex dynamical systems arising in renal anemia therapy Submitted, 2021. See `https://arxiv.org/abs/2106.11733`

[2] D.H. Fuertinger. A model of erythropoiesis. PhD thesis, Karl-Franzens University Graz, 2012.

[3] D.H. Fuertinger, F. Kappel, S. Thijssen, N.W. Levin, and P. Kotanko. A model of erythropoiesis in adults with sufficient iron availability. *J. Mathematical Biolology*, vol. 66, 1209-1240, 2013.

[4] L. Mechelli, J. Rohleff and S. Volkwein. Model order reduction for optimality systems through empirical gramians. In preparation, 2022.

[5] S. Rogg, D.H. Fuertinger, S. Volkwein, F. Kappel, P. Kotanko. Optimal EPO dosing in hemodialysis patients using a non-linear model predictive control approach. *J. Mathematical Biolology*, vol. 79, 2281-2313, 2019.

Data-driven MPC for linear descriptor systems based on Willems' fundamental lemma

Timm Faulwasser (1), Philipp Schmitz (2), Karl Worthmann (2),

(1) TU Dortmund, Dortmund, Germany;
(2) Optimization-based control group, Technische Universität Ilmenau, Ilmenau, Germany;

Willems' fundamental lemma allows for deriving a non-parametric description of the input-output behaviour of linear control systems. In this talk (see also [1] for details), we first tailor the fundamental lemma to linear descriptor systems and discuss some details of particular interest for this system class. Based on the gained insight, we propose an MPC scheme, for which recursive feasibility and asymptotic stability of the origin is guaranteed.

[1] P. Schmitz, K. Worthmann, and T. Faulwasser. Willems' fundamental lemma for linear descriptor systems and its use for data-driven output-feedback MPC. *IEEE Control Systems Letters*, accepted for publication.

ArXiv preprint available: https://arxiv.org/abs/2202.07930

27. Population ageing and inequality — 11:10 – 12:50

***Chair*: A. Prskawetz, M. Sanchez** — **FH HS 6**

An Earned Income Pension Credit as a means to reduce old-age poverty risk

Fabian Kindermann (1,2,3,4), Veronika Püschel (1),

(1) University of Regensburg; (2) CEPR; (3) HCEO; (3) Netspar;

In this paper, we study the optimal design of pension systems that aim at reducing old-age poverty risk. We argue that a pension scheme that grants subsidies to the earnings poor based on their annual earnings is superior to a redistribution scheme that is based on the life-time earnings history of individuals, like e.g. US social security. In particular, we find that pension subsidies for individuals with low earnings that are designed in a similar way as the Earned Income Tax Credit provide both insurance against old-age poverty and incentives for labor force participation. As such, the Earned Income Pension Credit (EIPC) generates substantial long-run welfare gains, especially for single women. To arrive at this conclusion, we evaluate the individual and macroeconomic consequences of redistributive pension reforms in a quantitative overlapping generations model that accounts for a rich set of demographics (gender, marital status, and family size), permanent labor market characteristics, idiosyncratic labor productivity shocks, individual savings choices and labor supply decisions at the extensive and the intensive margin.

On the optimal reform of income support for couples with children

Viola Garstenauer (1), Nawid Siassi (2),

(1) ECON, TU Wien, Vienna, Austria; (2) ECON, TU Wien, Vienna, Austria;

We characterize the optimal reform of U.S. income support for non-college educated couples with children. We develop a heterogeneous-agents model with uninsurable idiosyncratic productivity, fertility and child care risk, and incomplete asset markets. In our model, married couples evolve through a life cycle and make optimal consumption-savings and labor supply decisions. We calibrate the model using microeconomic household survey data from the Current Population Survey. Importantly, our model matches salient data moments on participation rates, the level and dispersion of hours worked, earnings, and expenditures on child care. Using our calibrated model, we then assess welfare-enhancing reforms to the current U.S. tax-transfer system. Our preliminary findings indicate that there is ample room to improve the well-being for low-income families. For instance, treating individuals instead of households as the relevant units of taxation allows the policymaker to lower participation tax rates for secondary earners and render the trade-off between guaranteed income and work incentives more favorable.

Anticipation of deteriorating health and information avoidance

Johannes Schünemann (1), Holger Strulik (2), Timo Trimborn (1),

(1) Aarhus University; (2) University of Göttingen; (3) University of Fribourg;

We integrate anticipatory utility and endogenous beliefs about future negative health shocks into a life-cycle model of physiological aging. Individuals care about their future utility derived from their health status and form endogenous beliefs about the probability of a negative health shock. We calibrate the model with data from gerontology and use the model to predict medical testing decisions of individuals. We find that anticipation in combination with endogenous beliefs provides a quantitatively strong motive to avoid medical testing for Huntington's disease which explains the low testing rates found empirically. We also study the case of breast and ovarian cancer and provide an explanation for why testing rates depend on the individual's income when treatment is available.

Optimal retirement with disability pensions

Hans Fehr (1), Adrian Fröhlich (1),

(1) University of Würzburg, Würzburg, Germany;

This paper develops a general equilibrium life-cycle model with endogenous retirement and disability risk, which captures the interaction between disability pensions (DP) and old-age pensions (OAP) in Germany. At certain ages households may either apply for DP or for OAP depending on the eligibility rules and generosity of the two programs. Consequently, reforms which affect only one program typically have spill-over effects to the other program which dampens the intended impact of the reform. Our simulation results indicate that such effects are also quantitatively important in Germany where the OAP normal retirement age will rise up to age 67 until 2030 while at the same time the generosity of disability pensions will dramatically increase. We show that the reform of the DP rules may offset the financial gains from the increase in the normal retirement age if current eligibility rules would prevail.

28. Dynamics of firm

11:10 – 12:50

***Chair*: P.M. Kort**

FH HS 7

Investment in zero-emission buses

Maximilian Brill (1), Tine Compernolle (2), Bruno De Borger (1), Peter M. Kort (3),

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Zero-Emission Buses (ZEB) are considered a critical step towards reducing greenhouse gas emissions and air pollution from current bus transit systems and creating a resilient transport infrastructure that is less dependent on fossil fuels. For this reason, many governments are subsidizing the adoption of ZEB. This paper employs a real options model following [1]: We consider a public transport operator that is maximizing its profit and holds the option to invest in ZEB, but faces irreversible investment costs and uncertain demand. We are interested in jointly determining the optimal timing and the optimal size of the investment. We consider two cases: The transit operator can either add the ZEB to its current diesel fleet or replace the entire diesel fleet. In addition, we are interested in how the government can increase social welfare with one of the following three policy instruments: first, by imposing diesel *taxes* and thus reducing the profit from the operation of diesel buses. Second, by *subsidizing* the purchase of ZEB, thereby lowering the high upfront costs. Third, by enacting a *regulation* requiring the use of ZEB from a specific date. A numerical implementation of the model illustrates the theoretical results. We have the following results. First, adding ZEB to the diesel fleet occurs at an earlier investment time than replacing the entire fleet. Therefore, the total fleet size in the adding scenario is smaller than in the replacing strategy. Second, profit maximization leads to a welfare loss of about 20%. This means that less ZEB are made available at a later point in time than in the case of social welfare maximization. Third, all three policy instruments help accelerate the adaptation of the ZEB in the replacement strategy. Fourth, subsidizing the purchase of ZEB is the most welfare-maximizing instrument, which decreases the welfare loss of the profit-maximizing operator by around 3,4%. In contrast, the diesel tax or regulation can only reduce the loss by about 1.1%.

[1] Hagspiel, Verena, Peter M. Kort, Cláudia Nunes, Rita Pimentel, and Kristian Støre. Capacity Optimization of an Innovating Firm. *International Journal of Production Economics*, **233**(March):108021, 2021.

Strategic investment under uncertainty: second mover advantage in the duopoly

Berend J. Stofferis (1), Peter M. Kort (2), Jacco J.J. Thijssen (3),

(1) TISEM, Tilburg University, Tilburg, the Netherlands; (2) TISEM, Tilburg University, Tilburg, the Netherlands; (3) University of York Department of Mathematics, University of York, Heslington, UK;

We study a strategic duopoly investment game. Initially the firms are potential entrants. The decision to enter implies choosing the optimal timing and size of a capacity investment. The literature until now typically admits a linear demand structure, and then finds that a preemption equilibrium results. Our main message is that, when replacing the linear demand function by an iso-elastic one, a second mover advantage can arise. This especially occurs in case of high demand uncertainty, a low discount rate, a high market trend and a high demand elasticity. Simulations reveal that the second mover advantage could result in inefficiently long waiting with investment and a lower expected welfare level.

Perpetual American standard and lookback options with event risk and asymmetric information

Pavel Gapeev (1), Libo Li (2),

(1) Department of Mathematics, London School of Economics and Politiical Science, London, United Kingdom; (2) School of Mathematics and Statistics, University of New South Wales, Sydney, Australia;

We derive closed-form solutions to the perpetual American standard and floating-strike lookback put and call options in an extension of the Black-Merton-Scholes model with event risk and asymmetric information. It is assumed that the contracts are terminated by their writers with linear or fractional recoveries at the last hitting times for the underlying asset price process of its ultimate maximum or minimum over the infinite time interval which are not stopping times with respect to the reference filtration. We show that the optimal exercise times for the holders are the first times at which the asset price reaches some lower or upper stochastic boundaries depending on the current values of its running maximum or minimum. The proof is based on the reduction of the original optimal stopping problems to the associated free-boundary problems and the solution of the latter problems by means of the smooth-fit and normal-reflection conditions. The optimal exercise boundaries are proven to be the maximal or minimal solutions of some first-order nonlinear ordinary differential equations.

The case of zero recoveries was recenly studied in:

[1] P.V. Gapeev and L. Li. Optimal stopping problems for maxima and minima in models with asymmetric information. Published online first in *Stochastics*, 28 pp, 2021.

A multi-layered learning approach for sequential decision problems with multiple uncertainties

Reidar B. Bratvold (1), Lone A. S. Erikson (2), Verena Hagspiel (2), Tonje J. Olsen (2),

(1) Department of Petroleum Engineering, University of Stavanger, Stavanger, Norway; (2) Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology, Trondheim, Norway;

In this paper we present a new methodology for solving sequential decision making problems under multiple uncertainties that allows for multi-level learning. We extend the classical real option valuation problem by allowing the hyperparameters of the stochastic processes underlying the decision problem to be updated in a Bayesian manner. The need for parameter updates may arise from regime shifts as indicated by the signals available to the decision maker. The solution approach extends the classical Least-Squares Monte Carlo (LSM) approach to update the model parameters of the stochastic processes using Bayesian inference. The methodology allows to derive the optimal decision policy and value of the investment opportunity. We provide further insights into the decision context by examining how different beliefs and parameter choices affect the optimal decision policy for an illustrative example using sensitivity analysis.

We find that receiving signals and updating the decision maker's beliefs can notably impact the decision policy and the investment value. If the signals differ sufficiently from the prior beliefs, a different decision policy and investment value are often reached. The signals will have a more prominent effect when the decision maker's uncertainty is high. The approach provides a more realistic value assessment and decision policy than traditional real option valuation as it accounts for possibilities not included in the original hyperparameters. It is flexible and versatile and applicable to a broad set of problems.

29. Semi Plenary — 14:00 – 14:50

***Chair*: A. Rösch** — **FH HS 5**

Infinite Horizon Optimal Control Problems with Discount Factor on the State

Eduardo Casas (1), Karl Kunisch (2),

(1) Dept. of Applied Mathematics and Computer Science, University of Cantabria, Spain; (2) Institute for Mathematics and Scientific Computing, University of Graz, Austria;

This talk deals with infinite horizon optimal control problems subject to semi-linear parabolic equations. A discount fact on the state variable is introduced in the cost. This allows the treatment of some nonlinearities for infinite horizon problems without stabilizability assumptions. These nonlinearities can be of polynomial type, thus covering reaction diffusion equations which are important for applications. Existence of a solution is proven, and first as well as second order optimality conditions are derived. They are used to analyze the approximation of the infinite horizon problem by finite horizon problems.

29. Semi Plenary — 14:00 – 14:50

***Chair*: P.M. Kort** — **FH HS 6**

The "new" epi-econ literature: Remarkable traits and open questions

Raouf Boucekkine (1), Luca Gori (2), Piero Manfredi (3),

(1) Rennes School of Business, Rennes, France; (2) Department of Law, University of Pisa, Pisa, Italy; (3) Department of Economics and Management, University of Pisa, Pisa, Italy;

We (partially) overview the recent huge burst in the epi-economic literature, induced by the Covid crisis, identifying real (and other much less real) novelties. To this end, we build up a unified planner epi-econ model with lockdown, testing/tracing and some additional interesting aspects introduced gradually like partial immunization, mental health, frictions, etc. A key aspect discussed is the internalization of individual behavior (in response to Non-Pharmaceutical Interventions, NPIs) by the planner. We specifically discuss the realistic case where, given uncertainty, individuals learn and modify their beliefs and their behavioral response to NPIs accordingly. We also discuss some of the potential research lines related to asymmetric information (planner vs individuals) leading to potentially interesting game-theoretic problems.

30. Variational analysis in optimization and control

15:10 – 16:25

***Chair*: R. Cibulka**

FH HS 8

Ranges of nonlinear mappings

Radek Cibulka (1), Marián Fabian (2), Tomáš Roubal (1),

(1) University of West Bohemia, Pilsen, Czech Republic; (2) Mathematical Institute of Czech Academy of Sciences, Prague, Czech Republic;

We study conditions ensuring that the range of a (nonlinear) mapping on a compact convex set covers a prescribed set via an approximation by a linear/nonlinear mapping and by a bunch of linear mappings in Fréchet spaces and Banach spaces, respectively.

Further, we provide sufficient conditions for directional and constrained (semi)regularity of single-valued mappings. In Banach and finite-dimensional spaces, we focus on approximations determined by a convex set of bounded linear mappings such as Páles-Zeidan Jacobian, e.g. [1], and Clarke's generalized Jacobian.

[1] Z. Páles and V. Zeidan. Infinite dimensional generalized Jacobian: properties and calculus rules. *Journal of Mathematical Analysis and Applications*, **344**(1):55-75, 2008.

Inverse mapping theorem in Fréchet spaces

Milen Ivanov (1), Nadia Zlateva (2),

(1) Radiant Life Technologies Ltd, Nicosia, Cyprus;
(2) Faculty of Mathematics and Informatics, Sofia University St. Kliment Ohridski, Sofia, Bulgaria;

We consider the classical Inverse Mapping Theorem of Nash and Moser from the angle of some recent development by Ekeland and the authors.

Geometrisation of tame estimates coupled with certain ideas coming from Variational Analysis when applied to a directionally differentiable mapping, produce very general surjectivity result and, if injectivity can be ensured, Inverse Mapping Theorem with the expected Lipschitz-like continuity of the inverse.

We also present a brief application to differential equations.

Nonlinear metric subregularity in optimal control

Alberto Domínguez Corella (1),

(1) Vienna University of Technology, Vienna, Austria;

Regularity is one of the fundamental concepts in modern analysis. The particular concept of nonlinear metric regularity, introduced in [1], aims at estimating (in a nonlinear way) the distance between preimages of set-valued mappings in terms of the residuals.

We will consider optimal control problems constrained by evolution equations. The talk will begin with a preliminary discussion on how the concept of nonlinear metric regularity applied to the optimality mapping associated with an optimal control problem relates with the local stability of the solutions of the problem. Later on, we will discuss some results concerning sufficient conditions for the optimality mapping of a problem to have the nonlinear regularity property. Finally, we will discuss some consequences of the results in optimal control as well as applications.

[1] Ioffe, Alexander D. (2013) *Nonlinear regularity models*, Math. Program., 139 **1-2, Ser. B**, 223–242.

31. Numerical analysis for PDE constrained optimization 15:10 – 16:25

***Chair*: J. Pfefferer, A. Rösch** **FH HS 5**

On optimal control problems with quasilinear parabolic PDEs

Fabian Hoppe (1), Ira Neitzel (1),

(1) INS, University of Bonn, Bonn, Germany;

In this talk we will present recent results for optimal control problems governed by quasilinear parabolic partial differential equations. Optimal control problems governed by nonlinear PDEs are in general non-convex. Many associated challenges have been studied in detail for semilinear PDEs, while the research on the quasilinear setting is less complete. We will focus on specific challenges of this problem classe due to the different structure of the nonlinearity.

Sparse optimal control of a quasilinear elliptic PDE in measure spaces

Fabian Hoppe (1),

(1) Insitut für Numerische Simulation, Rheinische Friedrich-Wilhelms-Universität Bonn, Bonn, Germany;

We consider sparse optimal control of a quasilinear elliptic equation in measure spaces. Our prototypical model problem reads as follows:

$$\min_{u\in\mathcal{M}(\Omega\cup\Gamma_N)} J(y,u) := \frac{1}{2}\|y-y_d\|^2_{L^2(\Omega)} + \gamma\|u\|_{\mathcal{M}(\Omega\cup\Gamma_N)}$$

$$\text{s.t.} \quad \begin{cases} -\nabla\cdot\xi(y)\rho\nabla y = u, & \text{in } \Omega\cup\Gamma_N, \\ y = 0, & \text{on } \Gamma_D, \end{cases}$$

where $\Omega\subset\mathbb{R}^n$, $n\in\{2,3\}$, is a domain, and $\Gamma_D\neq\emptyset$ and Γ_N denote the Dirichlet and Neumann boundary, respectively. By $\mathcal{M}(\Omega\cup\Gamma_N)$ we denote the space of regular Borel measures on $\Omega\cup\Gamma_N$ that serves as our control space.

Under rather general assumptions we prove existence of optimal controls and derive first-order necessary optimality conditions. Under additional assumptions also second-order necessary and sufficient optimality conditions are obtained. The key technique of our analysis is the application of the so-called Kirchhoff transform, i.e. a nonlinear superposition operator that allows to transform the quasilinear state equation into a linear one.

Optimal control of sliding droplets using the contact angle

H. Bonart (1), C. Kahle (2), J.-U. Repke (3),

(1) Technische Universität Darmstadt; (2) Universität Koblenz–Landau; (3) Technische Universität Berlin;

We present results on optimal control of sliding droplets. Here the contact angle between droplet and solid serves as a control variable. The fluid is modeled by a thermodynamically consistent diffuse interface model with a suitable contact line model.

In earlier work [1] we compared different time discretization schemes for this model that mimics the energy behaviour of the continuous model. We now [2] employ a particular scheme to derive existence of optimal controls for a time discrete optimal control problem and also first order neccessary conditions.

As controls we consider finite dimensional controls for the contact angle distribution. We test our approach by driving a droplet up an inclined plate.

[1] H. Bonart, C. Kahle. Optimal Control of Sliding Droplets Using the Contact Angle Distribution. *SIAM Journal on Control and Optimization*, **59**(2):1057–1082, 2021

[2] H. Bonart, C. Kahle, J.-U. Repke. Comparison of Energy Stable Simulation of Moving Contact Line Problems using a Thermodynamically Consistent Cahn–Hilliard Navier–Stokes Model. *Journal on Computational Physics*, **399**:108959, 2019

32. Dynamic games and applications 15:10 – 16:25

***Chair*: S. Wrzaczek** **FH HS 6**

Payment schemes for sustaining cooperation in dynamic games

Elena Parilina (1), Georges Zaccour (2),

(1) Saint Petersburg State University, Saint Petersburg, Russia; (2) Chair in Game Theory and Management, GERAD and HEC Montréal, Canada;

It is a challenge to sustain cooperation in a finite-horizon dynamic game. Since players generally have an incentive to deviate to their noncooperative strategies in the last stage, a backward induction argument leads them to defect from cooperation in all stages. In this paper, we propose two payment schemes having some desirable properties, namely, individual rationality and stability, which ensure that the players cooperate throughout the entire planning horizon. The setup and the results are general, that is, they do not rest on particular specifications of the payoff functionals or the state dynamics. We illustrate our results with a linear-quadratic dynamic game of pollution control.

The work of the first author was supported by the Russian Science Foundation grant No. 22-11-00051, https://rscf.ru/en/project/22-11-00051/

Sharks, squalene and COVID vaccines - when a scarce renewable resource becomes strategic in short run

Agnieszka Wiszniewska-Matyszkiel (1), Rajani Singh (2),
(1) Institute of Applied Mathematics and Mechanics, University of Warsaw, Warsaw, Poland; (2) Department of Digitalization, Copenhagen Business School, Copenhagen, Denmark;

In this paper, we model the problem of "the tragedy of the commons" that appears in the context of current pandemic, describing one of the aspects whose potential consequences hasn't been investigated. It is related to the fact that some of COVID-19 vaccines at various stages of development use adjuvants based on squalene obtained from endangered deep-sea shark species.

We model the squalene market, with such COVID-19 vaccines producers as a part of it, as a compound dynamic game taking into account various aspects of this market. One of them is the privileged position of relatively small number of those vaccine producers.

The game describes a market consisting of pharmaceutical, cosmetic and fishing sector. We calculate Nash and Stackelberg equilibria in which COVID-19 vaccine producers do not take into account their influence on the population of sharks. We prove the existence and uniqueness of equilibria together with deriving the formula to find it. We discuss the consequences of relaxing the assumption of vaccine producers myopia concerning sharks.

"The tragedy of the commons" in both cases results in endangering of the vaccination programme, including COVID vaccines: either because of depletion of the shark population or its reduction to a level at which the cost of squalene production exceeds the maximal price that can be paid for it. So, the solution of "the tragedy" by a regulatory institution is suggested, with various remedies that can be used by it.

Despite using terminology related to a specific problem of high current importance, this may be treated as a starting point to a general theory emphasizing the need to indicate other potential scarce renewable resources for which similar phenomena are likely to appear in the future, in order to counteract such risks a priori.

Reverse revenue sharing contract in a dynamic closed-loop supply chain with uncertain parameters

Elnaz Kanani Kuchesfehani (1), Elena Parilina (2), Georges Zaccour (3),

(1) GERAD, HEC Montréal, Canada; (2) Saint Petersburg State University, Saint Petersburg, Russia; (3) Chair in Game Theory and Management, GERAD and HEC Montréal, Canada;

We represent a closed-loop supply chain made up of one manufacturer and one retailer, as a stochastic dynamic game, which allows us to capture the dynamic nature of the returns of past-sold products for remanufacturing along with uncertainty in the parameter values. We characterize and compare the solutions in three scenarios. In the no-sharing scenario, we assume that the manufacturer alone incurs the cost of the green activities aimed at incentivizing consumers to return previously purchased products at the end of their useful life. In the second scenario, namely, reverse revenue sharing contract (RRSC), the retailer shares the cost of the green activities and the manufacturer transfers part of its revenues to the retailer. Finally, for completeness and comparative purposes, we also determine the solution in the vertically integrated supply chain. Numerical experiments are provided.

33. Dynamics of firm — 15:10 – 16:25

***Chair*: P.M. Kort** — **FH HS 7**

Product life cycles and investment: a real options analysis

Anne G. Balter (1), Kuno J.M. Huisman (1), Peter M. Kort (1),

(1) Department of Econometrics and Operations Research, Warandelaan 2, 5037AB, Tilburg, The Netherlands;

Typically product demand follows a product life cycle (PLC). This means that after a product is introduced, demand for this product first starts to grow, which after some time is followed by a decline in demand. Moreover, in most cases demand is stochastic. This paper combines these two characteristics by employing a geometric Brownian motion process with a first increasing and afterwards decreasing trend. Our aim of the paper is to investigate the optimal investment decision of a firm in production capacity. The investment decision involves deciding about the timing and the size of the investment. We make a distinction between firms being a *product-life-cycle leader* and a *product-life-cycle follower*. For a PLC-leader the growth stage starts at the moment this firm invests. In case of a PLC-follower, the firm enters an existing product life cycle, implying that the decline can already start before this firm even has invested.

One of the interesting results is that a PLC-leader waits for a higher demand level before it invests with the same amount when the expected length of the growth interval is shorter. For the PLC-follower it holds that it may be optimal to invest earlier because of this probability that the decline could already start before the firm invests. In such a case the expected future demand is lower, which makes it optimal that the firm attracts less capacity. This makes the investment cheaper and then the firm does not need to wait for a high demand level to make the investment profitable.

Investments in R&D and production capacity with uncertain breakthrough time: private versus social incentives

Martijn Ketelaars (1), Peter M. Kort (1),

(1) Tilburg University, Department of Econometrics and Operations Research, Tilburg, The Netherlands;

The paper considers a sequential investment project which starts with a product innovation phase, and subsequently, once the R&D project is completed, a production phase. The investment decision of the R&D phase involves choosing the time and the size of the R&D investment. The time to breakthrough is stochastic where the instantaneous probability of innovation is increasing in the R&D investment size. Once the R&D stage is completed the firm starts producing the new product. To do so, the firm first needs to invest in production capacity, the size of which must be determined. We compare the optimal investment decisions of the firm with those of the social planner and conclude that the firm invests too late in R&D and not enough in production capacity. We find that a proper subsidy policy, consisting of an R&D investment and a productive investment subsidy can make up for that. However, taking into account a budget constraint such that subsidy expenses cannot exceed the resulting increase in total surplus, learns that a first-best solution can only be reached if the demand situation is relatively stable, i.e., when growth and demand uncertainty are limited, or when the price elasticity of demand is low.

Optimal abandonment under a spectrally negative Lévy process

Laura Delaney (1), Jacco J.J. Thijssen (2),

(1) Kings Business School, London, UK; (2) Department of Mathematics, University of York, York, UK;

This paper considers an abandonment problem in which the underlying uncertainty is modelled as a spectrally negative Lévy jump diffusion. We show that the solution to the corresponding optimal stopping problem may not be a threshold policy found by the usual "value-matching" and "smooth-pasting" conditions. We show that smooth pasting may fail when the jumps are (in expectation) large and/or frequent. We provide a verification theorem for such cases in terms of viscosity solutions to the Hamilton–Jacobi–Bellman equation. It turns out that the optimal abandonment trigger is either the trigger obtained from value-matching and smooth pasting, or the "zero-NPV" trigger. In the latter case, smooth pasting leads to a sub-optimal abandonment rule in which the decision maker abandons too early. We show that there always exists and expected jump size beyond which the latter case applies. This means that the usual trigger obtained by applying smooth pasting is only valid when jumps are not too big (in expectation).

34. Variational analysis in optimization and control 16:40 – 17:55

***Chair*: R. Cibulka** **FH HS 8**

Robust feedback stabilization by means of Lyapunov-like functions determined by Lie brackets

Giovanni Fusco (1),

(1) University of Padova, Department of Mathematics Tullio Levi-Civita, Padova, Italy;

We use Lie brackets of unbounded vector fields to consider a dissipative relation that generalizes the differential inequality which defines classic control Lyapunov functions (see [1]). Under minimal regularity assumptions, we employ locally semiconcave solutions of this extended relation, called *degree-k control Lyapunov functions*, in order to design *degree-k Lyapunov feedbacks*, that is, particular discontinuous feedback laws that stabilize the underlying system to a given closed target with compact boundary, in the sample and hold sense. We also prove that this feedback construction is robust when small measurement errors and external disturbances occur.

The talk is based on the results in [2].

[1] Clarke F., Ledyaev Y., Sontag E., Subbotin A., *Asymptotic controllability implies feedback stabilization*, IEEE Trans. Automat. Control, vol. 42, no. 10, pp. 1394–1407, 1997.

[2] Fusco G., *Robust feedback stabilization by means of Lyapunov-like functions determined by Lie brackets.* J. Differential Equations, vol. 287, pp. 88–112, 2021.

Goh and Legendre-Clebsch conditions for non-smooth optimal control problems

Francesca Angrisani (1), Franco Rampazzo (1),

(1) Dept. of Mathematics "T.Levi-Civita", University of Padova, Italy;

For end-point constrained optimal control problems like

$$\min_{u\in\mathcal{U}} \Psi(T,x(T)) \qquad \begin{cases} \dfrac{dx}{dt} = f(x(t)) + \displaystyle\sum_{i=1}^{m} g_i(x(t))u^i(t), \quad \text{a.e. } t\in[0,T], \\ x(0) = \hat{x}, \qquad (T,x(T)) \in \mathfrak{T}, \end{cases} \tag{P}$$

where the cost Ψ and *the vector fields* $f, g_1, \dots, g_m$ *are just Lipschitz continuous*, I will present some achievements in the direction of adding some higher order necessary conditions to the the (nonsmooth) Maximum Principle. In particular, I will discuss how one can generalize the classical *Goh condition*

$$p(t)\cdot[g_i,g_j](x(t)) = 0, \qquad \forall t\in[0,T]$$

and *Legendre-Clebsch condition* ($m=1, g=g_1$),

$$p(t)\cdot[f,g](x(t)) = 0, \quad p(t)\cdot[g,[f,g]](x(t)) \le 0 \qquad \forall t\in[0,T],$$

where $p(\cdot)$ is a solution of the adjoint inclusion. (For smooth vector fields X, Y, $[X,Y]$ denotes the Lie bracket, i.e. $[X,Y] = DY\cdot X - DX\cdot Y$.) The main tools for these extensions consist of the notion of *Quasi Differential Quotient* [2], together with a set-valued generalization $[X,Y]_{set}$ of the Lie bracket introduced in [3] (and expanded in [1]).

[1] Feleqi E., Rampazzo F. Iterated Lie brackets for nonsmooth vector fields, *NoDEA Nonlinear Diff. Eq. Appl., vol. 24, no. 6, Art. 61, 2017.*

[2] Palladino M., Rampazzo F.,*A geometrically based criterion to avoid infimum gaps in Optimal Control*, to appear in J.of Diff. Eqs.

[3] Rampazzo F., Sussmann H.J., *Set-valued differentials and a nonsmooth version of Chow-Rashevski's theorem*, Proceedings of the 40th IEEE Conference on Decision and Control, Orlando, Florida, (IEEE Publications, New York), vol. 3, pp. 2613-2618, 2001.

Markov decision process with observation costs

Christoph Reisinger (1), Jonathan Tam (1),

(1) Mathematical Institute, University of Oxford, Oxford, United Kingdom;

We present a framework for a controlled Markov chain where the state of the chain is only given at chosen observation times and of a cost. Optimal strategies therefore involve the choice of observation times as well as the subsequent control values. We show that the corresponding value function satisfies a dynamic programming principle, which leads to a system of quasi-variational inequalities (QVIs). Next, we give an extension where the model parameters are not known *a priori* but are inferred from the costly observations by Bayesian updates. We then prove a comparison principle for a larger class of QVIs, which implies uniqueness of solutions to our proposed problem. We utilise penalty methods to obtain arbitrarily accurate solutions. Finally, we perform numerical experiments on three applications which illustrate our framework.

35. Data-driven and learning-based control

16:40 – 17:55

***Chair*: T. Faulwasser, K. Worthmann**

FH HS 5

On the role of stable integration in optimal control and deep learning

Sofya Maslovskaya (1), Sina Ober-Blöbaum (1),

(1) Department of Mathematics, Paderborn University, Paderborn, Germany;

Deep Learning proved to be efficient in empirical learning from data. However, it still misses understanding of theoretical properties and behavior, and thus, may lead sometimes to significant errors. It was recently noted that a class of deep learning algorithms based on residual network (ResNet) can be seen as a discretized optimal control problem [1]. Indeed, propagation through layers of a given network coincides with integration by the Euler scheme of some control system and the loss function which is minimized in deep learning corresponds to a discretized cost function. Therefore, there exists a continuous counterpart corresponding to some given deep learning algorithm which takes form of an optimal control problem. Well developed analytical and numerical methods of optimal control theory can be used for construction of new deep learning architectures with predictable behaviour. On one hand, the continuous counterpart of a deep learning problem should be set in such a way that it has stable solutions. Such a formulation was recently proposed in [2] using the notion of turnpike property. On the other hand, the numerical methods should preserve the properties of these solutions. In this context, we analyse at which scale the techniques of the numerical optimal control and stable numerical integrators can provide convergence and stability for the associated deep learning algorithms.

[1] M. Benning, E. Celledoni, M. J. Ehrhardt, B. Owren, and C.-B. Schönlieb. Deep learning as optimal control problems: Models and numerical methods. *Journal of Computational Dynamics*, **6**(2):2158–2491, 2019.

[2] C. Esteve, B. Geshkovski, D. Pighin and E. Zuazua. Large-time asymptotics in deep learning. *arXiv preprint arXiv:2008.02491*, 2020.

A deep neural network approach for computing Lyapunov functions

Lars Grüne (1),

(1) Chair of Applied Mathematics, University of Bayreuth, Germany;

We propose a deep neural network architecture and a training algorithm for computing approximate Lyapunov functions of systems of ordinary differential equations. Under the assumption that the system admits a compositional Lyapunov function, we prove that the number of neurons needed for an approximation of a Lyapunov function with fixed accuracy grows only polynomially in the state dimension, i.e., the proposed approach is able to overcome the curse of dimensionality. We show that nonlinear systems satisfying a small-gain condition admit compositional Lyapunov functions. Numerical examples in up to ten space dimensions illustrate the performance of the training scheme.

[1] L. Grüne. Computing Lyapunov functions using deep neural networks. *Journal of Computational Dynamics*, Online First 2020, http://dx.doi.org/10.3934/jcd.2021006

Towards encrypted data-driven (M)PC via explicit controller representations

Philipp Binfet (1), Nils Schlüter (1), Dieter Teichrib (1), Manuel Klädtke (1), Moritz Schulze Darup (1),

(1) Control and Cyberphysical Systems, TU Dortmund University, Dortmund, Germany;

Encrypted control seeks confidential controller evaluations in cloud-based or networked systems. After the seminal realization of encrypted linear feedback in [1], various control schemes have been realized in an encrypted fashion using techniques such as homomorphic encryption or multi-party computation. In particular, encrypted model predictive control (MPC) has been addressed in numerous works (see [2] and references therein). Remarkably, the basis for secure MPC often is the explicit solution of the underlying optimal control problem (OCP). For instance, [2] approximates the explicit MPC solution using max-out neural networks and securely evaluates these networks with so-called garbled circuits.

Now, neglecting the focus on encrypted control for a moment, data-driven predictive control (DPC, [3]) is becoming a popular alternative to classical MPC. In DPC, the prediction of the systems' behavior is carried out based on collected input-output data instead of a model as in MPC. While deterministic DPC and MPC are equivalent with respect to the resulting control actions, the underlying OCPs (initially) show some crucial differences. Most importantly, the number of decision variables in the DPC-related OCP is usually significantly larger than for MPC. As a consequence, explicit DPC has been considered as "unattractive". However, we recently discovered that the OCP for deterministic DPC can always be transformed into a variant that offers exactly the same complexity (in terms of decisions variables and constraints) as the corresponding MPC. Moreover, also the explicit solutions of both OCPs are of the same complexity (w.r.t. the number of affine segments). In our talk, we exploit this feature in order to realize encrypted DPC by adapting the implementation of encrypted MPC in [2].

[1] K. Kogiso and T. Fujita. Cyber-security enhancement of networked control systems using homomorphic encryption. *54th Conference on Decision and Control*, pp. 6836–6843, 2015.

[2] K. Tjell, N. Schlüter, P. Binfet, and M. Schulze Darup. Secure learning-based MPC via garbled circuit, *60th Conference on Decision and Control*, pp. 4907-4914, 2021.

[3] J. Coulson, J. Lygeros, and F. Dörfler. Data-enabled predictive control: In the shallows of the DeePC. *18th European Control Conference*, pp. 307–312, 2019.

36. Dynamic games and applications

16:40 – 17:55

***Chair*: S. Wrzaczek**

FH HS 6

Closed-loop Nash equilibrium for a partial differential game with application to competitive personalized advertising

Dominika Machowska (1), Andrzej Nowakowski (2), Agnieszka Wiszniewska-Matyszkiel (3),

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This paper is devoted to an N-person partial differential game whose dynamics of the state variable is described by a hyperbolic differential equation with certain boundary and initial conditions while the

objective of each player is given by a finite horizon accumulated payoff functional with discounting. We extend the concept of a closed-loop Nash equilibrium for a partial differential game with the dynamics of the states described by a hyperbolic differential equation (a transport equation). We propose the definition of a dual closed-loop Nash equilibrium for which we give sufficient conditions. Moreover, we present the relationship between the Nash equilibria with the dual closed-loop and the classical closed-loop information structure. We apply the new results to the goodwill dynamics model in which the goodwill is influenced by personalized advertising and customers' recommendations for which we construct a dual closed-loop Nash equilibrium and we examine its economic properties.

On capital accumulation games in Cournot duopoly model of complements

Bertrand Crettez (1), Naila Hayek (1), Guiomar Martín-Herrán (2),

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We analyze a differential game of capital accumulation in the Cournot duopoly model of complements. In this model, consumers have a downward-sloping demand for a final product which is made out of n different complement goods. The firms producing these goods accumulate capital and set their prices facing capacity constraints. It turns out that in the price equilibrium, firm's instant profits are discontinuous functions of production capacities. As a consequence, one cannot use standard optimal control approaches to study the equilibria of the dynamic game. We nevertheless find that in contrast with the literature on capital accumulation games the open-loop Nash equilibrium does not exist generally. Existence of equilibrium only holds when firms cannot disinvest and have the same initial production capacity.

Shall fiscal policies be centralized in a monetary union? A dynamic game approach

Reinhard Neck (1), Dmitri Blueschke (1),

(1) Department of Economics, University of Klagenfurt, Klagenfurt, Austria;

In this paper we present an application of the dynamic tracking games framework to a monetary union. We use a small stylized nonlinear two-country macroeconomic model of a monetary union for analysing the interactions between fiscal (governments) and monetary (common central bank) policy makers, assuming different objective functions of these decision makers. Using the OPTGAME algorithm we calculate solutions for two game strategies: one cooperative (Pareto optimal) and one non-cooperative game type (the Nash game for the feedback information pattern). Applying the OPTGAME algorithm to the MUMOD2 model [1], we show how the policy makers react upon demand shocks according to these solution concepts. To this end we introduce two sequences of shocks on the monetary union. The first sequence of shocks aims at describing the dynamics in a monetary union in a situation similar to the economic crisis (2007-2010), the sovereign debt crisis (2010-2013) and the current Covid-19 crisis (2020-?) in Europe. The second sequence of shocks serves to discuss macroeconomic policy strategies for these shocks. In particular, we investigate the welfare consequences of two scenarios: decentralized fiscal policies by independent governments (the present situation), both under a non-cooperative and a cooperative mood of play, and a centralized fiscal policy under different assumptions about the joint objective function corresponding to different weights for the governments in the bargaining process assumed to precede the design of the common fiscal policy. We show the crucial importance of these weights (and hence of the regulations contained in the fiscal constitution of the union) for the results of the outcome in terms of sustainability of fiscal policies and main objective variables of the policy makers.

[1] R. Neck, D. Blueschke. Every county for itself and the central bank for us all? In *International Advances in Economic Research*, 26:377–389, 2020.

37. Stochastic control models in economics and finance 16:40 – 17:55

***Chair*: S. Federico, G. Ferrari** **FH HS 7**

Some results on optimization of controlled stochastic age-structured models of economic-epidemic dynamics

Giovanni Zanco (1),

(1) Luiss, Roma, Italy;

I will discuss a class of controlled age-structured SIR-type models and discuss how stochasticity can be introduced in such models. The epidemic dynamics are coupled with economic dynamics: the type of controls considered are those used addressing the recent pandemic, like lockdown measures and testing, and they are costly actions affecting both the epidemic and the economic dynamics.
I will describe the abstract set-up in which such models can be cast, together with some results on optimality and on the related Hamilton-Jacobi-Bellman equation.
The talk is based on some current work that extends previous results obtained in collaboration with Giorgio Fabbri and Fausto Gozzi.

Stochastic optimal growth through state-dependent probabilities

Davide La Torre (1), Simone Marsiglio (2), Frankline Mendivil (3), Fabio Privileggi (4),

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We extend the classical discrete time stochastic one-sector optimal growth model with logarithmic utility and Cobb-Douglas production á-la Brock and Mirman [1] to allow probabilities to be state-dependent. In this setting the probability of occurrence of a given shock depends on the capital stock, thus as the economy accumulates more capital the probability of occurrence of different shocks changes over time. We explicitly determine the optimal policy and its relation with state-dependent probabilities in two alternative scenarios in which the probability function, assumed to take a logarithmic form, is either decreasing or increasing with capital. We show that, by affecting the optimal policy, state-dependent probabilities act as an engine of capital accumulation, which, through its effects on the probability of shocks realization, impacts the evolution of economic inequality. In particular, whenever the probability is decreasing (increasing) in the capital stock the probability of the most (least) favorable shock increases, and this incentivizes the planner to increase (decrease) his capital investment, which in turn will generate a widening (reduction) in economic inequalities over time. We then show that the optimal solution can be converted into an affine iterated function system with affine state-dependent probabilities which converges to an invariant self-similar measure supported on a compact (eventually fractal) attractor. We also characterize the properties of such an invariant self-similar measure in terms of singularity and absolutely continuity with respect to the Lebesgue measure, which ultimately depends on the magnitude of the capital share.

[1] W. A. Brock and L. J. Mirman. Optimal Economic Growth and Uncertainty: the Discounted Case. *Journal of Economic Theory*, **4**:479–513, 1972.

Thursday, July 14^{th}

38. Infinite horizon optimal control and applications 08:30 – 09:20

***Chair*: N. Hayek, S. Pickenhain** **FH HS 8**

A model of optimal stoical growth with pollution

Bertrand Crettez (1), Naila Hayek (1),

(1) CRED, Université Paris Panthéon Assas, Paris, France;

We study a model of optimal stoical growth in a discrete-time, infinite horizon setting. In this model, a consumption good is produced according to a linear technology that employs labor. The production of the consumption good causes polluting emissions that increase the stock of pollution. The representative individual's preferences are described by a quadratic function that increases (up to a certain level) with the consumption good, and decreases both with labor and pollution. We depart from the literature by introducing a stoical effort in these preferences. The higher the stoical effort, the lower the pleasure derived from the consumption good. By lowering the pleasure derived from consumption, the representative individual reduces her consumption and thus the emissions. As result, the pollution stock of the next time period is lower and brings about an increase in her instant utility at every future date.

With the addition of the stoical effort the model is no longer convex and studying optimal paths is non trivial. We provide necessary, and also sufficient conditions, under which growth is stoical (the stoical effort is always positive). We show that the higher the effect of the stoical effort in reducing the desirability of consumption, the lower the stationary value of pollution. Finally, we also provide conditions under which growth is not stoical (the optimal path coincides with that of the standard optimal growth model with pollution).

On the equivalence of relaxations in a view of Infinite Horizon Optimal Control

Ilya Dikariev (1), Valeriya Lykina (1), Sabine Pickenhain (1),

(1) BTU Cottbus-Senftenberg, Cottbus, Germany;

In this paper we will consider a generalized Optimal Control Problem with Infinite Horizon and extend the already known relaxation methods, introduced for finite horizon like Γ-regularization, as in [3], convex combinations of Dirac-measures, as in [2] and Young-measures, also known as Gamkrelidzes relaxed controls, [4].

We succeed in proving equivalence results for these relaxations, so we can concentrate in using of Young-measures without losing of concept of generality. Under mild assumptions existence results follow for the relaxed problems, in which Weighted Sobolev-spaces are used as state spaces.

[1] Lykina, V.; Pickenhain, S. (2017). Weighted functional spaces approach to infinite horizon optimal control problems: a systematic analysis of hidden opportunities and advantages. *Journal of Mathematical Analysis and Applications* **454**, Issue 1, pp. 195 – 218.

[2] Carlson, D.A. Nonconvex and relaxed infinite – horizon optimal control problems. *Journal of optimization theory and applications*, 78(3):465–491, 1993.

[3] Ekeland, I.; Temam, R. *Convex Analysis and Variational Problems.* SIAM, 1999.

[4] Gamkrelidze, R.V. *Principles of Optimal Control Theory.* Plenum Press, New York and London, 1978.

39. Data-driven and learning-based control

08:30 – 09:20

***Chair*: T. Faulwasser, K. Worthmann**

FH HS 5

Learning motion primitives for dynamical control system representation

Kathrin Flaßkamp (1), Matheus V. A. Pedrosa (1), Tristan Schneider (1),

(1) Systems Modeling and Simulation, Systems Engineering, Saarland University, Saarland, Germany;

Continuous-time, controlled dynamical system behavior can be encoded into motion primitives, such that an abstraction as a finite automaton is obtained. The motion planning problem can then be addressed by graph-based techniques which have favorable properties when e.g. used in real-time planners. The design of the motion primitives library is crucial to the planner's performance.

Motion primitives can be computed analytically or numerically, but based on the dynamical system model. However, this approach does not take into account whether the obtained motion primitives are *likely* to be needed in the planning afterwards.

In this talk, we propose to base the automaton design on data from humanly controlled systems. Thereby, the chosen motion primitives represent best the human's control strategy. We illustrate the approach by an autonomous driving application using recorded driving data and solving planning problems based on real-world street layouts and traffic scenarios. The data-based library of motion primitives is shown to represent typical driving behavior. Moreover, its motion plans outperform those from a purely model-based automaton.

Efficient data-driven control using autonomous surrogate models

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(1) Department of Computer Science, Paderborn University, Paderborn, Germany; (2) Department of Mathematics, Paderborn University, Paderborn, Germany;

In recent years, significant progress has been made in the field of machine learning and data-based methods, including the modeling of dynamical systems with the help of data-based surrogate models. These have also found their way into the field of control engineering. However, the computation of an adequate surrogate model with an additional control input is usually much more complex and difficult than for autonomous systems.

To adress this issue, in this talk, we present the *QuaSiModO* framework (**Qua**ntization, **Si**mulation, **Mod**eling and **O**ptimization), which is based on a discretization of the control inputs such that modeling techniques for autonomous systems can be used. As a result, the modeling effort and also the amount of required data can be reduced. Relaxation and rounding techniques from the area of mixed integer control are then used to solve the new discrete control problem.

40. Mathematical models for the human impact on the environment 08:30 – 09:20

***Chair*: F. Gozzi, G. Fabbri, S. Faggian** **FH HS 6**

Biodiversity loss and multi-host reservoir epidemics:An evolutionnary approach

Emmanuelle Augeraud-Véron (1), Arnaud Ducrot (2),

(1) BSE, University of Bordeaux, Pessac, France; (2) LMAH, Iniversité de Normandie, Le Havre, France;

For many epidemics, the set of species constituting the succeptible reservoir to propagate the pathogen at stake is not clearly identified. This is particularly the case for rabies. Recent epidemiological studies, based on the major traits common to the species already identified, have highlighted species not yet listed as reservoirs but likely to be good candidates in the transmission of the pathogen [2]. The aim of our study is to investigate the impact of a reduction in biodiversity in the spread of an epidemic that is transmitted to species with similar traits. For this purpose, we built an integro-differential model with non-local mutation effects to describe the evolutionary dynamics of the epidemic [1]. Biodiversity impacts the natural host dynamics, but also the strength of the infection. Our mathematical study of the non-local integro-differential equation allows us first to calculate the R0 and to study the impact of biodiversity on this parameter. The dilution and amplification effects are highlighted. We then study the existence of endemic statiionary states, depending on threshold parameters.

[1] Djidjou-Demasse, R., Burie, J. B., Ducrot, A., Lion, S., Richard, Q., Fabre, F. (2020). An epi-evolutionary model to predict spore-producing pathogens adaptation to quantitative resistance in heterogeneous environments. bioRxiv, 423467.

[2] Worsley-Tonks, K. E., Escobar, L. E., Biek, R., Castaneda-Guzman, M., Craft, M. E., Streicker, D. G., ... , Fountain-Jones, N. M. (2020). Using host traits to predict reservoir host species of rabies virus. PLoS neglected tropical diseases, 14(12), e0008940. *PLoS neglected tropical diseases.* 14(12), e0008940.

Why and when coalitions break down? An alternative differential game-based approach

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(1) Rennes School of Business, Rennes, France; (2) CNRS, Paris School of Economics, Paris, France; (3) Department of mathematics, Purdue University, USA; (4) University of Luxembourg, Luxembourg;

Consider a group of players initially members of a given coalition. More concretely, consider such a coalition in the context of environmental agreements where initially members agree to manage cooperatively the public bad, the stock of pollution. As a shortcut to the constitutional aspects of the coalition, we assume that each country enters the coalition with a given fixed share of the (intertemporal) payoff of the coalition. A key assumption is that this constitutional weight is independent of the technological level (productivity in a broad sense) of the countries, in order to disentangle the implications of pure constitutional vs technological/economic effects. We allow indeed for constitutional and technological heterogeneities across countries. We further study under which conditions a given country of the initial coalition may eventually split at a given finite time T. If a country splits, a non-cooperative game sets in between the country and the group of countries remaining in the coalition. Within a full linear-quadratic

model, we characterize the optimal affine Markovian subgame perfect strategies for a given split time T. We later solve for the whole sequence (starting with the initial cooperative game phase) and uncover the conditions under which splitting occur at finite time. In particular, we study in depth the splitting time decision and the determinants of the duration of the coalition. Particular attention is paid to the role of technological vs constitutional heterogeneity across players.

41. Optimal control in production, logistics and marketing 08:30 – 09:20

***Chair*: G. Feichtinger, A. Seidl** **FH HS 7**

Panic buying: consumer stockpiling and the potential effect of regulations

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In the media, panic buying featured as one of the top headlines at the outset of the COVID-19 pandemic. This type of consumer behavior was clearly triggered by concern that supermarkets might run out of food and other basic goods. Could regulation or any other form of intervention mitigate panic buying? To study this question, we consider the dynamics of retailing operations in response to such behavior. Assuming that consumer stockpiling is triggered by a sudden drop in the level of retail stock, we show that the retailer will not necessarily employ all available resources to ease the situation. In particular, the optimal strategy for a profit maximizing retailer is to employ a "wait and see" (intentional scarcity) policy at the initial stage of panic buying. This stage is critical, as it could lead to stock inventories reaching a minimal level, which would in turn lead to major instabilities in the market. We show that income tax relief for retailers and the provision of subsidies to support overtime working are not useful tools; i.e., they do not mitigate panic buying and may not even lead to increases in the rate of supply. This is because these interventions induce counterproductive price cuts which serve to increase consumer stockpiling. On the other hand, tighter price-cap regulation does increase retail prices at the critical initial stage and decreases them thereafter. It also improves the supply rate, in contrast to the usual impact of imposing a price ceiling. However, price-cap regulation is a very delicate tool, which can only be used to a limited extend, ast he lower the cap, the less effective is the retailer's controlover consumer purchases.

Optimal offshore production policy under social and economic uncertainty

Alessandra Buratto (1), Luca Grosset (1), Maddalena Muttoni (1),

(1) Department of Mathematics "Tullio Levi-Civita", University of Padova, Padova, Italy;

We consider a firm that offshores part of its production to a foreign country. Although entailing transportation costs, production is cheaper abroad, and this allows setting lower unit prices.

At a random instant τ, due to social, political, or economic causes, a disruptive event occurs and provokes a rise in the costs associated to the offshoring.

The idea of this work is to investigate how the firm should prepare and adapt its offshoring policy to the regime shift. The problem is formulated and solved as a 2-stage optimal control problem with random switching time [1, 2].

[1] S. Wrzaczek, M. Kuhn, and I. Frankovic. Using age structure for a multi-stage optimal control model with random switching time. *Journal of Optimization Theory and Applications*, **184**(3):1065-1082, 2020.

[**2**] V. M. Veliov. Optimal control of heterogeneous systems: basic theory. *Journal of Mathematical Analysis and Applications*, **346**(1):227-242, 2008.

42. Infinite horizon optimal control and applications 09:40 – 10:55

***Chair*: N. Hayek, S. Pickenhain** **FH HS 8**

Infinite-dimensional multiobjective optimal control in continuous time

Naila Hayek (1), Hasan Yilmaz (2),

(1) CRED EA 7321, Université Panthéon-Assas Paris 2, Paris, France; (2) SAMM EA 4543, Université Paris 1 Panthéon Sorbonne, France;

This paper studies multiobjective optimal control problems in the continuous-time framework when the space of states and the space of controls are infinite-dimensional and with lighter smoothness assumptions than the usual ones. The paper generalizes to the multiobjective case existing results for single-objective optimal control problems in that framework. The dynamics are governed by difference equations and a finite number of terminal equality and inequality constraints are present. Necessary conditions of Pareto optimality are provided namely Pontryagin maximum principles in the strong form.

[**1**] J. Blot and H.Yilmaz, A generalization of Michel's result on the Pontryagin Maximum Principle, Journal of Optimization Theory and Applications, volume 183, p. 792-812, 2019.

Asymptotic stabilization of control constrained dynamical systems via optimal control - a duality based approach

Katharina Kolo (1), Sabine Pickenhain (2), Valeriya Lykina (2),

(1) Titus Research GmbH, Wildau, Germany;
(2) Mathematical Institute, Brandenburg University of Technology at Cottbus-Senftenberg, Cottbus, Germany;

The approach we follow here is based on optimal control techniques by Kalman/Letov. We consider an infinite horizon optimal control problem $(P)_\infty$, realizing the Lyapunov-exponential stability by the choice of corresponding state spaces as Weighted Sobolev spaces and the following regulator type objective.

$$\begin{aligned} J_\infty(x,u) &= \int_0^\infty \frac{1}{2}\left(x^T(t)Wx(t) + u^T(t)Ru(t)\right)e^{\beta t}dt \to \min! \\ x &\in W_2^{1,n}([0,\infty), e^{\beta t}), \quad \beta > 0 \end{aligned}$$

The presented work focuses on a dual-based treatment of the problem $(P)_\infty$. This dual problem, obtained by a Lagrange-type duality, leads to an infinite horizon variational problem, where the dual variables belong again to a weighted Sobolev space $W_2^{1,n}([0,\infty), e^{-\beta t})$ with a density function $e^{-\beta t}$. We succeeded to prove saddle point conditions as well as necessary optimality conditions for the dual problem. For the numerical treatment of the dual problem we present a direct pseudo-spectral method based on generalized Laguerre polynomials. Convergence results are proved for this numerical method. The application of the proposed method is illustrated by a Lotka-Volterra model with control constraints.

Linear programming estimates for Cesàro and Abel limits of value functions in optimal control

Vladimir Gaitsgory (1), Ilya Shvartsman (2),

(1) Department of Mathematics, Macquarie University, Sydney, Australia; (2) Department of Mathematics and Computer Science, Penn State Harrisburg, USA;

We consider infinite horizon optimal control problems with time averaging and time discounting criteria and derive estimates and representations for Cesàro and Abel limits of the value functions in the case when they depend on the initial conditions. We establish that these limits are bounded above by the optimal value of a certain infinite dimensional linear programming problem and that they are bounded from below by the optimal value of the corresponding dual problem. We show that Cesàro and Abel limits are equal if they are continuous functions of the initial condition, strengthening previously known results that require uniform convergence to ensure this equality. The obtained representations are used to derive optimality conditions for the long run average optimal control problem.

43. Data-driven and learning-based control

09:40 – 10:55

***Chair*: T. Faulwasser, K. Worthmann**

FH HS 5

Data-driven representations of stochastic LTI systems

Timm Faulwasser (1), Guanru Pan (1), Ruchuan Ou (1),

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Data-driven system descriptions based on the fundamental lemma by Willems et al. [1] are subject to continued research interest. Essentially, the lemma states that the trajectories of any controllable Linear Time Invariant (LTI) system can be represented without explicit knowledge of a state-space model. Specifically, provided persistency of excitation holds, over any finite horizon the manifest system behavior is contained in the column space of a Hankel matrix constructed from recorded input-output data.

However, to the best of the authors' knowledge, so far there appears to be no stochastic variant of the fundamental lemma. Hence, this note recalls the main results of a recent submission [2], wherein we present a stochastic fundamental lemma for LTI systems. To this end, we rely on Polynomial Chaos Expansion (PCE) which is an established method that can be applied in Markovian and non-Markovian settings. Its core idea is based on the observation that random variables can be regarded as elements of an $\mathcal{L}_2$ probability space and hence they admit representations in appropriately chosen polynomial bases [3]. This talk recapitulates main findings from [2] and illustrates the results with examples.

[1] Willems, Jan C., et al. "A note on persistency of excitation." Systems & Control Letters 54.4 (2005): 325-329.

[2] Pan, Guanru, Ruchuan Ou, and Timm Faulwasser. "On a Stochastic Fundamental Lemma and Its Use for Data-Driven MPC." arXiv preprint arXiv:2111.13636 (2021).

[3] Sullivan, Timothy John. Introduction to uncertainty quantification. Vol. 63. Springer, 2015.

Learning supported MPC with multiple tubes

Markus Kögel (1), Janine Matschek (2), Rolf Findeisen (2),

(1) Laboratory for Systems Theory and Automatic Control, Otto-von-Guericke-University Magdeburg, Germany;
(2) Control and Cyber-physical Systems Laboratory, TU Darmstadt, Germany;

Model predictive control (MPC) is a method to control multivariate systems subject to constraints, which solves at each sampling instant a finite horizon optimization problem. Often the underlying system dynamics are affected by disturbances and uncertainties, which are unknown at the time of the controller design. Utilizing worst case bounds on the disturbances/uncertainties allows, in principle, to guarantee specific closed loop properties such as stability, but might limit the performance. Therefore machine learning methods are frequently utilized to learn disturbances and their effect on the future system evolution in predictive control [3,4,5]. The aim of combining machine learning for disturbance estimation and MPC is to lower the conservatism related to worst case, robust controllers.

In this talk we outline how tube based MPC approaches utilizing multiple tubes [1,2], can be supported by machine learning to deal with a priori unknown uncertainties. We tackle problems of the form

$$x_{k+1} = f(x_k, u_k, w_k)$$

where x_k is the known state, u_k is the applied input and w_k describes unknown disturbance/noise. We show how machine learning can be utilized to learn different bounds for multiple tubes and how these multiple tubes can be used for efficient, robust, and less conservative online control with MPC.

[1] M. Kögel and R. Findeisen. Robust MPC with reduced conservatism by blending multiples tubes. In *Proc. American Control Conference*, 1949-1954, 2020.

[2] M. Kögel and R. Findeisen. Fusing multiple time varying tubes for robust MPC. In *Proc. IFAC World Congress*, pages 7137–7144, 2020.

[3] J. Matschek and Findeisen, R. Learning supported model predictive control for tracking of periodic references. In Learning for Dynamics and Control. In *Learning for Dynamics and Control, Proc. Machine Learning Research*, pages 511-520, 2021.

[4] L. Hewing, K.P. Wabersich, M. Menner, and Zeilinger, M.N. Learning-based model predictive control: Toward safe learning in control. In *Annual Review of Control, Robotics, and Autonomous Systems*, 3, pages 269-296, 2020.

[5] J. Matschek, A. Himmel, and R. Findeisen. Constrained learning for model predictive control in asymptotically constant reference tracking tasks. In *Proc. 19th IFAC Symposium for System Identification*, pages 244–249, 2021.

Very low-dimensional parametrizations of fluid flow for the use in nonlinear controller design

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The control of general nonlinear systems is a challenging task in particular for large-scale models as they occur in the semi-discretization of partial differential equations of, say, fluid flow. In order to employ methods from linear numerical algebra and linear control theory, one may embed the nonlinear system in the class of linear parameter varying (LPV) systems.

In this talk, we will discuss how convolutional neural networks can be used to design LPV approximations of incompressible Navier-Stokes equations. In view of a possibly low-dimensional approximations of the parametrization, we discuss the use of neural networks in a semi-discrete PDE context and compare their performance to other, more classical model reduction approaches for very low parameter dimensions.

If time permits, we will address how very low-dimensional parametrization can be used for detecting flow regimes and how this enables the use of nonlinear control techniques that, otherwise, would be out of reach for general semi-discrete flow models.

44. Mathematical models for the human impact on the environment 09:40 – 10:55

***Chair*: F. Gozzi, G. Fabbri, S. Faggian** **FH HS 6**

Optimal approaches to the control of an emerging pathogen

Andy Dobson (1), Cristiano Ricci (2), Mercedes Pascual (3), Ted Loch-Temzelides (4), Fausto Gozzi (5), Giorgio Fabbri (6), Raouf Boucekkine (7),

(1) Princeton University and Santa Fe Institute, USA; (2) Pisa University, Italy; (3) University of Chicago, USA; (4) Rice University, USA; (5) Luiss University, Roma Italy; (6) CNRS-GAEL, Grenoble, France; (7) CUT, Rennes School of Business, France;

Covid-19 is the most recent 'emerging pathogen' to cross from a wildlife reservoir into the human population and generate a major global pandemic. These events are becoming more frequent and have significant potential to disrupt the human economy through the mechanisms put in place to minimize transmission in the early stages of the epidemic. There are two main options available to control the spread of a novel pathogen: (1) locking down of a significant proportion of the economy and (2) testing and isolation of individuals confirmed to be infected. There is also always the 'laissez-faire' strategy of doing nothing. In this talk I will describe a set of hybrid epidemiological / economic models that the group of us have developed to compare the different strategies and their economic and demographic impact. Throughout we examine what should be done in the time before vaccines are available. Our principle goal is to develop a modelling framework that suggests approaches to epidemic control that minimize the number of deaths and the impact on the economy. The results suggest that while lockdown is effective at slowing the spread of the pathogen in the early stages of an epidemic outbreak, testing and isolation of exposed and potentially infectious individuals is considerably more effective and has a much smaller impact on the economy. In both cases delays at setting a response in place considerably increase both the number of deaths and the cumulative impact on the economy. Delay in setting control measures in place

ultimately leads to the costs of the control measures exceeding that of the damage caused by the 'laissez faire' approach. While our results are specifically derived for a Covid like pathogens, our model framework can be readily modified to examine emerging pathogens with different etiological characteristics. This would allow it to be used to developed polices for future emerging pathogens.

Climate change and the optimal design of international environmental policy in an heterogenous world

Raouf Boucekkine (1), Giorgio Fabbri (2), Fausto Gozzi (3), Ted Loch-Temzelides (4),

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We investigate the design of dynamic international climate policy taking into account several aspects of heterogeneity across countries. We use a dynamic integrated assessment model, incorporating climate damages in the spirit of Nordhaus. We build multi-country extensions in order to investigate the strategic implications posed by climate change. We study the relative effectiveness of a variety of transfers, including direct financial transfers, as well as technology sharing. Our framework allows us to compute both the Nash and the Stackelberg equilibria of the induced dynamic game. We compare planners' solutions to possible outcomes of non-cooperative Nash and Stackelberg games. The techniques we employ enhance tractability in the analysis of discrete-time dynamic games and allow us to compare several policies, including general transfers, transfers that target TFP, and transfers that target improvements in abatement technologies.

Sliding Modes in the Management of Renewable Resources

Anton Bondarev (1), Thorsten Upmann (2,3,4),

(1) International Business School Suzhou, Xi'an Jiaotong-Liverpool University, China; (2) Helmholtz-Institute for Functional Marine Biodiversity (HIFMB) Oldenburg, Germany; (3) University of Oldenburg, Oldenburg, Germany; (4) Bielefeld University, Bielefeld, Germany;

The presence of tipping points in ecological systems implies abrupt changes in the dynamics of the ecosystem. In these piecewise-smooth dynamical systems (PWS systems) sliding dynamics, i.e., dynamics on the switching boundary, have been reported for population models. However, the question whether or not, and if so under which conditions, sliding dynamics may occur in an *optimally controlled system* have not yet been studied. We explore this issue in a simple harvesting model with two regimes, and find that optimal sliding may occur if regular steady states do not exist. Hence, sliding dynamics may be part of an optimal policy.

45. Optimal control in production, logistics and marketing 09:40 – 10:55

***Chair*: G. Feichtinger, M. Kopel, A. Seidl** **FH HS 7**

Accumulation and Obsolescence of Research Knowledge

Gustav Feichtinger (1), Andreas Novák (2),

(1) ORCOS, Vienna University of Technology, Vienna, Austria; (2) BDA, University of Vienna, Vienna, Austria;

In this paper we study how the scientific production of a representative researcher develops over his/her career. Using Pontryagin's maximum principle we derive the optimal capital accumulation over the life cycle of a scientist. In particular, we are able to identify two regimes of human capital accumulation (say intensive versus normal) and to characterize their optimality depending on the initial human capital level. The paper includes a formal proof of the negative impact of the obscolescence rate as well as the discount rate on human capital production and we show that this reduction is largest at earlier ages.

The digital economy and advertising diffusion models: critical mass and the stalling equilibrium

Gustav Feichtinger (1), Dieter Grass (2), Richard F. Hartl (3), Peter M. Kort (4), Andrea Seidl (3),

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In the digital economy, it is frequently observed that products become more valuable the more people use it. To account for such network effects, we introduce a new diffusion equation in a dynamic model of the firm with the aim to obtain the advertising policy that maximizes firm profits. Also an advertising budget is introduced and we examine its effects.

Our main results are threefold. First, we find that introduction of the network effect leads to a critical mass of the sales level. Only if the sales are greater or equal than this level, it is optimal for the firm to have a substantial sales level in the long run. Second, introduction of an advertising budget could result in the existence of a Stalling Equilibrium. This new type of unstable equilibrium, where the solution can be abnormal, and at which the advertising amount is at its upper bound, serves as the critical mass threshold separating growth and decline. Third, the advertising policy is continuous in the sales level either when the discount rate is large and the rate of forgetfulness is low, when the Stalling equilibrium exists, or when the rate of forgetfulness is that high that the firm is always in decline. Otherwise, at the critical mass threshold the firm is indifferent between growth and decline where the initial advertising rate for the growth trajectory is significantly larger than in the decline case.

Optimal investment in the safety of autonomous vehicles: the effect of liability in mixed traffic scenarios

Sharon Di (1), Herbert Dawid (2), Peter M. Kort (3), Gerd Muehheusser (4),

(1) Department of Civil Engineering and Engineering Mechanics, Columbia University, New York; (2) Bielefeld University, Department of Business Administration and Economics, and Center for Mathematical Economics, Germany; (3) Department of Econometrics and Operations Research & CentER, Tilburg University, The Netherlands; (4) University of Hamburg, Department of Economics, and IZA, and CESifo;

We analyze the optimal investment of an autonomous vehicle (AV) producer into AV safety in a mixed traffic framework with endogenous demand for AVs. Consumers decide between purchasing an AV or a conventional car based on their (heterogeneous) preferences for the two types of products as well as on prices and on the respective expected liability payments they face. Increasing the AV safety does not only reduce the probability of accidents caused by AVs, but also helps to avoid accidents in interactions between AVs and conventional cars that would be caused by human drivers. We show that in such a setting under certain conditions an increase in AV safety might reduce the demand for AVs. In particular, we study the role of the allocation of liability between the producer and consumer in case of an AV caused accident for the occurrence of this phenomenon and analyze how the firm's optimal investment as well as the safety level and market share of AVs depends on the liability scheme.

46. Continuous optimization: theory and applications 11:10 – 12:25

***Chair*: R. Bot, A. Daniilidis** **FH HS 8**

Some convergence results about primal-dual algorithms in the presence of adjoint mismatch

Emilie Chouzenoux (1), Andrés Contreras (1), Jean-Christophe Pesquet (1), Marion Savanier (1),

(1) CVN, OPIS Inria, CentraleSupelec, Université Paris-Saclay, Gif-Sur-Yvette, France;

Many problems in signal/image processing and data science can be formulated as a penalized least squares optimization problem of the form:

$$\underset{x\in\mathcal{H}}{\text{minimize}} \ \frac{1}{2}\|y-Hx\|_{\mathcal{G}}^2 + f(x) + g(Dx), \tag{1}$$

where f and g are proper, convex and lower semicontinuous functions defined on real Hilbert spaces $\mathcal{H}$ and $\mathcal{L}$ respectively, and H and D are bounded linear operators on real Hilbert spaces. In recent years, primal-dual proximal splitting algorithms have appeared as efficient tools in solving (1) and its corresponding dual problem. The related iterations involve the adjoints H^* and D^* of H and D, respectively, and proximal steps performed on f and g. However, in some problems related to image processing, the exact computation of H^* and D^* is impossible or too computationally intensive. Inspired from the results obtained in [1] for an instance of the proximal-gradient algorithm, we present new convergence results for some primal-dual algorithms allowing to solve (1), in the presence of adjoint mismatch on both operators H^* and D^*. Our analysis includes a characterization of the fixed points corresponding to these mismatched primal-dual algorithms and conditions for converging to such fixed points. We also provide bounds on the error between the resulting limits and the exact solution to (1). Finally, we illustrate our theoretical results with image reconstruction examples in X-ray tomography.

[1] E. Chouzenoux, J.-C. Pesquet, C. Riddel, M. Savanier and Y. Trousset. Convergence of Proximal Gradient Algorithm in the Presence of Adjoint Mismatch. *Inverse Problem*, **37**(6): 065009, 2021.

Optimality conditions and regularization for stochastic optimization with almost sure state constraints

Caroline Geiersbach (1), Michael Hintermüller (1), Winnifried Wollner (2),

(1) RG8: Nonsmooth Variational Problems and Operator Equations, Weierstrass Institute, Berlin, Germany; (2) Mathematics Department, Technische Universität Darmstadt, Darmstadt, Germany;

In this talk, we present necessary and sufficient optimality conditions for convex stochastic optimization problems subject to almost sure equality and conical constraints. We refine classical results by Rockafellar and Wets from two-stage stochastic optimization to include states belonging to the Bochner space of essentially bounded random variables with images in a reflexive and separable Banach space. Under certain conditions, the optimality conditions given are necessary and sufficient. Lagrange multipliers exhibit different regularity depending on whether or not the assumption of relatively complete recourse is satisfied. We propose a Moreau–Yosida regularization for such problems and show consistency of the optimality conditions for the regularization problem as the regularization parameter is taken to infinity. Algorithmic approaches using stochastic approximation are discussed and an application to PDE-constrained optimization under uncertainty is presented. This talk is based on the publications [1] and [2].

[1] C. Geiersbach and W. Wollner. Optimality conditions for convex stochastic optimization problems in Banach spaces with almost sure state constraints. *SIAM Journal on Optimization*, **31**(4): 2455-2480, 2021.

[2] C. Geiersbach and M. Hintermüller. Optimality conditions and Moreau–Yosida regularization for almost sure state constraints. *WIAS Preprint no. 2862.* 2021.

An adaptive trust-region method without function evaluations

Geovani N. Grapiglia (1), Gabriel F.D. Stella (2),

(1) Université catholique de Louvain, INMA, Louvain-la-Neuve, Belgium; (2) Universidade Federal do Paraná, PPGM, Curitiba, Brazil;

In this talk we present an adaptive trust-region method for smooth unconstrained optimization. The update rule for the trust-region radius relies only on gradient evaluations. Assuming that the gradient of the objective function is Lipschitz continuous, we establish worst-case complexity bounds for the number of gradient evaluations required by the proposed method to generate approximate stationary points. As a corollary, we establish a global convergence result. We also present numerical results on benchmark problems. In terms of the number of calls of the oracle, the proposed method compares favorably with trust-region methods that use evaluations of the objective function.

47. Numerical analysis for PDE constrained optimization 11:10 – 12:25

***Chair*: J. Pfefferer, A. Rösch** **FH HS 5**

Optimal control of semilinear parabolic equations – Lipschitz stability of local minimizers and an example of their non-uniqueness

Eduardo Casas (1), Fredi Tröltzsch (2),

(1) Universidad de Cantabria, Santander, Spain; (2) Technische Universität Berlin, Berlin, Germany;

We consider a distributed optimal control problem for a semilinear parabolic partial differential equation with homogeneous Neumann boundary condition. First, we discuss results on the stability of locally optimal solutions with respect to perturbations of the initial data. Based on different types of sufficient optimality conditions for a local solution of the unperturbed problem, Lipschitz or Hölder stability with respect to the perturbations are presented. Second, an example with semilinear equation, constant initial data, and standard quadratic tracking type objective functional is constructed that has at least two different locally optimal solutions. By our perturbation analysis, the existence of a problem with non-constant initial data is shown that also has at least two different locally optimal solutions.

A priori error estimates for finite element discretizations of Transient Stokes optimization problems with pointwise state constraints in time

Boris Vexler (1), Dmitriy Leykekhman (2),

(1) Technical University of Munich, Munich, Germany; (2) University of Connecticut, USA;

We consider the following optimal control problem

$$\text{Minimize} \quad \frac{1}{2}\int_0^T \int_\Omega |\mathbf{u}(x,t) - \hat{\mathbf{u}}(x,t)|^2 \; dxdt + \frac{\alpha}{2}\int_0^T \int_\Omega |\mathbf{q}(x,t)|^2 \; dxdt$$

subject to transient Stokes equation

$$\begin{aligned} \partial_t \mathbf{u} - \Delta \mathbf{u} + \nabla p &= \mathbf{f} + \mathbf{q} && \text{in } (0,T]\times\Omega, \\ \nabla \cdot \mathbf{u} &= 0 && \text{in } (0,T]\times\Omega, \\ \mathbf{u} &= \mathbf{0} && \text{on } (0,T]\times\partial\Omega, \\ \mathbf{u}(0) &= \mathbf{u}_0 && \text{in } \Omega. \end{aligned}$$

and to state constraints:

$$\int_\Omega \mathbf{u}(x,t)\cdot\omega(x)dx \le b \quad \text{in} \quad [0,T],$$

for a given $\omega(x) \in L^2(\Omega)^d$ and $b \in \mathbb{R}$. We assume that Ω is a bounded convex polygonal or polyhedral domain in $\mathbb{R}^d$, $d = 2, 3$.

We discretize the problem with finite elements satisfying the discrete inf-sup condition in space and discontinuous piecewise constant Galerkin methods in time and the control $\mathbf{q}$ is discretized by piecewise constant functions. We establish quasi-optimal *a priori* error estimate for the fully discrete error of the control and state. To obtain such results we employ discrete maximal parabolic regularity results for the transient Stokes problem to obtain best type *a priori* error estimates of the fully discrete error in $L^\infty([0,T]; L^2(\Omega)^d)$ norm. In my talk I will discuss the above results and main ideas of the proof.

H^{curl} conformal shape optimization

Sven Beuchler (1), Philipp König (1),

(1) IfaM, Leibniz University of Hanover, Hanover, Germany;

This talk provides an introduction into H^{curl} based shape optimization and the necessary transformations, which are used for deformation and design purposes of models described by Maxwell's Curl-Curl-Problem

$$\nabla \times \nabla \times u - \kappa^2 u = f.$$

The process can be utilized to determine the optimal shape and structure of optical wave guides given a desired optimality condition. Numerical simulations and results are provided based on the optimization of a Y-beamsplitter.

48. Dynamic games and applications 11:10 – 12:25

***Chair*: S. Wrzaczek** **FH HS 6**

Optimal international policy with R&D-based growth and the risk of global environmental disaster

Tapio Palokangas (1),

(1) HeGSE, University of Helsinki, Helsinki, Finland;

Optimal policy is examined in a union of regions that produce goods from capital, labor and energy and perform R&D by specific labor. R&D generates economic growth and the extraction of energy contributes to the stock of pollution, increasing the risk of welfare-damaging environmental disaster.

The union-wide social planner can take the risk of disaster into account, but the regional governments are small enough to ignore it. The model is organized as a differential game where the social planner is the Stackelberg leader and the regional governments are the Stackelberg followers. The leader's behavior is solved by dynamic programming with a stochastic regime shift, but the follower's by dynamic programming.

The results are the following. The social optimum can be decentralized by imposing a precautionary tax on R&D and a precautionary subsidy to investment. The former is optimally equal to the marginal cost of R&D times the present value of the expected social loss due to the disaster, and the latter to the marginal cost of investment times the present value of the expected social loss due to the disaster. Energy or emissions should not be taxed or subsidized. The risk of the disaster decreases the socially optimal growth rate.

Differential game of renewable resources with sliding modes and hybrid limit cycles

Anton Bondarev (1), Thorsten Upmann (2),

(1) IBSS, Xi'an Jiaotong-Liverpool University, Suzhou, P. R. China.; (2) HIFMB, University of Oldenburg, Oldenburg, Germany.;

We explore the effects of multiple regimes in a standard renewable resource optimal harvesting model. We consider cooperative and non-cooperative solutions to the game with $n > 1$ fisheries. We argue that

thresholds of the state variable may yield tipping points, and show that these thresholds may result in optimal policies where harvesting takes place on the boundary; in this case, the *sliding mode* may emerge. We consider the switches of the growth rate of the stock at the threshold as defining the non-smooth nature of the problem. We observe, under certain conditions, the sliding mode as the equilibrium outcome (similar to the optimal control problem).

We next prove the necessary conditions for the crossing hybrid limit cycle trajectory to be the optimal (Nash) outcome of the game. This novel result uses Hamiltonians' continuity and Stoke's theorem as main tools for the proof. We shortly discuss environmental and policy implications of these novel types of dynamics.

On discrete-time approximations to infinite horizon differential games

Javier De Frutos (1), Víctor Gatón (1), Julia Novo (2),

(1) IMUVA, Universidad de Valladolid, Spain; (2) Dept. Matemáticas, Universidad Autónoma de Madrid, Spain;

In this paper we study a discrete-time semidiscretization of an infinite time horizon noncooperative N-player differential game. We prove that as the discretization time step approaches zero the discrete-time value function approximate the value function of the differential game. Furthermore, the discrete-time Nash equilibrium is an ε-Nash equilibrium for the continuous-time differential game.

Value of information and cooperation in a differential game of pollution control

Ekaterina Gromova (1), Ekaterina Marova (1),

(1) Baltic International Academy, Riga, Latvia;

In the presentation we consider cooperative differential game of pollution defined in [1]. First, we focus on evaluating how profitable it is to create coalitions or play individually for players. To characterize the profit from entering a coalition we introduce the value of cooperation and normalized value of cooperation [2]. Further, we study the influence of knowing accurate information about a certain game parameter and characterize the accuracy of this information by the value of information and normalized value of information [3]. These characteristics were introduced in [4], where the unknown parameter was assumed to be the initial state (level of emissions). In this work we study the influence of the absorption coefficient corresponding to the natural purification of the atmosphere.

The theoretical results are supported by the numerical evidence obtained for real data from the city of Penza (Russia).

[1] Haurie, A., Zaccour, G. Differential game models of global environmental management. *Annals of Dynamic Games*, Boston, 1994 , pp. 124-132.

[2] Chebotareva, A., Su, S., Voronina, E. and Gromova, E. Value of cooperation in a differential game of pollution control, to be published in LNCS, 2022.

[3] Tur A, Gromova E, Gromov D. On the Estimation of the Initial Stock in the Problem of Resource Extraction. *Mathematics*, **9(23)**:3099, 2021.

[4] Chebotareva A., Su S., Tretyakova S., Gromova E. On the value of the preexisting knowledge in an optimal control of pollution emissions, *Contributions to Game Theory and Management*, **14**, pp.49-58, 2021.

49. Dynamic Games in Industrial Organization — 11:10 – 12:25

Chair: **M. Kopel** — **Saal**

On Uniqueness of Linear Markov Perfect Equilibria in Linear-Quadratic Differential Games

Markus Eigruber (1), Franz Wirl (2),

(1) Vienna University of Technology, Institute of Management Science, Vienna, Austria; (2) University of Vienna, Faculty of Business, Economics and Statistics, Vienna, Austria;

Even tough the issue of multiple nonlinear equilibria in linear-quadratic, differential games is well known (see the seminal paper of Tsutsui & Mino, 1990, on sticky prices and Dockner & van Long, 1993, on a dynamic tragedy of the commons) the possibility of multiple *linear* Markov perfect equilibria (LMPE) is, by and large, ignored in the literature. And indeed, almost all papers confined to a single state (a very large majority of the application of differential games to economic problems) identify a unique LMPE. This paper explains the finding of uniqueness, defines necessary conditions for multiplicity and discusses the two counterexamples found in the literature (Lockwood, 1996 and Engwerda, 1999f). We derive our conclusions from an analysis of the phase plane in the state and the derivative of the value function as well as an analysis of the respective auxiliary system. The two (pathological) examples from the literature are arithmetical in nature only and are not derived from an underlying economic problem which we provide. In our IO-setting of learning by doing multiple symmetric LMPEs naturally arise. Our game leads, in contrast to the above mentioned ones, to different long run outcomes but require a higher dimensional state space.

Environmental and economic consequences of a market with green and brown firms

Francisco Cabo (1), Lucia Sbragia (2),

(1) IMUVa, Universidad de Valladolid, Valladolid, Spain; (2) Department of Economics and Finance, Durham University Business School, Durham, UK;

In this paper we consider an industry where firms which compete à la Cournot are divided into green and brown firms. Firms are motivated to be green by the willingness of consumers to pay a premium price for green products. However being green requires some "sacrifice" compared to the business as usual: a production practice more considerate of the environment and/or an attitude towards firms' contribution to the global environmental damage caused by the their production activity. In particular, we consider three different definitions of being green. In the first case, individual green firms adopt a cleaner but more expensive production technology (Green Tech); in the second case, green firms further to adopting a more expensive and less polluting technology form a cartel and control the price (Green Cartel); finally, green firms also recognize that, as a group, they have an impact on the accumulated pollution (Green Club).

Firms can change behavior over time based on the relative performance of the other group. This performance is defined by the value function of a differential game played from this given time on playing either like green or brown firm. We first analyze the conditions under which different equilibrium market compositions can be observed, namely only brown, only green, or a mix of brown and green firms. We then look at the environmental and economic consequences of each of the three possible market composition and compare them for the three different definitions of being green to understand which "type" of green is better and under what conditions.

On the interplay between environmental CSR practices, green innovation and pollution tax

Gianluca Iannucci (1), Alessandro Tampieri (1),

(1) Department of Economics and Management, University of Firenze, Florence, Italy;

This paper analyses the long run market structure in an industry where production is polluting and firms may be profit seeking (PS) or adopt environmentally and socially responsible practices (ECSR). To control for pollution, the government sets an environmental tax on production, while firms may invest in innovation to abate pollution. Firms' behaviour (either PS or ECSR) is endogenously determined through an indirect evolutionary approach. Our findings show that the market size plays a role at explaining the industry configuration. In particular, ECSR firms are favoured when the market is large. In the analysis of welfare, a trade-off emerges between the level of taxation and the share of ECSR firms in the industry.

50. Plenary

14:00 – 14:50

***Chair*: A. Daniilidis**

FH HS 8

Variational convexity at a local minimum in nonconvex optimization

R. Tyrrell Rockafellar (1),

(1) Department of Mathematics, University of Washington, Seattle, USA;

Variational convexity of a function is a primal-dual localized property according to which the function values and associated subgradients can't be distinguished from those coming from a convex function. With its strong version, mimicking behavior of a strongly convex function, it can hold even when the effective domain isn't convex locally. That leads to key insights into sufficient conditions for local optimality that promote numerical methodology. In classical nonlinear programming, for instance, the "strong second-order sufficient condition" corresponds to strong variational convexity of an extended-real-valued function that incorporates canonical perturbation variables into the objective and dualizes to the usual augmented Lagrangian. It is equivalent then to a characterization of optimality in terms of that Lagrangian having a local saddle point which combines strong convexity in the primal variable with concavity in the dual variable and effectively reduces the nonconvex optimization to convex optimization in a local primal-dual sense.

That paradigm has been shown moreover to generalize to finite-dimensional problem models far beyond just nonlinear programming. Whether it can extend somehow to problems in optimal control and other applications, is tantalizing but not yet known.

51. Continuous optimization: theory and applications 15:10 – 16:25

***Chair*: R. Bot, A. Daniilidis** **FH HS 8**

Necessary and sufficient conditions for existence of quadratic Lyapunov functions in first-order methods

Manu Upadhyaya (1), Sebastian Banert (1), Pontus Giselsson (1),

(1) Deptartment of Automatic Control, Lund University, Lund, Sweden;

Two methodologies for automated algorithm analysis, the performance estimation framework (PEP) and the integral quadratic constraints framework (IQC), have recently been instrumental in advancing the understanding and performance of first-order methods. In this work, we propose a methodology that takes inspiration from the tightness results in the PEP framework and the general algorithm formulation of the IQC framework to show necessary and sufficient conditions for the existence of a quadratic Lyapunov function for first-order methods. We show that such a Lyapunov function exists if and only if a small scale convex semi-definite program is feasible. A key strength of our method is that it is easily adaptable to a wide range of algorithms. The methodology is constructive in the sense that it also provides a quadratic Lyapunov function in the case it exists.

Escaping limit cycles: Global convergence for constrained nonconvex-nonconcave minimax problems

Thomas Pethick (1), Puya Latafat (2), Panagiotis Patrinos (2), Olivier Fercoq (3),
Volkan Cevher (1),

(1) Laboratory for Information and Inference Systems (LIONS), EPFL, Lausanne, Switzerland; (2) Department of Electrical Engineering (ESAT-STADIUS), KU Leuven, Leuven, Belgium; (3) Laboratoire Traitement et Communication d'Information, Télécom Paris, Institut Polytechnique de Paris, Paris, France;

We introduce a new extragradient-type algorithm for a class of nonconvex-nonconcave minimax problems. It is well-known that finding a local solution for general minimax problems is computationally intractable. This observation has recently motivated the study of structures sufficient for convergence of first order methods in the more general setting of variational inequalities when the so-called *weak Minty variational inequality* (MVI) holds. This problem class captures non-trivial structures as we demonstrate with examples, for which a large family of existing algorithms provably converge to limit cycles. Our results require a less restrictive parameter range in the weak MVI compared to what is previously known, thus extending the applicability of our scheme. The proposed algorithm is applicable to constrained and regularized problems, and involves an adaptive stepsize allowing for potentially larger stepsizes. Our scheme also converges globally even in settings where the underlying operator exhibits limit cycles.

Conjugate dualities for relative smoothness and strong convexity under the light of generalized convexity

Emanuel Laude (1), Andreas Themelis (2), Panagiotis Patrinos (1),

(1) ESAT, KU Leuven, Belgium; (2) ISEE, Kyushu University, Fukuoka, Japan;

Relative Bregman smoothness and strong convexity have recently gained considerable attention in optimization. However, conjugate dualities for Bregman smoothness and strong convexity remain an open problem as noted earlier by Lu, Freund, and Nesterov, *Relatively smooth convex optimization by first-order methods, and applications*, SIAM Journal on Optimization, 28(1):333–354, 2018. In this paper we address this question by introducing the notions of relative anisotropic strong convexity and smoothness as the respective dual counterparts of Bregman smoothness and strong convexity. In essence, the duality holds between tilt- and shift-parametrized families of upper and lower bounds and can thus be examined under the light of generalized convexity. In the Euclidean case this specializes to the well-known conjugate duality between Lipschitz smoothness and strong convexity. The two notions here introduced can be thought of as anisotropic generalizations of the well-known descent lemma and the strong convexity subgradient inequality. Alternatively, in the context of generalized convexity these characterizations can be interpreted as generalized subgradient inequalities. In the Euclidean case, the class of strongly convex functions can be described in terms of pointwise maxima over quadratics with uniform curvature. Surprisingly, in contrast, the class of anisotropically strongly convex functions, in general, only forms a proper subset of the corresponding class of pointwise maxima, unless a certain saddle-point property holds. Aside from the Euclidean case, this saddle-point property is shown to hold automatically in the one-dimensional or the essentially smooth case.

52. Data-driven and learning-based control — 15:10 – 16:25

***Chair*: T. Faulwasser, K. Worthmann** — **FH HS 5**

Data-driven modelling of nonlinear dynamics by barycentric coordinates and memory

Niklas Wulkow (1), Péter Koltai (2), Vikram Sunkara (1), Christof Schütte (1),

(1) Zuse Institute Berlin, Germany; (2) Freie Universität Berlin, Germany;

We present a method to model dynamical systems from data. We use the recently introduced method Scalable Probabilistic Approximation (SPA) to project points from a Euclidean space to convex polytopes and represent these projected states of a system in new, lower-dimensional coordinates denoting their position in the polytope. We then introduce a specific nonlinear transformation to construct a model of the dynamics in the polytope and to transform back into the original state space. To overcome the potential loss of information from the projection to a lower-dimensional polytope, we use delay-embedding. By construction, our method gives linear dynamics in the nonlinear feature space, and it produces stable models. We illustrate the capacity of the method to reproduce even chaotic dynamics and attractors with multiple connected components on various examples.

Online convex optimization for control of dynamical systems

Marko Nonhoff (1), Matthias A. Müller (1),

(1) Leibniz University Hannover, Institute of Automatic Control, Hannover, Germany;

In this talk, we study the application of online convex optimization (OCO) to the problem of controlling a linear time-invariant system. In OCO, the goal is to design an algorithm which minimizes the sum of time-varying cost functions $\sum_{t=0}^{T} L_t$ over a finite horizon T, where the cost functions L_t are a priori unknown to the algorithm. Specifically, at every time instance t, the algorithm only has access to the previous cost functions $L_0, \dots, L_{t-1}$. However, no underlying dynamical system is considered in the existing standard OCO literature. We apply the OCO framework to control a dynamical system subject to a priori unknown time-varying cost functions, which commonly arise in practice, e.g., due to changing energy prices. To this end, we propose a controller based on OCO [1] and make use of the notion of regret, i.e., the difference between the controller's closed-loop performance and the optimal closed-loop cost in hindsight when knowing all cost functions, to obtain performance guarantees for the closed loop. Moreover, we extend our results to systems subject to input and state constraints [2] as well as unknown systems with output feedback [3]. Finally, we present a numerical example to illustrate our algorithm's closed-loop performance.

[1] M. Nonhoff and M. A. Müller. Online Gradient Descent for Linear Dynamical Systems. *IFAC-PapersOnLine: 21st IFAC World Congress*, 2020, pp. 945-952.

[2] M. Nonhoff and M. A. Müller. An online convex optimization algorithm for controlling linear systems with state and input constraints. *Proc. of the 2021 merican Control Conference (ACC)*, 2021, pp. 2523-2528.

[3] M. Nonhoff and M. A. Müller. Data-driven online convex optimization for control of dynamical systems. *Proc. of the 2021 60th IEEE Conference on Decision and Control (CDC)*, 2021, pp. 3640-3645.

Finite-data error bounds for Koopman-based control

Feliks Nüske (1), Sebastian Peitz (2), Friedrich Philipp (3), Manuel Schaller (3), Karl Worthmann (3),

(1) Max Planck Institute for Dynamics and Complex Systems, Magdeburg, Germany; (2) Data Science for Engineering, University of Paderborn, Germany; (3) Optimization-based Control, Technische Universität Ilmenau, Germany;

The Koopman operator has become an essential tool for data-driven approximation of dynamical (control) systems via extended dynamic mode decomposition. We provide first probabilistic error bounds for the approximation of control-affine systems by means of bilinear data-driven surrogate models with finite-data both for ordinary and stochastic control systems while using either ergodic trajectories or i.i.d. samples. We demonstrate the effectiveness of this bilinear approach by means of numerical examples with a Duffing oscillator and an Ornstein-Uhlenbeck process.

53. Dynamic games in environmental and resource economics 15:10 – 16:25

***Chair*: L. Lambertini** **FH HS 6**

Public policy design and common property resources: A social network approach

Jorge Marco (1), Renan Goetz (2),

(1) Universidad de los Andes, Bogotá, Colombia; (2) University of Girona, Girona, Spain;

This paper analyzes the influence that social networks and the state of a common property resource have on compliance with social norms. The analytical framework allows to signpost the maximal and minimal influence different types of social networks have for supporting full cooperation, thereby allowing policy-makers to compare the effectiveness of legal (e.g., fines or subsidies) and informal (e.g., network-oriented) enforcement policies. The study also shows to what extent social networks allow reducing the stringency of legal enforcement policies. Yet, cooperation is unlikely to prevail if the agents do not perceive the scarcity of the common property resource as severe. We find that stable steady-states where compliers and defectors coexist (partial cooperation) are supported and their stability depends on the type of functions representing social pressure and sanctioning costs respectively. The analytical framework is employed for a numerical study that is empirically calibrated for an aquifer in Spain. The study shows that depending on the type of network and the scarcity of the resource the critical mass of compliers for cooperation can be reduced, but at most only by 20-25%. Similarly, the necessary number of defectors to be targeted by legal enforcement can be cut in the two-digit percent range. Moreover, it shows that subsidizing the compliers' sanctioning costs may impede to attain a steady state based on partial cooperation.

Related literature:

[1] Calvó-Armengol, A. and M. O. Jackson. Peer Pressure. *Journal of the European Economic Association*, **8(1)**: 62-89, 2010.

[2] C. Hilbe, S. Šimsa, K. Chatterjee and M. Nowak. Evolution of cooperation in stochastic games. *Nature*, **559**: 246-249, 2018.

[3] G. Tabellini. The Scope of Cooperation: Values and Incentives. *Quarterly Journal of Economics*, **123**: 905-950, 2008.

Collective self-restraint in dynamic common-pool resource games through voting

Matthias Hettich (1), Simon G. Haastert (2),

(1) ECDF, Technical University Berlin, Berlin, Germany; (2) Institute for Econometrics and Economic Statistics, University of Münster, Münster, Germany;

The rapid degradation of natural common-pool resources such as common fisheries, rain forests or the global greenhouse gas budget is one of the greatest threats to human wellbeing. Static, non-cooperative models in game theory like the Prisoner's dilemma predict that such non-excludable, finite resources are exposed to socially inadequate appropriation strategies - a phenomenon often termed the "tradegy of commons". However, the extension of these models to a dynamic setting shows that cooperation between multiple agents is possible. In fact, many lab-experiments and real life case studies confirm that cooperation in terms of a sustainable use of a common-pool resource is feasible. Certain conditions like reciprocal

punishment or reward mechanisms, preferences such as inequity-aversion or ways of communication can facilitate cooperation. We propose voting on a per-unit tax rate as another instrument to enable multiple agents to coordinate their appropriation efforts. In order to model the complex social interdependencies in a dynamic setting we choose an agent-based model. Agents act individually in a model-free, partially observable Markov game in a trial-and-error-fashion. The agents are trained via a deep reinforcement algorithm that allows a continuous state and action space. They influence each other by reducing the common-resource stock and by voting on a tax rate. Further, individual appropriation efforts have an impact on the resource price that is determined via market clearing with an exogenous demand. In this setting, agents are able to indirectly punish agents for above-average use and overexploitation of the common-pool resource by voting for a non-zero tax rate. We find that introducing the tax voting mechanism facilitates cooperative behavior and improves both resource sustainability and social welfare.

Sharing profit in a partnership: an optimal control approach

Eugene Khmelnitsky (1), Yigal Gerchak (1),

(1) Dept. of Industrial Engineering, Tel Aviv University, Tel Aviv, Israel;

The setting up of a new partnership involves negotiation. Would-be partners must agree on a scheme for dividing uncertain future profits (or losses). We consider partnerships of two or more partners where the negotiated division depends on the partners' attitudes toward risk, their beliefs concerning future profit, and their alternatives (i.e., the disagreement point). We propose two schemes. First, an asymmetric approach starts with one party making a decision that maximizes its expected utility that respects the other's individual rationality [1]. The second scheme is symmetric and based on Nash bargaining solution. We use the optimal control theory and numerical methods based on the maximum principle in order to formulate the problem and obtain the optimal contract terms. We show that, in general, the optimal shares can exhibit non-linear and even non-monotone-increasing behavior. We focus on particular cases where the optimal contract takes the form of a linear or piece-wise linear function of future profit, and show the advantage of the optimal contract compared to a best linear contract.

[1] Y. Gerchak, and E. Khmelnitsky. Partnership's profit sharing: linear and nonlinear contracts. *International Game Theory Review*, **21**:1940008, 2019.

54. Economic dynamics with regime switching

15:10 – 16:25

***Chair*: J. Haunschmied, W. Semmler**

FH HS 7

Delayed monetary policy effects in a multi-regime cointegrated VAR (MRCIVAR)

Pu Chen (1), Willi Semmler (2), Helmut Maurer (3),

(1) Melbourne Institute of Technology, Australia ; (2) New School for Social Research, New York, USA; (2) University of Bielefeld, Germany; (2) IIASA, Laxenburg; (3) Department of Applied Mathematics, Münster, Germany,;

The effectiveness of monetary policies under delayed policy impacts is explored. Initially the monetary policy efficacy in the context of a differential delay system of the macro-finance link is investigated. The nonlinear macro system with delays gives rise to a time-delayed optimal control problem. The optimality

conditions are analyzed, and the control problem is numerically solved by discretization and optimization methods. The solutions suggest that with too long a delay destabilizing financial conditions may emerge, rendering the policy ineffective. The possibility of asymmetric adjustments to some long-run steady state in a non-stationary environment using Multi-Regime-Cointegration-VAR (MRCIVAR) for an interest rate cut and a non-rate-cut regime is explored. Though the rate cuts may not perform well with too long of a delay, given diverse shocks, monetary policy still performs better in a rate cut than a non-rate cut regime. Given the perils of deteriorating financial conditions, the better stabilization properties in a rate cut regime are empirically validated through data for European countries and the US.

The irreversible pollution game

Raouf Boucekkine (1), Weihua Ruan (2), Benteng Zou (3),

(1) Rennes School of Business, France ; (2) Purdue University Northwest, USA; (3) University of Luxembourg, Luxembourg;

We study a 2-country differential game with irreversible pollution. Irreversibility is of a *hard* type: above a certain threshold level of pollution, the self-cleaning capacity of Nature drops to zero. Accordingly, the game includes a non-concave feature, and we characterize both the cooperative and non-cooperative versions with this general non-LQ property. We deliver full analytical results for the existence of Markov Perfect Equilibria. We first demonstrate that when pollution costs are equal across players (symmetry), irreversible pollution regimes are more frequently reached than under cooperation. Second, we study the implications of asymmetry in the pollution cost. We find far nontrivial results on the reachability of the irreversible regime. However, we unambiguously prove that, for the same total cost of pollution, provided the irreversible regime is reached in both the symmetric and asymmetric cases, long-term pollution is larger in the symmetric case, reflecting more intensive free-riding under symmetry.

(In memory of Ngo van Long.)

Titans that clash and a state that buffers

Serhat Dogan (1), Emin Karagozoglu (1), Kerim Keskin (2), Cagri Saglam (1),

(1) Bilkent University, Ankara, Turkey; (2) ADA University, Baku, Azerbaijan;

We present a game-theoretic approach to the analysis of the emergence or survival of buffer states. We analyze a two-stage game with three players orderly located on a linear territory, where the player in the middle is passive, and the players on the two ends are aggressive with options to declare war against the others. We conduct an equilibrium analysis and characterize the conditions under which the passive player acts as a buffer state between the aggressive players. We find various equilibrium outcomes, which can be grouped into the following categories: (i) peace with buffer, (ii) peace without buffer, and (iii) the last man standing. Our comparative static analyses reveal valuable insights regarding the factors affecting the existence of buffer states.

55. Continuous optimization: theory and applications — 16:40 – 17:55

***Chair*: R. Bot, A. Daniilidis** — **FH HS 8**

Recent advances in global convergence of Newton method

Konstantin Mishchenko (1),

(1) ENS-CNRS-INRIA, Paris, France;

In the talk, we will cover several recent results on global convergence of Newton method. All methods discussed in the talk will be related to the following update rule:

$$x^{k+1} = x^k - (\nabla^2 f(x^k) + \lambda_k \mathbf{I})^{-1} \nabla f(x^k),$$

where x^k is the algorithm's iterate, $\lambda_k > 0$ is a regularization parameter and $\mathbf{I}$ is the identity matrix. We will star the discussion with a recent result on global convergence of this algorithm for convex objectives when λ_k is chosen as $\lambda_k = \sqrt{H\|\nabla f(x^k)\|}$ and the Hessian of f is $(2H)$-Lipschitz. We will then move on to discuss generalizations of this scheme to monotone operators and more general smoothness conditions. We will also touch upon other choices of λ_k that can potentially lead to faster convergence. Finally, we will discuss potential future directions.

Two steps at a time — taking GAN training in stride with Tseng's method

Axel Böhm (1), Michael Sedlmayer (2), Ernö Robert Csetnek (1), Radu Ioan Boţ (1),

(1) Faculty of Mathematics, University of Vienna, Vienna, Austria; (2) Research Network Data Science @ Uni Vienna, University of Vienna, Vienna, Austria;

Motivated by the training of Generative Adversarial Networks (GANs), we study methods for solving minimax problems with additional nonsmooth regularizers. We do so by employing *monotone operator* theory, in particular the *Forward-Backward-Forward* method, which avoids the known issue of limit cycling by correcting each update by a second gradient evaluation and does so requiring less projection steps compared to the Extragradient method in the presence of constraints. Furthermore, we propose a seemingly new scheme which recycles old gradients to mitigate the additional computational cost. This way we rediscover a known method, related to the *Optimistic Gradient Descent Ascent* method. For both schemes we prove novel convergence rates for convex-concave minimax problems via a unifying approach. The derived error bounds are in terms of the gap function for the ergodic iterates. For the deterministic and the stochastic problem we show a convergence rate of $\mathcal{O}\left(\frac{1}{k}\right)$ and $\mathcal{O}\left(\frac{1}{\sqrt{k}}\right)$, respectively. We complement our theoretical results with empirical improvements in the training of Wasserstein GANs on the CIFAR10 dataset.

Hessian barrier algorithms for non-convex conic optimization

Pavel Dvurechensky (1), Mathias Staudigl (2),

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(2) Maastricht University, Department of Data Science and Knowledge Engineering, P.O. Box 616, NL–6200 MD Maastricht, The Netherlands;

We consider the minimization of a continuous possibly non-convex function over the intersection of a regular cone with an affine set via a new class of adaptive first- and second-order optimization methods, building on the Hessian-barrier techniques introduced in Bomze, et al. [Hessian barrier algorithms for linearly constrained optimization problems. SIAM Journal on Optimization, 2019]. Our approach is based on a potential-reduction mechanism and attains a suitably defined class of approximate first- or second-order KKT points with the optimal worst-case iteration complexity for both first-order and second-order schemes. A key feature of our methodology is the use of self-concordant barrier functions to construct strictly feasible iterates via a disciplined decomposition approach and without sacrificing on the iteration complexity of the method. To the best of our knowledge, this work is the first which achieves these worst-case complexity bounds under such weak conditions for general conic constrained optimization problems.

56. Data-driven and learning-based control 16:40 – 17:55

***Chair*: T. Faulwasser, K. Worthmann** **FH HS 5**

Control Lyapunov functions, optimal control problems, and deep neural networks

Mario Sperl (1),

(1) Chair of Applied Mathematics, University of Bayreuth, Germany;

It is known that compositional functions can be approximated by deep neural networks without suffering from the curse of dimensionality. In [1] it was shown that systems satisfying a small-gain condition admit compositional Lyapunov functions, which can be approximated by deep neural networks with a number of neurons that grows only polynomially in the state dimension. In this talk, we discuss how the approach in [1] can be extended to control Lyapunov functions. To this end, we investigate structural properties of the underlying nonlinear control system that yield the existence of a compositional control Lyapunov function. On the other hand, we also characterise situations where a simple form of compositionality does not exist. Moreover, suitable network architectures and training algorithms for the computation of control Lyapunov functions are presented. We also discuss aspects of the more general case of an optimal control problem and the approximation of a compositional optimal value function. Further, we demonstrate how these approaches perform in practice.

[**1**] L. Grüne. Computing Lyapunov functions using deep neural networks. *Journal of Computational Dynamics*, 8 (2021), 131-152, http://dx.doi.org/10.3934/jcd.2021006

Polynomial approximation based learning techniques for high-dimensional feedback control

Karl Kunisch (1,2), Donato Vásquez-Varas (1), Daniel Walter (1),

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We consider the problem of computing an optimal feedback law for a nonlinear and high dimensional continuous time control system. Our approach consists in approximate the problem as a learning task in the space of multivariate polynomials. We provide some results concerning the convergence and the generalization power of the solutions of the learning problem. Further, we exploit the structure of multivariate polynomials to solve efficiently the learning task. Finally, we perform some numerical test on nonlinear and high dimensional problems.

$\mathcal{H}_\infty$ Controller design for port-Hamiltonian systems

Paul Schwerdtner (1), Matthias Voigt (2),

(1) Institute of Mathematics, Technische Universtität Berlin, Berlin, Germany; (2) UniDistance Suisse, Brig, Switzerland;

The $\mathcal{H}_\infty$ optimal control problem plays an important role in attenuating undesired external noise and dealing with model uncertainties in dynamical systems. Solving this problem classically is based on determining solutions of multiple algebraic Riccati equations. This approach often suffers from numerical difficulties, in particular in the vicinity of the optimal performance level. Furthermore, if the underlying system dynamics is of high complexity, then the controller is of the same order as the plant model which is undesired from the practical point of view.

Therefore, we propose a new approach for port-Hamiltonian plant models for which we construct low-order port-Hamiltonian controllers. Using a structured parametrization of the controller we can connect the $\mathcal{H}_\infty$ control problem to a regression-type optimization problem [1] based on frequency domain samples of the plant model. This strategy is particularly well-suited for large-scale systems and does not suffer from the problems of the Riccati equation approach.

[1] P. Schwerdtner and M. Voigt. SOBMOR: Structured optimization-based model order reduction, March 2022. Revision submitted for publication, preliminary version also available as Preprint arXiv:2011.07567.

57. Optimal control and applications on biology and medicine 16:40 – 17:55

***Chair*: P. Bettiol, V. Milisic, J.Rouot** **FH HS 6**

Optimal control of gene networks

Nicolas Augier (1), Yabo Agustìn Gabriel (2),

(1) CNRS, Laboratory for Analysis and Architecture of Systems, Toulouse, France; (2) Université Côte d'Azur, Inria, INRAE, CNRS, Sorbonne Université, Sophia-Antipolis, France;

We present a study of the minimal-time problem for piecewise affine multistable systems started in [1], modeling the dynamics of gene networks. Motivated by applications in synthetic biology and biotechnology, our aim is to minimize the time needed for a system to achieve transitions between two stable steady states. The latter represents the possible states of a genetic network, playing a fundamental role in biocomputing and gene therapy. In the case of bidimensional systems, results show that a time-optimal transition between states should pass by an undifferentiated state, which is well known in cell biology for its importance in fate differentiation of cells. In order to characterize the capacity of gene networks to achieve transitions, we provide lower bounds on the minimal time, whose knowledge become relevant when considering realistic systems involving subsystems evolving on different time scales.

[1] N.Augier, A.Yabo. Time-optimal control of piecewise affine bistable gene-regulatory networks. *International Journal of Robust and Nonlinear Control*, doi: 10.1002/rnc.6012, 2022.

Optimal control problems involving parameters and applications

Piernicola Bettiol (1),

(1) University of Brest, Brest, France;

Control systems involving parameters appear a natural framework for applications such as some biological models: these might have a bi-level in which an optimal control problem is coupled with a nonlinear programming problem. We shall provide necessary optimality conditions for a class of optimal problems which involve parameters, and show how these conditions can be applied to derive necessary optimality conditions for problems having a bi-level structure.

Game theory and optimal control for a groundwater pollution problem

Emmanuelle Augeraud-Véron (1), Catherine Choquet (2), Éloïse Comte (3), Moussa Mory Diédhiou (4),

(1) GREThA, University of Bordeaux, Bordeaux, France; (2) MIA, La Rochelle University, La Rochelle, France; (3) LISC, INRAE, Aubière, France; (4) LMA, University Cheikh Anta Diop, Dakar, Sénégal;

We study a spatial differential game in the context of the non cooperation between two polluters for an optimal control problem of groundwater pollution. Two spatio-temporal objectives are considered, taking into account the trade-off between polluter's private benefits and environmental damages due to the pollution. They are constrained by the spread of the pollutant in the underground and the velocity of the flow, modeled by a convection-diffusion-reaction equation coupled with an elliptic equation. The main result is

the existence of a Nash equilibrium. Proving its uniqueness is a complex issue due to the nonlinearities of the model, but a characterization of the Nash equilibrium by a system of partial differential equations can be derived from a Pontryagin's approach. According to some additional assumptions, a uniqueness result follows ([1]). We produce numerical simulations, comparing cooperative and non-cooperative cases between the polluters.

[1] E. Augeraud-Véron, C. Choquet, É. Comte and M. M. Diédhiou, A game theory approach for the groundwater pollution control. *Accepted in SIAM Journal on Control and Optimization.*

58. Economic dynamics with regime switching — 16:40 – 17:55

***Chair*: J. Haunschmied, W. Semmler** — **FH HS 7**

Regime switching in dynamic games with hyperbolic discounting

Jesús Marín-Solano (1), Jorge Navas (1),

(1) Departament de Matemàtica Econòmica, Financera i Actuarial and BEAT, Universitat de Barcelona;

Dynamic optimization problems with endogeneous regime shifts have been widely studied in the literature. In the case of dynamic games, the characterization of optimal switching times as Markovian strategies presents some difficulties, derived from the fact that the strategies of one player over a regime also affect the timing of transition of the other player to a new regime. Some of these situations have been addressed for different classes of strategies, such as mixed open-loop / feedback or piecewise closed loop strategies. In the case of time inconsistent preferences, problems with one decision maker can be seen as (dynamic) sequential games with many agents. This introduces additional problematic issues in the search of "optimal" switching times. In this work, our aim is to extend previous results on regime shifts previously derived, independently, in the literature of dynamic games and nonconstant discounting. Both discrete and continuous time settings are explored. The main results are illustrated with a resource extraction model with technology adoption. In this model, we analyze first the problem with one decision maker with time-inconsistent preferences, and extend later on the analysis to the case of two players.

A time-dependent switching mean-field game on networks

Fabio Bagagiolo (1), Luciano Marzufero (2),

(1) University of Trento, Trento, Italy; (2) University of Trento, Trento, Italy;

Motivated by an optimal visiting problem, we investigate a switching mean-field game model on a network, where both a decisional and a switching time-variable are controls at disposal of the agents for what concerns, respectively, the instant to decide and to perform the switch. Every switch between the nodes of the network corresponds to a flip from 0 to 1 of one component of the string $p = (p_1, \dots, p_n)$ which, in the optimal visiting view, possibly represents the visited targets, being labeled by $i = 1, \dots, n$. The goal is to reach the final string $(1, \dots, 1)$ (i.e., to visit all the targets) within a fixed final time T, minimizing a switching cost also depending on the congestion on the nodes. We show the existence of a suitable approximated ε-mean-field equilibrium and then address the limit when ε goes to 0.

The main reference is preprint [1], submitted in December 2021. In the *References* section, you can find all the bibliography concerning the topic.

[1] F. Bagagiolo, L. Marzufero. A time-dependent switching mean-field game on networks motivated by optimal visiting problems. Preprint 2021, *arXiv:2201.00260*.

Selling with product recommendation and efficient below-cost pricing

Wenji Xu (1), Shuoguang Yang (2),

(1) City University of Hong Kong, Hong Kong, China; (2) Hong Kong University of Science and Technology, Hong Kong, China;

A long-lived seller sells a product by setting prices and making product recommendations to short-lived consumers arriving in continuous time. The seller receives consumer feedback about the product value, whose arrival rate depends on the instantaneous sales volume. Under a constant pricing regime, the seller sets a price *above* the production cost and *spams* consumers with non-authentic product recommendations; under a flexible pricing regime, the seller *never spams* the consumers and may set the price *below* the production cost for an initial or interim period of time. The outcome under flexible pricing Pareto dominates the outcome under constant pricing.

Friday, July 15^{th}

59. Optimal control and calculus of variations on metric spaces 08:30 – 10:10

***Chair*: H. Frankowska, M. Quincampoix** **FH HS 8**

When HJB meets pontryagin in mean-field control

Benoît Bonnet (1), Hélène Frankowska (2),

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It has been known for a long time that optimality conditions in control theory usu- ally come in either of the two following forms : on the one hand Hamilton-Jacobi-Bellman equations, which are global necessary and sufficient optimality conditions, and on the other hand the Pontryagin Maximum Principle, which is a local necessary optimality condition. Both approaches have been extensively used over the years to produce a wealth of theoretical and numerical results, but to this day very few results do provide actual connections between them. This talk will present fine analytical properties of the value function associated with mean-field optimal control problems, and will show how these latter can used in order to bridge between the two families of optimality conditions. More specifically, the talk will show that by using these properties in conjunction with the maximum principle, it is possible to derive sufficient optimality conditions for Pontryagin extremals, prove propagation of regularity results along optimal trajectories, and to characterise optimal feedbacks in terms of adequate set-valued mappings.

Weak KAM theory for sub-Riemannian control systems

Piermarco Cannarsa (1), Cristian Mendico (1),

(1) Department of Mathematics, University of Rome Tor Vergata, Rome, Italy;

The long-time average behavior of the value function in the classical calculus of variations is known to be connected with the existence of solutions of the so-called critical equation, that is, a stationary Hamilton-Jacobi which includes a sort of nonlinear eigenvalue—called the *critical costant* (see, e.g., [2] for an overview).

Here, we will address similar issues for *sub-Riemannian systems*, that is, control systems associated with a family of vector fields which satisfies the Lie Algebra rank condition on the whole Euclidean space. First, we will describe the way how, in [1], the critical constant of the problem has been obtained by studying the convergence of the time-averaged value function as time horizon goes to infinity. The construction of one viscosity solution of the critical equation, which coincides with its Lax-Oleinik evolution, is another result from [1] that will be presented.

Then, we will discuss the latest progress of the theory, including a variational representation of the critical constant by an adapted notion of closed measures and the horizontal differentiability of critical solutions on the Aubry set.

[1] P. Cannarsa and C. Mendico. Asymptotic analysis for Hamilton-Jacobi-Bellman equations on Euclidean space. Available in arXiv:2012.09099v3

[2] A. Fathi. Weak KAM Theory: the connection between Aubry-Mather theory and viscosity solutions of the Hamilton-Jacobi equation. In *Proceedings of the International Congress of Mathematicians.* Seoul, 2014.

A variational approach to evolution equations driven by dissipative operators in Wasserstein spaces

Giulia Cavagnari (1), Giuseppe Savaré (2), Giacomo E. Sodini (3),

(1) Department of Mathematics, Politecnico di Milano, Milan, Italy; (2) Dipartimento di Scienze delle Decisioni, Bocconi University, Milan, Italy; (3) IAS, Technische Universität München, Munich, Germany;

In this talk, we present well-posedness of so-called *Measure Differential Equations*. These are evolution equations in the Wasserstein space $\mathcal{P}_2(\mathsf{X})$ of Borel probability measures on a Hilbert space X, driven by a suitably extended notion of dissipative probability vector fields. We take inspiration from the theory of dissipative operators in Hilbert spaces and of Wasserstein gradient flows of geodesically convex functionals. Our approach is based on a measure-theoretic version of the Explicit Euler scheme. We characterize the limit solutions by a suitable *Evolution Variational Inequality* (EVI), inspired by the Bénilan notion of integral solutions to dissipative evolutions in Banach spaces. We then get existence, uniqueness and stability of EVI solutions and finally compare this notion of solution with the weaker barycentric/distributional one introduced by B. Piccoli. If time allows, we introduce a Lagrangian counterpart: a characterization of the measure-valued EVI solution as the time-dependent law of the unique solution of a corresponding evolution equation in the lifted Hilbert space $L^2(\Omega)$.

Hamilton-Jacobi inequalities on metric spaces

Zoé Badevin (1), Hélène Frankowska (2),

(1) CNRS, IMJ-PRG, Paris, France; (2) CNRS and Sorbonne Université, Paris, France;

In some applied models (of flocking or of the crowd control) it is more natural to deal with elements of a metric space (as for instance probability measures endowed with the Wasserstein distance or compact subsets of a vector space endowed with the Hausdorff distance) rather than with vectors in a normed vector space. For a generalized (mutational) control system stated on a metric space we consider the Mayer type optimal control problem. Then its value function satisfies two contingent Hamilton-Jacobi-Bellman inequalities involving metric directional derivatives. We provide sufficient conditions for the existence and uniqueness of solutions to these inequalities. As an example of applications we consider controlled continuity equations on the metric space of probability measures with compact support. In this case metric Hamilton-Jacobi-Bellman inequalities can be equivalently written in terms of metric Hadamard generalized differentials. Then continuous solutions are unique whenever we focus our attention on solutions defined on time dependent compact valued tubes of probability measures (described via the growth bounds on the support of solutions to controlled continuity equations).

[1] Badreddine Z. & Frankowska H. Viability and invariance of systems on metric spaces. submitted.

[2] Badreddine Z. & Frankowska H. Solutions to Hamilton-Jacobi equation on a Wasserstein space. Calculus of Variations and PDEs, 61:9, 2022.

60. Mean field games

08:30 – 10:10

***Chair*: Scientific Committee**

FH HS 5

Hybrid control approach for optimal visiting problems

Fabio Bagagiolo (1), Adriano Festa (2), Luciano Marzufero (1),

(1) Università di Trento, Italy; (2) DISMA, Politecnico di Torino, Italy;

In an optimal visiting problem, we want to control a trajectory that has to pass as close as possible to a collection of target points or regions. We introduce a hybrid control-based approach for the classic problem where the trajectory can switch between a group of discrete states related to the targets of the problem [1]. The model is shown to be effective to solve the "Orienteering Problem" a framework which originates from the sport of orienteering [3]. The model is subsequently adapted to a mean-field game framework to study viability and crowd fluxes to model a multitude of indistinguishable players [2].

[1] Bagagiolo, F., Festa, A., Marzufero, L.: A hybrid control framework for an optimal visiting problem. *IFAC-PapersOnLine*, 54(5), pp. 241–246, (2021).

[2] Bagagiolo, F., Festa, A., and Mazufero, L. Hybrid control for optimal visiting problems for a single player and for a crowd. *Nonlinear Differential Equations and Applications*, 29(1), 4, (2022).

[3] Bagagiolo, F., Festa, A., Marzufero, L.: The orienteering problem: a hybrid control formulation. *IFAC-PapersOnLine*, 54(5), pp. 175–180 (2021).

Evolutionarily stable sets in stochastic games

Divya Murali (1), K.S. Mallikarjuna Rao (2), A.J. Shaiju (1),

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Evolutionarily stable set (ESSet), introduced in [3] is a set of strategies which score equally against each other and will drive all other strategies away. In other words, each strategy in the ESSet will be an ESS, if other strategies of the ESSet are not present. In this work, we extend the notion of ESSets to stochastic games using the framework developed in [2]. We show that the set of all population distributions, inducing strategies of the ESSet, is asymptotically stable with respect to the associated replicator dynamics for the stochastic game. We also consider the notion of a strict equilibrium set (SESet)[1], which is a set based generalisation of strict Nash equilibrium, in the context of evolutionary stochastic games, and discuss its relation with ESSets.

[1] D. Balkenborg and K. Schlag, On the evolutionary selection of sets of Nash equilibria. *Journal of Economic Theory*, 133:295-315, 2007.

[2] J. Flesch, T. Parthasarathy, F. Thuijsman, P. Uyttendaele. Evolutionary stochastic games. In *Dynamic Games and Applications*, (3):207-219, 2013.

[3] B. Thomas, On evolutionarily stable sets. *Journal of Mathematical Biology*, 22:105-115, 1985.

Finite state mean field games with common shocks

Berenice Anne Neumann (1), Frank Thomas Seifried (1),

(1) FB IV - Mathematics, University of Trier, Trier, Germany;

We present a new framework for mean field games with finite state space and common noise, where the common noise is given through shocks that occur at random times. We first analyse the game for up to n shock times in which case we are able to characterize mean field equilibria through a family of parametrized and coupled forward-backward systems and prove existence of solutions to these systems for a small time horizon. Thereafter, we show that for the case of an unbounded number of shocks the equilibria of the game restricted to n shocks are approximate mean field equilibria.

Large ranking games with diffusion control

Stefan Ankirchner (1), Nabil Kazi-Tani (2), Julian Wendt (1), Chao Zhou (3),

(1) Institute for Mathematics, University of Jena, Ernst-Abbe-Platz 2, 07743 Jena, Germany; (2) Nabil Kazi-Tani, Institut Elie Cartan de Lorraine, Université de Lorraine, UFR MIM, 3 rue Augustin Fresnel. 57073 Metz Cedex 03, France; (3) Department of Mathematics and Risk Management Institute, National University of Singapore, 10 Lower Kent Ridge Road, 119076 Singapore;

In this talk we consider a symmetric stochastic differential game where each player can control the diffusion intensity of an individual dynamic state process, and the players whose states at a deterministic finite time horizon are among the best $\alpha \in (0, 1)$ of all states receive a fixed prize. Within the mean field

limit version of the game we compute an explicit equilibrium, a threshold strategy that consists in choosing the maximal fluctuation intensity when the state is below a given threshold, and the minimal intensity else. We show that for large n the symmetric n-tuple of the threshold strategy provides an approximate Nash equilibrium of the n-player game. We also derive the rate at which the approximate equilibrium reward and the best response reward converge to each other, as the number of players n tends to infinity. Finally, we compare the approximate equilibrium for large games with the equilibrium of the two player case.

61. Optimal control and applications on biology and medicine 08:30 – 10:10

***Chair*: P. Bettiol, V. Milisic, J.Rouot** **FH HS 6**

Optimal Darwinian selection of microalgae

Walid Djema (1),

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In this talk, we investigate the problem of species separation in minimal time. Droop model is considered to describe the evolution of two distinct populations of microorganisms that are in competition for the same resource in a photobioreactor. We focus on an optimal control problem (OCP) subject to a five-dimensional controlled system in which the control represents the dilution rate of the chemostat. The objective is to select the desired species in minimal-time and to synthesize an optimal control [**1**]. This is a very challenging issue, since we are are dealing with a ten-dimensional optimality system. We provide properties of optimal controls allowing the strain of interest to dominate the population. The analysis is mainly based on the Pontryagin Maximum Principle (PMP), along with a thorough study of singular arcs that is crucial in the synthesis of optimal controls [**1**]. These theoretical results are also extensively illustrated and validated using a direct method in optimal control (via the `Bocop` software). The approach is illustrated with numerical examples with microalgae, reflecting the complexity of the optimal control structure and the richness of the dynamical behavior. In particular, a *turnpike*-like feature of the optimal solutions is highlighted on a slightly different OCP [**2**]. We provide sufficient conditions for the existence of the *turnpike* property associated with the optimal control and state-trajectories, as well as their respective co-state trajectories [**2**]. The proof is mainly based on the hyperbolicity of the linearized Hamiltonian-system around the solution of the so-called *static*-OCP.

[1] **Djema, W., Bayen, T. and Bernard, O.** *Optimal Darwinian Selection of Microorganisms with Internal Storage.* Processes, 10(3), p.461., 2022.`https://doi.org/10.3390/pr10030461`

[2] **Djema, W., Giraldi, L., Maslovskaya, S. and Bernard, O.** *Turnpike features in optimal selection of species represented by quota models.* Automatica, 132, p.109804., 2021. `https://doi.org/10.1016/j.automatica.2021.109804`

Contraction and pattern formation in actomyosin networks

Cecilia Gonzalez-Tokman (1), Alex Mogilner (2), Dietmar Oelz (1), Alex Tam (3),

(1) The University of Queensland (UQ), Brisbane, Australia; (2) New York University (NYU), USA; (3) University of South Australia (UniSA), Australia;

We investigate dynamics and emergent phenomena in disordered actomyosin networks such as the cell cortex. We derive a mathematical model for the evolution of two-dimensional networks which enables direct calculation of the network stress tensor. Simulations confirm that both protein friction and actin filament bending are required for contraction. Analysis of a toy-model version of the model for only two filaments immersed in an actomyosin network shows that bending induces a geometric asymmetry that enables motors to move faster close to filament plus-ends, inhibiting expansion. Finally we consider minimal models for pattern formation in such a network. We suggest a minimal model for F-actin turnover which is a point process reminiscent of the classical voter model and compute its asymptotic distribution.

Geometric optimal control of the Generalized Lotka-Volterra equation with application to the intestine microbiote

Bernard Bonnard (1), Jérémy Rouot (2), Cristiana F. Silva (3),

(1) UBFC & INRIA Sophia Antipolis, Équipe McTAO, France; (2) Université de Bretagne Occidentale, LMBA, Brest, France; (3) Université Aveiro, Portugal;

In this talk we present the Generalized Lotka–Volterra dynamics associated to the model of C-difficile infection of the intestine microbiote and aiming to transfer the system from an infected state to an healthy state. The control inputs are of two types : fecal injection or bactericides which act as Dirac pulses and prebiotics or antibiotics which act as continuous controls. An uniform frame can be introduced using the tools from geometric control to analyze the accessibility set as the orbit of a pseudo-semi group. Optimal control can be considered in the frame of permanent control or sampled-data control. The later being adapted to the practical constraints of a finite set of medical interventions. In both case the optimal control problems can be analyzed using direct and indirect schemes aiming to reach an healthy state. Such methods are tested on toys models in dimension 2 and 3 related to the construction of reduced dynamics. Even those simple situations lead to interesting questions of accessibility and integrability issues in relation with the study of dynamical systems.

Some adhesion models and related mathematical results in the context of cell motility

Vuk Milisic (1),

(1) Univ Brest, UMR CNRS 6205, Laboratoire de Mathématiques de Bretagne Atlantique, 6 Avenue Le Gorgeu, 2920 Brest, France;

We present a model of adhesion that accounts for the fine mechanics of linkages at the cell scale level. We present a scaling that transforms this model into friction. We describe various convergence results obtained when gradually complexifying the mathematical context in which this adhesion term appears.

We point out several aspects that might be of interest for the control community.

[1] V. Milisic and C. Schmeiser Asymptotic limits for a non-linear integro-differential equation modelling leukocytes' rolling on arterial walls *Nonlinearity* 35 (2022) p. 843-869

[2] V. Milisic From delayed minimization to the harmonic map heat equation *Journal of Functional Analysis* Volume 279, Issue 2, 2020,

[3] V. Milisic and D. Oelz On the asymptotic regime of a model for friction mediated by transient elastic linkages. *Journal de Mathematiques Pures et Appliquées* (9) 96 (2011), no. 5, p. 484–501.

62. Dynamic economic policy

08:30 – 10:10

***Chair*: B. Heijdra, P. Heijnen**

FH HS 7

Rent seeking, capital accumulation, and macroeconomic growth

Ben J. Heijdra (1,2), Pim Heijnen (1),

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We study the effects of time-using rent-seeking activities on the macroeconomic allocation and the economic growth rate. We formulate a highly stylized three-sector general equilibrium model with overlapping generations of individuals. The production side features one sector producing the capital good and two consumption goods sectors. All sectors operate under constant returns to scale technology with human and physical capital as inputs. One of the consumption goods sectors is a monopoly, where a continuum of agents compete for a share of monopoly profits. Agents are heterogeneous in their (intrinsically useless) rent-seeking ability. In the benchmark model each agent decides during youth on how much time to spend on lobbying activities, education, and production work. An intergenerational human capital externality of the 'shoulders of giants' type ensures that the model features endogenous growth. The rewards to rent-seeking accrue during youth and part of the additional income is saved. Interestingly, a move from a perfectly competitive economy to one involving monopolization and rent-seeking increases the steady-state economic growth rate in the benchmark model. We identify three main mechanisms affecting the growth rate under monopoly and rent-seeking, namely (a) the kind of inputs used in the rent-seeking competition (raw time or education level), (b) the type of growth engine (human or physical capital externality), and (c) the time of life at which the rent-seeking booty is received (youth or old-age). The conclusions for the benchmark model are robust to changes in the mechanisms for (a) and (b) but not for (c). If rent-seeking rewards accrue during old-age then the move from a perfectly competitive economy to one involving monopolization and rent-seeking decreases the steady-state economic growth rate.

Efficiency and equity: a general equilibrium analysis of rent-seeking

Pim Heijnen (1), Ben J. Heijdra (1),

(1) University of Groningen, Groningen, the Netherlands;

We study the rent-seeking phenomenon using a simple, static general equilibrium model. The economy consists of two sectors, both employing a constant returns-to-scale technology with labor as its sole input. One of the sectors is a monopoly, where a continuum of agents compete for a share of monopoly profits (i.e. rent). Agents are heterogeneous in labor productivity and rent-seeking ability: they face a choice between engaging in (productive) work or vying for a share of the rent (i.e. a contest against other rent-seekers). At the aggregate level, rent-seeking reduces the available amount of labor in the economy and

thereby lowers output and welfare (rent-seeking is inefficient). At the individual level, rent-seeking shifts income towards rent-seekers. Consequently, an economy with few rent-seekers tends to have high income inequality: an effect that is exacerbated by the fact that rent is decreasing in the number of rent-seekers (low levels of rent-seeking increase inequity). This tradeoff between efficiency and equity is the primary focus of this paper. We investigate how the distribution of rent-seeking ability and the correlation between labor productivity and rent-seeking ability shape this tradeoff.

Revisiting the effects of a money-financed fiscal stimulus

Christiaan van der Kwaak (1),

(1) University of Groningen, the Netherlands;

In the standard New Keynesian model, a money-financed fiscal stimulus is more effective in expanding output than a debt-financed stimulus. This result relies on the fact that the resulting expansion of the money supply decreases the interest rate on government debt. In recent years, however, several central banks have started paying interest on reserves. In that case, I find that the effectiveness of a money-financed fiscal stimulus is the same as that of a debt-financed stimulus. This result continues to hold when the interest rate on government debt is higher than that on reserves as a result of financial frictions. A money-financed stimulus only becomes more effective when balance-sheet-constrained financial intermediaries simultaneously finance credit to the real economy or when government debt is subject to default risk. However, I find the quantitative impact to be small.

Distortions in investment timing and quantity in real options with asymmetric information

Maarten van Oosterhout (1), Gijsbert Zwart (1),

(1) Department of Economics, Econometrics and Finance, University of Groningen, the Netherlands;

We analyze real options investment under asymmetric information on investment costs, where decisions not only involve investment timing, but also investment quantity. A principal, who could be a regulator, offers a menu of contracts to the agent (the regulated firm). The regulated firm has better information on costs than the regulator, and the optimal regulation trades off distortions in investment decisions and informational rents left to the firm. In a non-dynamic situation, it is well known that optimal contracts involve downward distortions on investment quantity. In the dynamic, real options situation, distortions also occur in investment timing: a high-cost firm's investment will be delayed beyond the optimal time, until revenues reach a higher investment threshold.

We explore the effect on investment quantity in this real option regulation under various assumptions on the stochastic process for revenues. On the one hand, the higher investment threshold tends to increase investment quantities, whereas screening of high-cost firms would favour reducing their investment quantity. We find a simple sufficient condition for the latter, quantity-reducing, effect to dominate, and show that it is satisfied for a wide range of commonly used stochastic processes.

63. Optimal control and calculus of variations on metric spaces 10:30 – 11:45

***Chair*: H. Frankowska, M. Quincampoix** **FH HS 8**

Control systems with constraints in metric spaces: Theory and examples

Thomas Lorenz (1),

(1) RheinMain University of Applied Sciences, Wiesbaden Rüsselsheim, Germany;

The following type of control problems arouses the interest in set-valued states: Some control systems are not just characterized by the state x and the control u, but they also depend on bounded "unknown noise" for covering aspects of imprecision and uncertainty. In a deterministic framework we suggest to formulate it as a further time-dependent parameter $v(t) \in V$ in the control equation $x' = f(t, x, u, v)$ and focus on the attainable set of states x over all measurable $v(t) \in V$. Hence, the traditional state vectors are now replaced by state *sets* whereas $u(t) \in U$ still serves as an open-loop control. Under suitable assumptions these attainable sets are compact, but not necessarily convex and so the dynamics has now left the classical setting of linear spaces.
In this talk we suggest how control equations and (closely related) differential inclusions can be formulated in a metric space. These so-called mutational inclusions extend Aubin's mutational equations. The focus is on new results about inclusions with state constraints and we sketch their consequences on miscellaneous examples (like hyperbolic inclusions or structured population models). It is partly joint work with Hélène Frankowska.

Random lift of set-valued maps and applications

Rossana Capuani (1), Antonio Marigonda (2), Michele Ricciardi (3),

(1) University of Tuscia, Viterbo, Italy; (2) University of Verona, Verona, Italy; (3) King Abdullah University of Sciences and Technologies (KAUST), Thuwal, Saudi Arabia;

Multi-agents systems are systems where the number of possibly interacting agents is so large that only a statistical description is viable. We present some recent results concerning the dynamics and the control of multi-agent systems, where the macroscopical trajectory can be seen as a suitable lift of the solution set-valued map of a differential inclusion expressing the microscopical dynamics of the agent. This allows to directly transfer relevant properties of the set-valued map to the dynamics in the set of probability measures. In particular, we will show an application concerning a leader-follower type of model for the confinement of a population by mean of the interaction with discrete agents.

Multiagent control systems with set-driven evolution

Marc Quincampoix (1),

(1) Laboratoire de Mathematiques de Bretagne Atlantique, Univ Brest, CNRS UMR 6205, Brest, France;

We investigate an optimal control problem with a large number of agents (possibly infinitely many). At each time, the set of available velocities available to each agent depends on its current position and also by a the position of the whole crowd of agents. So, the multi-agent dynamical system has two levels:

A microscopic level which describes the dynamic of a individual agent which trajectory $s \mapsto x(s) \in \mathbb{R}^d$ satisfies

$$\dot{x}(s) \in F(s, x(s), E(s)),\ s \in [0, T] \tag{1}$$

where $E(s) \subset \mathbb{R}^d$ is the set of positions at time s of the other agents.

The state variable of the macroscopic system is $E(s)$ the set of positions of the agents at time s. A trajectory $s \mapsto E(s)$ could be viewed as the superposition of a subset of trajectories $s \mapsto x(s)$ satisfying (1).

We study the evolution of such a bi-level control systems and also investigate the relations with multiagent control systems modelized through a dynamic on probability measures in $\mathbb{R}^d$ (used for instance in the mean field approach). When the mean field control dynamic depends only on the support of the probability measure, we discuss the links with the above set driven bi-level control system.

[1] M. Bivas, M. Quincampoix. Optimal control for the evolution of deterministic multi-agent systems *Journal of Differential Equations*, **269**(3):2228-2263, 2020.

[2] R. Capuani, A. Marigonda, M. Quincampoix. Set-driven evolution for multiagent system *In preparation.*

[3] C. Jimenez , A. Marigonda, M. Quincampoix. Optimal Control of Multiagent Systems in the Wasserstein Space *Calculus of Variations and Partial Differential Equations*, **2**, 2020.

64. Numerical analysis for PDE constrained optimization 10:30 – 11:45

***Chair*: J. Pfefferer, A. Rösch** **FH HS 5**

Convex relaxations of parabolic optimal control problems with combinatorial switching constraints

Christoph Buchheim (1), Alexandra Grütering (1), Christian Meyer (1),

(1) Faculty of Mathematics, TU Dortmund, Dortmund, Germany;

We consider optimal control problems for partial differential equations where the controls take binary values but vary over the time horizon, they can thus be seen as dynamic switches. The switching patterns may be subject to combinatorial constraints such as, e.g. an upper bound on the total number of switchings or a lower bound on the time between two switchings. While such combinatorial constraints are often seen as an additional complication that is treated in a heuristic postprocessing, the core of our approach is to investigate the convex hull of all feasible switching patterns in order to define a tight convex relaxation of the control problem. The convex relaxation is built by cutting planes derived from finite-dimensional projections, which can be studied by means of polyhedral combinatorics, and solved by an outer approximation algorithm. However, both the relaxation and the algorithm are independent of any fixed discretization and can thus be formulated in function space. Preliminary numerical results illustrate the efficiency of our approach.

Sparse Dirichlet Optimal Control Problems

Mariano Mateos (1),

(1) Departamento de Matemáticas, Universidad de Oviedo, Gijón, Spain;

In this talk, we analyze optimal control problems governed by an elliptic partial differential equation, in which the control acts as the Dirichlet data. Box constraints for the controls are imposed and the cost functional involves the state and possibly a sparsity-promoting term, but not a Tikhonov regularization term. Two different discretizations are investigated: the variational approach and a full discrete approach. For the latter, we use continuous piecewise linear elements to discretize the control space and numerical integration of the sparsity-promoting term. It turns out that the best way to discretize the state equation is to use the Carstensen quasi-interpolant of the boundary data, and a new discrete normal derivative of the adjoint state must be introduced to deal with this. Error estimates, optimization procedures and examples are provided.

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Optimal control problems in H^s with L^p cost with $s \in (0,1)$, $p \in (0,1)$

Harbir Antil (1), Daniel Wachsmuth (2),

(1) George Mason University, Fairfax, VA; (2) University of Wuerzburg, Wuerzburg, Franconia;

We investigate optimization problems in $H^s(\Omega)$ spaces, where $s \in (0,1)$ is chosen small to facilitate discontinuous solutions. The cost functional includes a L^p-cost term with $p \in (0,1)$, which promotes sparsity of solutions. We prove existence of solutions. Necessary optimality conditions are obtained by means of a smoothing procedure. This procedure is also used to derive an optimization algorithm. Weak limit points satisfy conditions weaker than the necessary optimality conditions. Results of numerical experiments are presented, which demonstrate that solutions are sparse and discontinuous.

65. Covid-19: optimal control approaches

10:30 – 11:45

***Chair*: D. Grass, S. Wrzaczek**

FH HS 6

Infinite horizon optimal control problems and their application to epidemiology

Katharina Kolo (2), Valeriya Lykina (1), Sabine Pickenhain (1),

(1) RG Optimization, Brandenburg University of Technology Cottbus-Senftenberg, Cottbus, Germany; (2) TITUS Research Ltd, Wildau, Germany;

In this talk we consider the application of optimal control problems with infinite horizon to epidemiology. In particular, a SEIR model with infinite horizon in a special setting with weighted functional spaces is under investigation. The key idea is to use weighted Sobolev and Lebesgue spaces as state and control spaces respectively. Both optimization and asymptotic stabilization problems are covered by this setting and are treated by a dual approach. Optimality conditions for this problem are formulated.

Optimal timing of non-pharmaceutical interventions during an epidemic

Nick F.D. Huberts (1), Jacco J.J. Thijssen (2),

(1) Management School, University of York, Heslington, York YO10 5ZF, United Kingdom; (2) Department of Mathematics, University of York, Heslington, York YO10 5ZF, United Kingdom;

In response to the recent outbreak of the SARS-CoV-2 virus governments have aimed to reduce the virus's spread through, *inter alia*, non-pharmaceutical intervention. We address the question when such measures should be implemented and, once implemented, when to remove them.

These issues are viewed through a real-options lens and we develop an SIRD-like continuous-time Markov chain model to analyze a sequence of options: the option to intervene and introduce measures and, after intervention has started, the option to remove these. Measures can be imposed multiple times.

We implement our model using estimates from empirical studies and our main conclusions are that: (1) measures should be put in place almost immediately after the first infections occur; (2) if the epidemic is discovered when there are many infected individuals already, then it is optimal never to introduce measures; (3) once the decision to introduce measures has been taken, these should stay in place until the number of susceptible or infected members of the population is close to zero; (4) it is never optimal to introduce a tier system to phase-in measures but it is optimal to use a tier system to phase-out measures; (5) a more infectious variant may reduce the duration of measures being in place; (6) the risk of infections being brought in by travelers should be curbed even when no other measures are in place. These results are robust to several variations of our base-case model.

Interplay of increased infectivity and vaccine resistance in the evolution of novel SARS-CoV-2 strains

Simon Rella (1), Yuliya Kulikova (2,3), Fyodor Kondrashov (1),

(1) IST Austria, Vienna, Austria; (2) OIST, Okinawa, Japan; (3) IIASA, Vienna, Austria;

With the ongoing SARS-CoV-2 pandemic an issue of controling the evolution and spread of novel variants is becoming very important. There are three main factors of particular epidemiological concern: higher infectivity, immunogenic drift (vaccine resistance) and increased virulence. Unless higher virulence comes together with higher infectivity (pleiotropy), higher virulence strains are not expected under the natural selection, we thus focus on the interplay of infectivity and vaccine resistance in this paper. So far we have seen a number of new strains that have emerged, with the most striking epidemiological factor of increased infectivity. Indeed, delta appears more infectious than the original strain, which has further been trumped by omicron. By contrast, vaccine escape of these new strains has not been as drastic, although still present. The emergence of a new strain is an inherently stochastic process under extensive influence of genetic drift. Therefore, it is important to model the dynamics of the new strains at their onset in a stochastic way to allow for genetic drift in the early phases of population dynamics of the new strains. We build a SIR-derived model (as in our previous work, [1]) with initial stochastic dynamics for the new strains to study the probability of their emergence and establishment. Our setup allows us to quickly assess the dynamics of emerging strains while maintaining realism of the stochastic nature of population genetic processes that determine the fates of rare alleles in the population. At the same time our setup allows to test the effect of vaccine hesitancy on the evolution of new strains and the effect of different non-pharmaceutical interventions to control the spread of pandemic. Which one of the two strains will win in the population? It is not a simple question because an emerging pandemic is not in equilibrium state, with the number of infections coming in waves, the number of vaccinated and recovered people increasing over

time, immunity vanishing over time, different countries imposing different policies to control the spread of the pandemic. We study how infectivity, vaccine-resistance, vaccination rates and non-pharmaceutical interventions interact in the process, affecting selective advantage of different strains. Our main result is that until the virus reaches some higher limit of infectivity, vaccine-resistant variants will continue to be eliminated from the population.

[1] Rella, S.A., Kulikova, Y.A., Dermitzakis, E.T. et al. Rates of SARS-CoV-2 transmission and vaccination impact the fate of vaccine-resistant strains. *Sci Rep*, **11**:15729, 2021. https://doi.org/10.1038/s41598-021-95025-3

66. Dynamics of firm

10:30 – 11:45

***Chair*: P.M. Kort**

FH HS 7

Investment strategy and capacity optimization under stochastic innovations

Peter M. Kort (1), Maria Lavrutich (2), Afonso Moreira (3,4), Cláudia Nunes (3), Carlos Oliveira (5,6),

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In this paper we extend the framework developed in [1] by including capacity optimization. In line with the original model, the firm faces an investment problem in which it can either buy the current technology or wait for the new one. We prove that the solution of the original investment model satisfies the associated HJB equation. Additionally, we provide a complete comparative statics considering the model with and without capacity optimization which allows us to highlight the effect of capacity in the model. We retain all the characteristics introduced in the original model, in particular the likelihood of each investment strategy that now depends on the chosen capacity. The results are tested for both linear and isoelastic demand functions.

[1] Steven R. Grenadier, Allen M. Weiss. Investment in technological innovations: An option pricing approach *Journal Of Financial Economics*, **44**(3):397-416, 1997.

Investment under a disruptive risk with costly Bayesian learning: the optimal choice of the learning rate

Verena Hagspiel (1), Roel L. G. Nagy (1), Jacco J.J. Thijssen (2), Sebastian Sund (1),

(1) Norwegian University of Science and Technology, Department of Industrial Economics, Trondheim, Norway;
(2) University of York, The York Management School, York, United Kingdom;

We study a sequential problem in which a firm decides on a learning rate by a learning investment first, and faces an investment/abandonment decision for a project secondly. The firm learns about the rate at

which a disruptive event arrives. Once the disruptive event arrives, the project is worthless. The option to learn is most valuable when the profit flow is average; when the profit flow is low (high), the firm abandons (invests) without doing a significant learning investment. The firm's optimal investment decision and the value of the option both depend on whether the costs associated with learning are paid upfront at once or continuously over time. The firm invests sooner in the project under a larger continuous learning cost than under a large upfront learning cost. We also study the role of the learning efficiency on the option value and the firm's investment. Both the option value and the investment threshold increase when the firm is able to learn more from a single signal, while the abandonment threshold decreases.

Investment timing under uncertainty in oligopoly markets with many firms

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We analyze the investment decisions under uncertainty in oligopoly markets by considering a market of many firms with asymmetric cost structure in a discrete time setting where the demand evolves stochastically and follows a linear structure. Considering competitive strategies, increasing the number of firms imposes inevitable complexities to the model structure. We propose a new approach, based on reinforcement learning, to numerically solve the investment timing problem when the number of firms increases. Our approach can be more generally applied to investigate markets of many firms with several investment opportunities.

67. Continuous optimization: theory and applications 11:55 – 13:10

***Chair*: R. Bot, A. Daniilidis** **FH HS 8**

Splitting and projection methods for control-constrained linear-quadratic optimal control problems

Regina S. Burachik (1), Bethany I. Caldwell (1), C. Yalçın Kaya (1),

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Splitting and projection-type algorithms have been applied to many optimization problems due to their simplicity and efficiency but the application of these algorithms to optimal control is less common. In this talk we utilize these methods to solve control-constrained linear-quadratic optimal control problems. Instead of the traditional approach where we discretize the problem and solve it using large-scale finite-dimensional numerical optimization techniques we split the problem in two and use projection methods to find a point in the intersection of the solution sets of these two subproblems hence giving the solution to the original problem. Promising numerical results with this approach for a double integrator problem have been illustrated in [1]. Here we will extend this work to more general control-constrained linear-quadratic optimal control problems and provide numerical results and comparisons using the Method of Alternating Projections, Dykstra's algorithm, Douglas–Rachford algorithm and Aragón Artacho–Campoy algorithm.

[1] H. H. Bauschke, R. S. Burachik, and C. Y. Kaya. Constraint splitting and projection methods for optimal control of double integrator In *Splitting Algorithms, Modern Operator Theory, and Applications. H.H. Bauschke et al., Springer, 2019.*

ProxSkip: Breaking the communication barrier of local gradient methods

Konstantin Mishchenko (1,2), Grigory Malinovsky (2), Sebastian Stich (3), Peter Richtárik (2),

(1) CNRS, ENS, Inria Sierra, Paris, France; (2) Computer Science, King Abdullah University of Science and Technology, Thuwal, Saudi Arabia; (3) CISPA Helmholtz Center for Information Security, Saarbrücken, Germany;

We introduce **ProxSkip**—a surprisingly simple and provably efficient method for minimizing the sum of a smooth (f) and an expensive nonsmooth proximable (ψ) function. The canonical approach to solving such problems is via the proximal gradient descent (**ProxGD**) algorithm, which is based on the evaluation of the gradient of f and the prox operator of ψ in each iteration. In this work we are specifically interested in the regime in which the evaluation of prox is costly relative to the evaluation of the gradient, which is the case in many applications. **ProxSkip** allows for the expensive prox operator to be skipped in most iterations: while its iteration complexity is $(\kappa \log \frac{1}{\varepsilon})$, where κ is the condition number of f, the number of prox evaluations is $(\sqrt{\kappa} \log \frac{1}{\varepsilon})$ only. Our main motivation comes from federated learning, where evaluation of the gradient operator corresponds to taking a local **GD** step independently on all devices, and evaluation of prox corresponds to (expensive) communication in the form of gradient averaging. In this context, **ProxSkip** offers an effective *acceleration* of communication complexity. Unlike other local gradient-type methods, such as **FedAvg**, **SCAFFOLD**, **S-Local-GD** and **FedLin**, whose theoretical communication complexity is worse than, or at best matching, that of vanilla **GD** in the heterogeneous data regime, we obtain a provable and large improvement without any heterogeneity-bounding assumptions.

A parallel Secant-like equation quasi-Newton method for unconstrained optimization

Issam A.R. Moghrabi (1),

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The quasi-Newton technique is one of the most well-known iterative methodologies for tackling unconstrained optimization problems. The quasi-Newton techniques are known for their rapid convergence and high accuracy. To give a more exact approximation of the objective function's second curvature, we suggest a modified secant relation based on a quadratic model. We next provide a novel BFGS approach based on this modified secant relationship for tackling unconstrained optimization issues. The suggested technique uses both gradient and function values, whereas the traditional secant relation just employs gradient values. This is motivated by the ambitious success of the methods derived here. The new derivation is based on a quadratic model of the objective function that is exploited in building a new BFGS update that satisfies a new variant of the Secant equation.The approach generates two search directions that can be examined in parallel to and a better solution at each iteration [1,2]. Such methods have proven useful in industrial, business, and engineering applications. Comparative testings reveal that the suggested technique is computationally efficient in terms of iteration count and function/gradient evaluations. On average, around 68 percent of the test functions were minimized with a smaller number of iterations and evaluations. We show that the suggested strategy is globally convergent without requiring a convexity assumption on the objective function under certain conditions.The convergence of the derived methods is addressed for quadratic functions as well. The primary insight from this research is that scaling the terms of the modified BFGS update may result in more efficient algorithms than the classical BFGS algorithm. Choosing the values for the Secant equation scaling factors, on the other hand, is a non-trivial process. One approach would be to scale the update equation during a subset of the iterations and apply another scaling on other iterations as an outcome of some conditions. The next step is to look at a

new strategy prompted by the idea of combining more of the data available on each iteration than what normally standard quasi-Newton methods utilize and which lends itself to parallelization for accelerated convergence.

[1] J.A. Ford and I.A.R. Moghrabi. Multi-step methods for optimization. *Journal of Computational and Applied Math.*, **476**(50):305-316, 1994.

[2] J.A. Ford and I.A.R. Moghrabi. Alternative parameter choices for multi-step methods. *Optimization Methods and Software.* **476**(2):357-370, 1993.

68. Numerical analysis for PDE constrained optimization 11:55 – 13:10

***Chair*: J. Pfefferer, A. Rösch** **FH HS 5**

Finite element methods with boundary concentrated meshes for PDEs with irregular boundary data

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We study the Laplace equation with inhomogeneous Dirichlet boundary conditions. There are many applications, where the boundary data are very irregular, belonging to $H^t(\Gamma)$ with some $t \in [1/2, 3/2)$ only. This is for instance the case, when the boundary data are obtained from noisy measurements or are non-smooth for design or physical reasons. Similar effects arise when studying control-constrained Dirichlet boundary control problems.

We want to investigate a finite element discretization for these problems and show that the optimal convergence rates in the usual norms can be restored if the computational meshes are appropriately refined towards the boundary. The required refinement accuracy depends clearly on the regularity of the data. The refinement does not significantly increasing the computational effort. In numerical experiments we confirm the behavior predicted by our analysis.

On the stability of affine optimal control problems governed by semilinear ellipitc PDE's

Nicolai Jork (1),

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In this talk, we give a survey of recent results regarding the stability of affine optimal control problems governed by semilinear elliptic PDE's. We are especially interested in the stability of the controls and states with respect to perturbations in the normal cone corresponding to the optimal control problem. An application of these results can be found in obtaining error estimates for the Tikhonov regularization and numerical discretization schemes. The main purpose of this presentation is the introduction of new assumptions that guarantee stability and that are weaker compared to the ones used in the classical literature. Further, they enable us to obtain new results as well as easier proofs for some known facts.
This talk is based on the joint work of the speaker with Eduardo Casas Rentería, Alberto Domínguez Corella, and Vladimir Veliov.

69. Covid-19: optimal control approaches

11:55 – 13:10

***Chair*: D. Grass, S. Wrzaczek**

FH HS 6

Game with COVID-19 transmission, vaccination and budged

Radoslaw Matusik (1), Andrzej Nowakowski (2),

(1) University of Lodz, Lodz, Poland; (2) University of Lodz, Lodz, Poland;

We build a mathematical game model of COVID-19 transmission including vaccinations of population and budged costs of different acting to eliminate pandemic. We assume the interactions among different groups: vaccinated, susceptible, exposed, infectious, super-spreaders, hospitalized, fatality defining a system of ordinary differential equations which describes compartment model of disease and cost of treatment. The goal of the game is to describe the development disease under different type of treatment, but including cost of them and social restrictions, during the shortest time period. To this effect we construct a dual dynamic programming method to describe closed loop Nash equilibrium for treatment, a group of people having antibodies and budged costs. Next, we calculate numerically an approximate closed loop Nash equilibrium.

On the stochastic engine of transmittable diseases in exponentially growing populations

Torsten Lindström (1),

(1) Linnæus University, Växjö, Sweden;

The purpose of this paper is to analyze the mechanism for the interplay of deterministic and stochastic models for contagious diseases. Deterministic models for contagious diseases are prone to predict global stability. Small natural birth and death rates in comparison to disease parameters like the contact rate and the removal rate ensures that the globally stable endemic equilibrium corresponds to a tiny average proportion of infected individuals. Asymptotic equilibrium levels corresponding to low numbers of individuals invalidate the deterministic results.

Diffusion effects force frequency functions of the stochastic model to possess similar stability properties as the deterministic model. Particular simulations of the stochastic model predict, however, oscillatory patterns. Small and isolated populations show longer periods, more violent oscillations, and larger probabilities of extinction.

We prove that evolution maximizes the infectiousness of the disease as measured by the ability to increase the proportion of infected individuals. This holds provided the stochastic oscillations are moderate enough to keep the proportion of susceptible individuals near a deterministic equilibrium.

We close our paper with a discussion of the herd-immunity concept and stress its close relation to vaccination-programs.

[1] T. Lindström, On the stochastic engine of transmittable diseases in exponentially growing populations. `https://arxiv.org/pdf/2104.03254.pdf`, April 2021.

Vaccination and waning immunity model for Covid-19

Georgi Angelov (1), Raimund Kovacevic (1), Nikolaos I. Stilianakis (2), Vladimir M. Veliov (1),

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Waning immunity models used in epidemiology can provide insights for long term vaccination strategies and can help public health decision making. An optimal control epidemiological model is presented for the the evolution of Covid-19 disease. It is described in the setting of a changing immunity dynamics, for which the immunity protection of an individual against the disease evolves over time. Increase in immunity protection is provided either by infection or from vaccination, but the protection level wanes over time due to the diminishing antibodies and other factors. The protection level of an individual determines not only his susceptibility and recovery, but also his infectiousness. The model uses a system of first order hyperbolic PDEs that describe the immunity protection dynamics and the transitions between different compartments of susceptible, infected and vaccinated individuals. Based on antibody data and epidemiological characteristics of the waning immunity for COVID-19 we analyze optimal vaccination policies. Numerical simulations are provided for the quantitative estimation of optimal vaccination strategies.

70. Dynamics of firm — 11:55 – 13:10

***Chair*: P.M. Kort** — **FH HS 7**

Smart products with reputation loss and R&D : effects of the option to update

Nick F.D. Huberts (1), Herbert Dawid (2), Peter M. Kort (3),

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This paper considers the problem of a monopolist that can invest in R&D to improve the quality of a smart product (e.g., AVs). A higher quality directly results in a lower frequency of incidents caused by the product. Each incident leads to reputation damage for the firm. The R&D process is uncertain both in terms of duration and outcome. In addition, the firm holds two (nested) options: the option to launch the product on the market and the option to update the product after launch. The firm chooses not only the timing to exercise its options, it also chooses its R&D intensity and production capacity size. We analyze the problem of the firm as well as the problem of the social planner. In particular, we are interested in the impact of the option to update on the firm's R&D and investment strategy.

Flexible or dedicated production: strategic investment under uncertainty

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An investment decision involves choices about timing, size, and technology. Concerning the latter, this paper focuses on volume flexibility in the sense that the investing firm can choose between a dedicated technology, where the firm always has to produce up to capacity, and a volume flexible technology where the firm can produce below capacity.

The present paper considers such investment decisions in a duopoly framework with demand uncertainty. Clearly, choosing for a flexible technology has the advantage that the firm can adjust its production amount to different demand realizations. On the other hand, choosing a dedicated technology implies that the firm is committed to produce a certain amount, which is advantageous from a strategic point of view.

Our main results are threefold. First, firms choose different technologies in equilibrium, i.e., in case the first investor chooses to be dedicated, the second investor is better off to be flexible, and vice versa. Second, in case the first investor chooses for a dedicated technology, we show that the optimal time and size of the investment is not influenced by the follower's choice regarding a dedicated or flexible technology. Third, we find that in equilibrium there is rent equalization, the first investor is flexible and the second investor is dedicated.

Health care rationing, sick pay, and the socioeconomic health gradient

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This paper examines the effects of health care rationing and sick pay on the socioeconomic health gradient and welfare in a calibrated multi-period overlapping generations model with stochastic survival. In line with modern gerontology research, survival probabilities depend on accumulated health deficits. We advance the health deficit approach in two directions. First, we capture the dynamic interaction between health deficits and the event of illness. Second, we distinguish between curative health care that reduces average sick time of individuals and preventive health care that slows down the acquisition of bodily impairments and thereby reduces the risk of illness. Individuals can respond to rationing of curative care in health systems by adjusting private expenses for preventive and curative care. We show that these substitution effects substantially raise differences in longevity, morbidity and welfare between income groups. By contrast, more generous sick pay significantly reduces the socioeconomic health gradient and welfare differences.

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