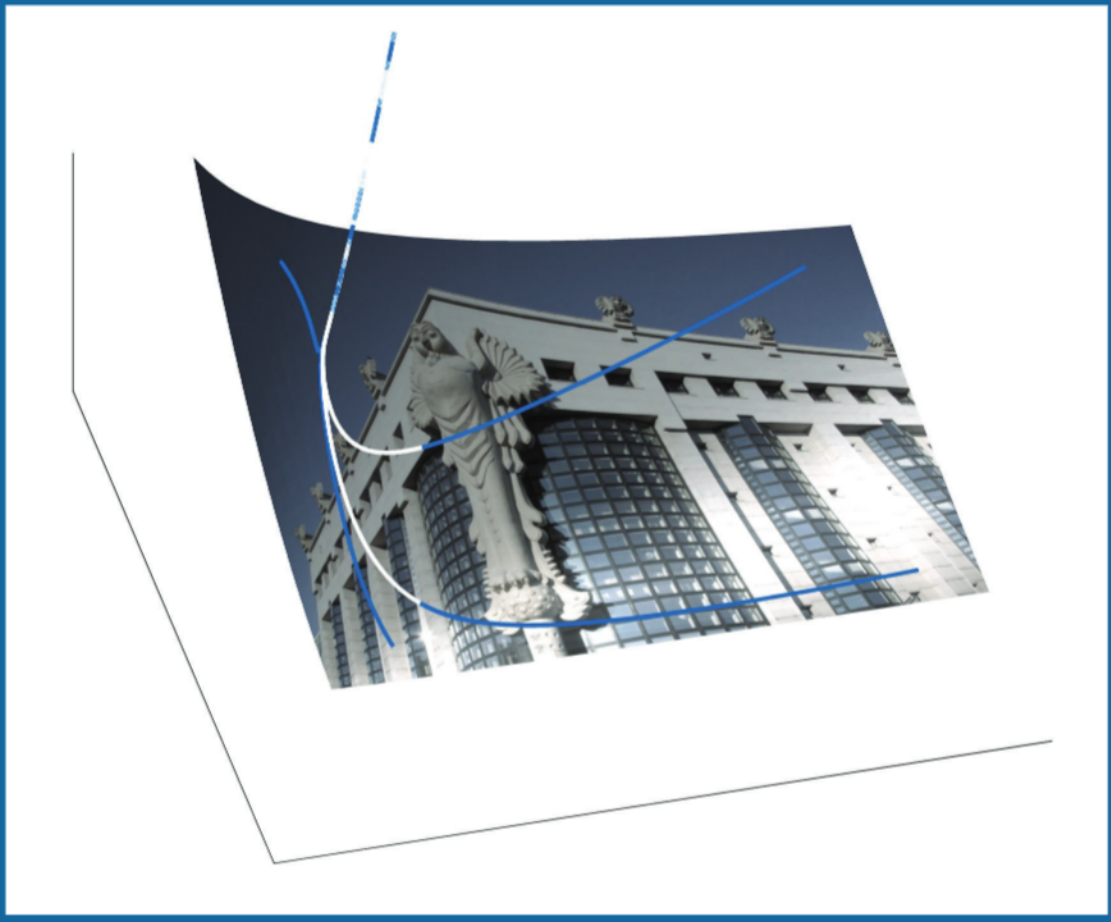
TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

16th Viennese Conference Optimal Control and Dynamic Games



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Contents

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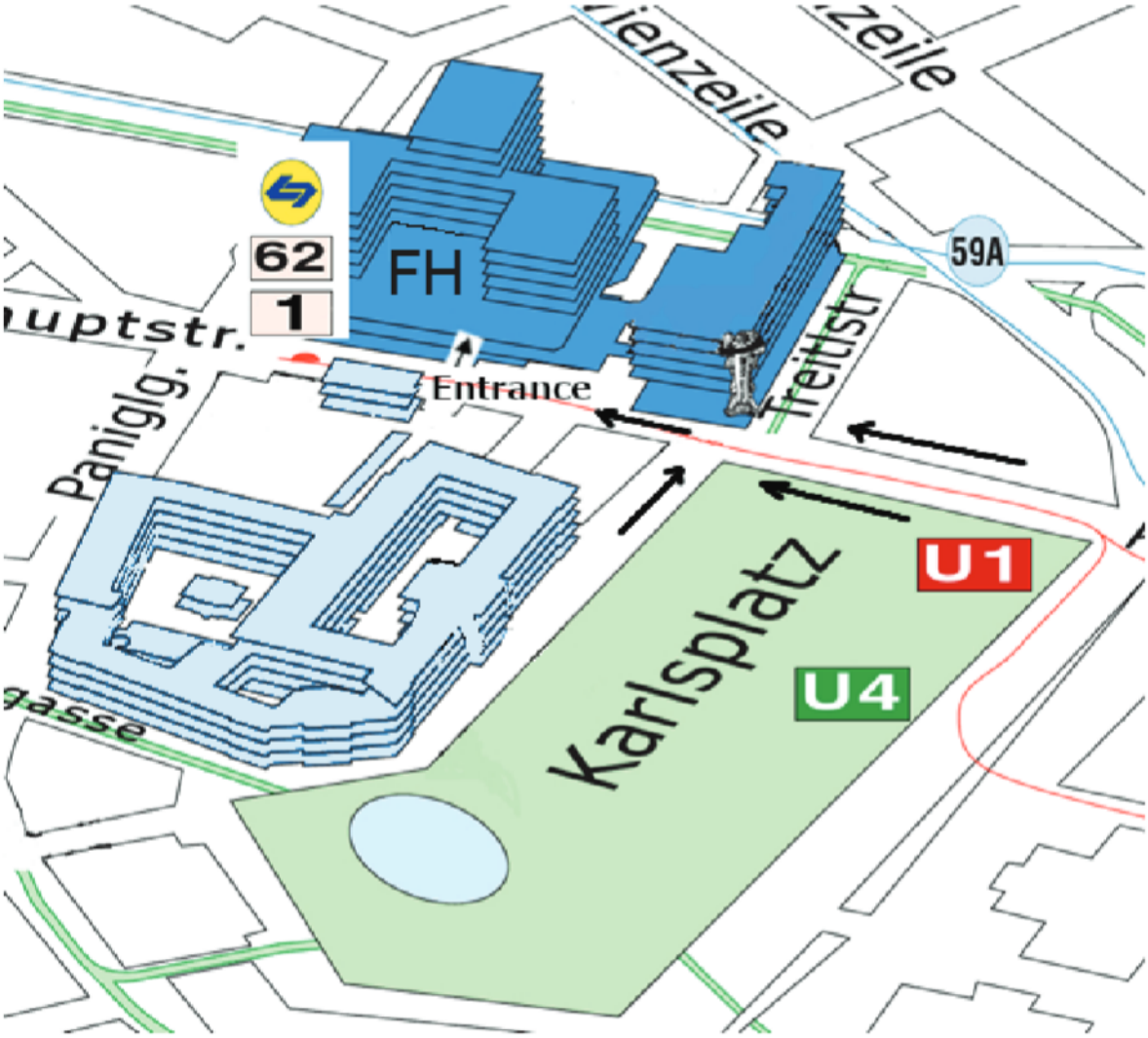
Raouf Boucekkine (FR)
Pierre Cardaliaguet (FR)
Xiaojun Chen (HK)
Barbara Kaltenbacher (AT)
Andreas Kugi (AT)
Gerd Wachsmuth (DE)

I. Practical Information

1. Conference Venue

Vienna University of Technology (TU Wien)

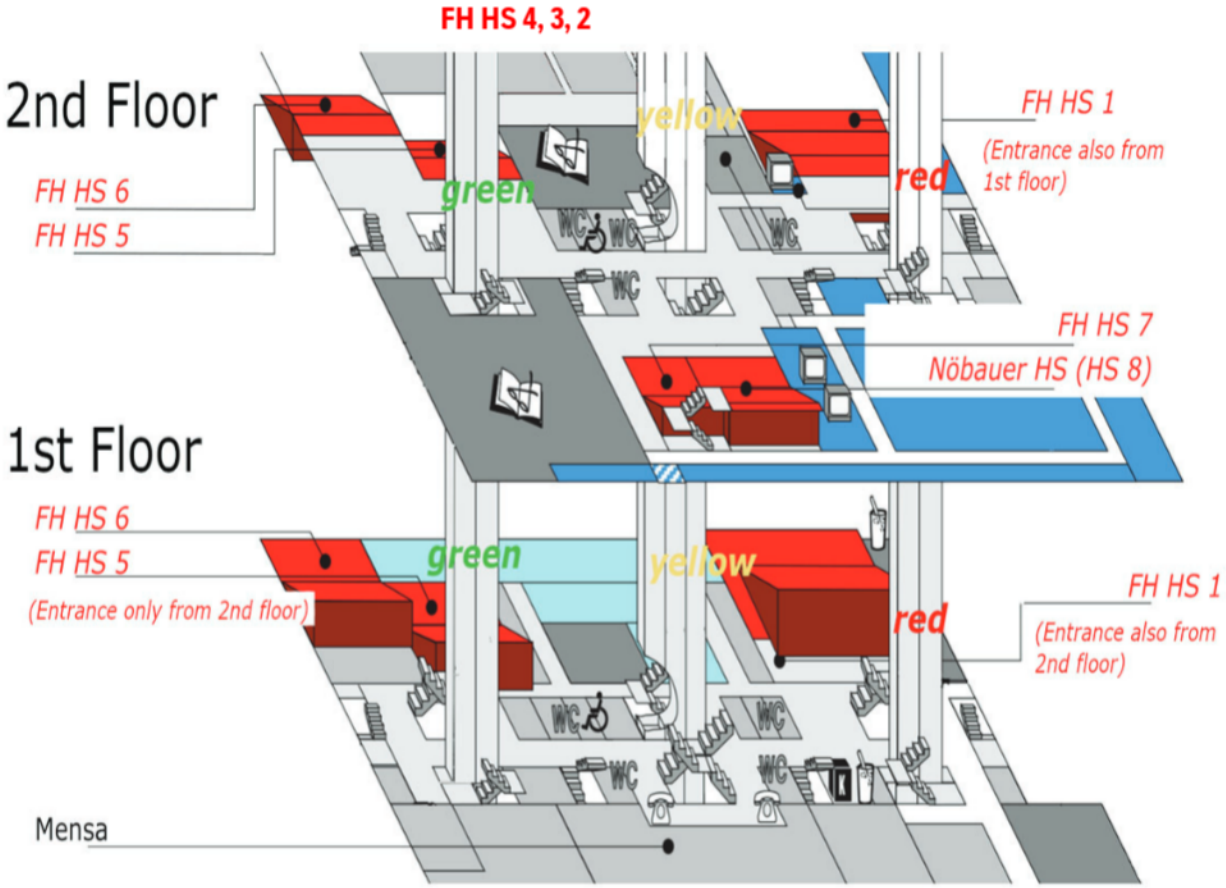
Vienna University of Technology is located in the heart of Vienna. The conference takes place in the „**Freihaus**“ (building FH), Wiedner Hauptstrasse 8-10, 1040 Vienna. As indicated on the map, the TU Wien can be reached by taking underground lines U1, U2 and U4 (NB closed between Schottenring and Friedensbrücke) to station „Karlsplatz“, or tram lines 1 or „Badner Bahn“ to station „Resselgasse“(Wieden).



Lecture Rooms

All lecture rooms are in the **Freihaus (FH)**, which is divided into obvious 3 areas - red, yellow and green.

- **HS1:** The main entrance is on the first floor of the red area. (Tues 15th only, registraton desk at 1st floor entrance). It is also accessible from the second floor
- **HS 2, 3 and 4:** From the 2nd floor yellow area aula, through the glass doors to the right of the lift.
- **HS 5:** entrance on the second floor of the green area
- **HS 6:** entrance behind the glass doors on the second floor of the green area
- **HS 7:** is located on the second floor of the yellow area (opposite the lifts)
- **HS 8 (Nöbauer):** is located on the second floor of the yellow area (opposite the lifts)
- **The Mensa** is located on the first floor in the yellow area
- **Yellow and Green Aula** coffee breaks & Conferenc Desk (yellow from Tuesday 15th 10am)



2. Conference Desk

The conference desk will be open on Monday 14th July from 17:00 until 19:00 in the Freihaus, first floor, yellow area, in front to the Mensa, where the welcome reception will begin at 19:00. Registraton will also be possible in front of HS1 first floor, red area on Tuesday 15th July from 07:45 until 09:30 and thereafter during morning coffee breaks at the conference desk in the 2nd floor aula in the yellow area.

Upon registration you will receive an information package including, vouchers for 3 onsite lunches in the university mensa and invitations for the social events. If you intend to stay for lunch on Friday 18th July, please advise the conference desk staff. Those who pre-prdered the conference booklet in print can collect them here also.

Conference4me App

This year, for the first time, we are trialing a conference app in the hope of aiding scheduling and communication.

Type ´Conference4me´ in Play Store/iTunes App Store/ Windows Phone Store or scan this code to download app.

3. Guidelines for Speakers and Session Organizers

All rooms are equipped with notebooks and projectors. Please prepare your presentation as pdf-file (recommended) or ppt-file on a USB memory-stick and copy it on the notebook **before** the session has started. Your presentation may last **20 minutes, including preparation and discussions**.

The session organizers are responsible for chairing their own sessions or providing an alternate. They must ensure they keep striclty to the schedule.

4. WLAN and Internet, charging of devices

We encourage all participants to use their eduroam accounts, if provided by their university. A limited number of TU-based internet accounts is available at the conference desk. It is possible to charge devices and work in HS2 from Tuesday to Thursday.

5. Lunches and Coffee Breaks

Upon registration you receive vouchers which can be redeemed at the mensa for 3 fixed menu lunches, one meat/one vegan option daily, with should or salad and one small bottle of water. The mensa is located at Freihaus, yellow area, first floor (see the map of Freihaus for details). It is self-service and will be open Tuesday to Friday over lunchtime only - see daily schedule for details. If you are going to be on site for lunch over 4 days, please let the conference desk know and we will issue a 4th voucher.

Lunch Menu

Tuesday 15.07.2025, 13:00 – 14:00

Veggie Menu with soup or salad:
BIO-Penne with paprika-leek-Ragout (A, F, O) VEGAN

Hearty Menu with soup or salad:
Lemon-coriander-chicken with Cous cous (A, O)

Wednesday 16.07.2025, 12:45 – 14:00

Veggie Menu with soup or salad:
Lentil curry mit basmati rice VEGAN

Hearty Menu with soup or salad:
„Mexiko“ wrap with beef mince and salad (A, C, G, M, O)

Thursday 17.07.2025, 12:45 – 14:00

Veggie Menu with soup or salad:
been goulash with BIO-Tofu and Kornspitz (bread) (A, F, L, O) VEGAN

Menü Herzhaft inkl. Suppe oder Salat:
Chicken rice (A, C, G, L, O)

Freitag 18.07.2025, 12:45 – 13:45

Veggie Menu with soup or salad:
Krautfleckerl (cabbage pasta) (A, C, O) VEGETARISCH

Hearty Menu with soup or salad:
Turkey goulash mit Spätzle (A, C, L, O)

Teas, coffees and water will be available during the programme breaks, morning and afternoon, until Friday
lunchtime.

Additionaly, there are many small shops and restaurants in the vicinity of the conference location.

6. Getting Around in Vienna

Airport transfer
There are frequent buses and trains going to and from the airport. The cheapest option is the "S-Bahn" (see summary of rates below).

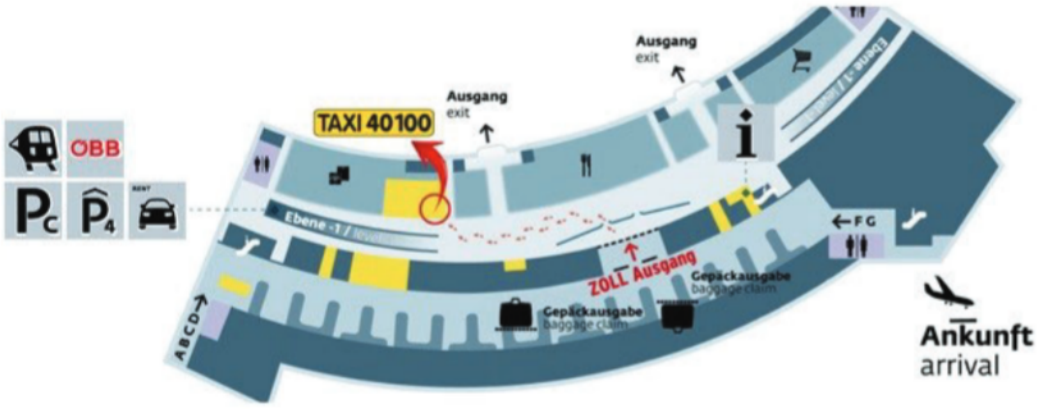
www.cityairporttrain.com,www.postbus.at, www.oebb.at.

	Place of Departure	Interval (min)	Duration (min)	Price (€)
Railjet	Vienna Central Station (Wien Hauptbahnhof— Südtiroler Platz)	~30	15	4.60
City Airport Train	Wien Mitte	~30	16	14,90
Airport Bus 1187	Westbahnhof (Europaplatz)	~30	45	11,-
Airport Bus 1185	Schwedenplatz (Morzinplatz)	~30	20	11,-
S-Bahn	Wien Mitte	~30	25	4,.60

Airport Taxi:

A taxi from/to the center costs about € 60.00, see www.taxi40100.at/flughafen (Tel: +43-(1) 40100).

The closest taxi rank to the university is Linke Wienzeile 4



All maps, timetables and changes to published schedules can also be found on www.wienerlinien.at.

Please note in Summer 2025 there is no service on the U4 between Friedensbrücke and Schottenring and no S-Bahn service between Wien Pratersten and Wien Floridsdorf. The underground runs from 5am until 12:20 am midweek, longer at weekends.

7. Social Program

Welcome Reception

The Welcome Reception will take place Monday, July 14th, from 18:00-21:00 in the Mensa on the first floor, yellow area. This event is covered by your registrations fee. You have the opportunity to collect your conference pack at the conference desk outside the mensa between .

Conference Dinner

The Mayor and Governor of Vienna requests the pleasure of your company at a drinks reception and buffet dinner on Tuesday July 15th, at the "Wiener Rathaus (City hall) - Wappensaal". Doors open 19:30, the event begins at 20:00.

Entrance: 1010 Vienna, Lichtenfelsgasse 2.

Public Transportation:

Tram 1 , D, U2 to station Rathausplatz/Burgtheater

Walking distance:

About 2 km (25 min) from TU Wien.

This event is covered by your registration fee. Please bring the invitation provided on registration.

Industry Session

An afternoon of talks by invited speakers, connecting science with industry, will take place after lunch on Wednesday, July 16th in HS 7 (2nd floor, yellow area).

Achieving great scientific/research results is one of the main drivers in academia, while having the results applied in the respective areas in the industry reinforces the success in a very organic way. Why then it is not easy to make them go hand in hand?

At VC2025 we would like to explore the opportunities for scientific conferences like this one, to bridge the gap between science and its application. This year we have invited 6 guest speakers, based in Vienna, to tackle this topic during a special session, chaired by Professor Filzmoser, TU Wien.

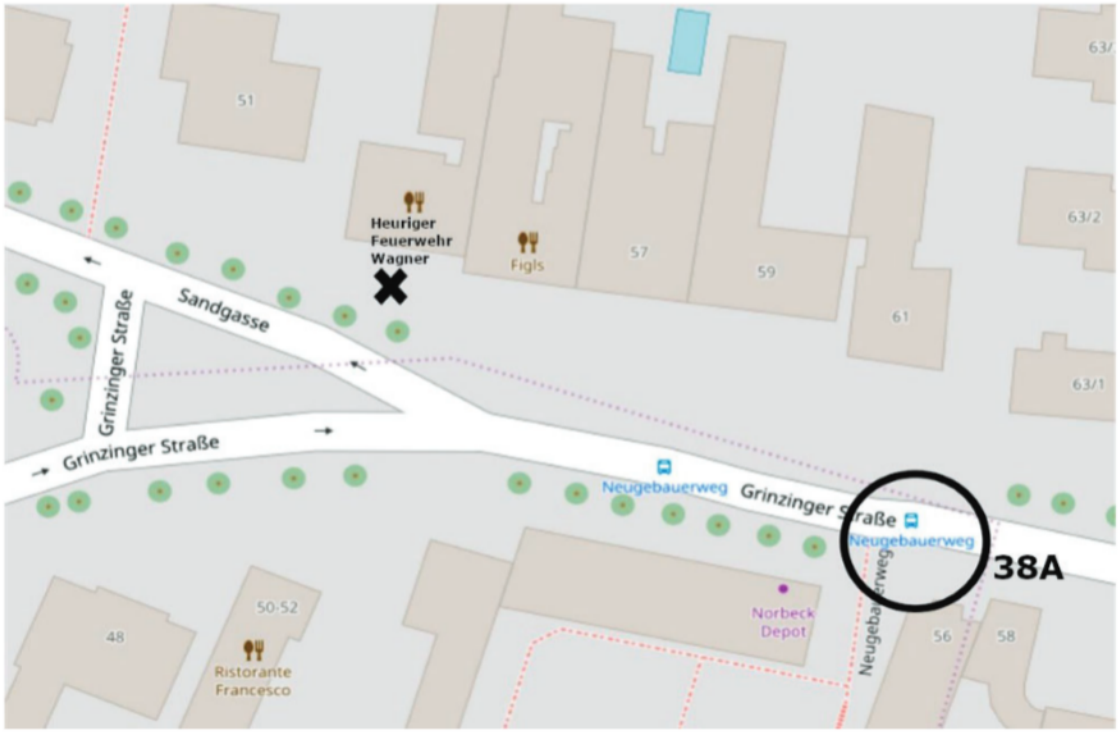
Discussion Dinner

This dinner offers the possibility for further discussion in the relaxed setting of "Feuerwehr Wagner", 1190 Vienna, Grinzinger Straße 53. Participants who paid extra for this dinner, which will be held on Thursday 17th July at 19:00 will receive and invitation in their conference pack.

Getting there:

Option 1: Take tram D from Schwarzenbergerplatz to Grinzinger Straße and then bus 38A to station Neugebauerweg. (Approx 1 hour)
Option 2: Take U2 from Karlsplatz to Schottentor, then tram 37 or 38 to Hohe Warte, this route is quicker, but involces a 7 minute walk to get to the Heuriger.

We will delegate members of staff to guide groups to this destination. Please note a valid public transport ticket is required before boarding.



8.Vienna Info

The tourist information office can be found the corner Albertinaplatz/Maysedergasse, 1010 Vienna
www.wien.info or www.wien.gv.at/tourismus

Tickets for Concerts and Theater

WIEN-TICKET Pavillon in the State Opera House, 1010 Vienna (5 minutes from Vienna University of
Technology)
www.wien-ticket.at/en

9. Supporters

We are grateful to Stadt Wien, Vienna Meeting Fund and TU Wien for their support.

II. Scientific Program

List of Sessions

Session name (time-slot number)	*Organizers*
Semi-Plenaries (2,3,48,49,80,81)	Scientific committee
Uncertainty Quantification in Optimization, Optimal Control, and Identification (4,9,14,19,24,30,36,42)	A. Khann, C. Tammer
Optimal control, backward stochastic differential equations and related topics (5,10,15)	S.Tang, K. S. Zhang
Hamilton-Jacobi equations for optimal control and games: new trends in numerical and analytical aspects (6,11,16,21,32)	M. Akian, D. Vasquez-Varas
Mean Field Control and Mean Field Games and Economic Applications (7,12,47,55)	D. Ghilli, M. Leocata
Dynamics of the firm (8,13,18,23,41)	P. Kort
Dynamic games in economics (17,22,27)	S.Wrzaczek, R. Neck
Control of dynamic systems and games with applications (20,25,89)	N. Hayek, S. Pickenhain
Advances in shape optimization (26)	M. Dambrine
Modelling disruptive changes and resilience in economic-environment systems (29)	M. Kuhn
Recent progress in PDE Constrained optimization (31,37,43,51,59,65)	P. Rösch, J. Pfefferer
Infinite dimensional optimal control and differential games in Economic (33,39,45,99)	F. Gozzi, F. Masiero, A. Zanco
Heterogeneity and inequality in population economics	A. Prskawetz, M. Sanchez-Romero (34)
Optimization in inverse problems (38)	K. Bredies
Recent advances in economic dynamics (40)	L. Deng
Operator and algebraic methods for games (44,52,60,66)	S. Gaubet, G. Vigeral
Do non linear feedback Nash equilibrium strategies really exist ? (46)	F. El Ouardighi, G. Feichtinger, S. Wrzaczek
Variational methods and transportation problems (50,57)	L. Mallozzi
Learning methods in optimal and predictive control (53,61,67,62,85,90)	L. Grüne, K. Worthmann
Industry session (54,62)	P. Filzmoser
Dynamic macroeconomics (56,63)	R. Neck
Set-valued mappings in approximation, control and optimization (58,83,88)	R. Baier, E. Farkhi
Control of systems in Wasserstein spaces (64,69,74,81)	H. Frankowska, M. Quincampoix
Mathematical models for human impact on the environment (68,73,78)	G. Fabbri, S. Faggian, F. Gozzi
Feedback control and stabilization of nonlinear PDE (70,75,96,101)	S. S. Rodrigues

Session name (time-slot number)	*Organizers*
Topology optimization: theory, numerics and applications (71,76,97,102)	K. Sturm, P. Gangl
Using Model Predictive Control, Reinforcement Learning, and related methods for risk (77)	W. Semmler
Continuous optimization: theory and applications (82,87,92,95,100,107)	R. Bot, Y. Malitsky, A. Daniilidis
Dynamic management of natural resources (86, 91)	T. Upmann, D. Gromov
Stochastic control and applications (93,98,103)	A. Calvia, E. Bandini
Economics of climate change (106)	S. Wrzaczek, M. Freiberger, M. Kuhn
Supply chain (28)	Scientific committee
Industry dynamics (35,94)	Scientific committee
Health (84) Environmental (104,108)	Scientific committee Scientific committee
Networks (105)	Scientific committee
Poster session	Scientific committee

TUESDAY 15th	SAAL 8	SAAL 6	SAAL 4	SAAL 5	SAAL 7
07:30 - 08:30	Registration				
08:30 - 09:10	**1 Opening (SAAL 1)**				
09:10 - 11: 00		**2 P. Cardaliaguet (SAAL 1)**		**3 A. Kugi**	
COFFEE					
Session Chair	**4 Khan, Tammer**	**5 Tang, Zhang**	**6 Akian, Vasquez**	**7 Ghilli, Leocata**	**8 Kort**
10:20 - 11:35	O. Scherzer	Q. Zhang	R. Ferretti	Ş.-L. Aniţa	H. Jeon
	J. Chirinos-Rodriguez	J. Zhang	B. Höveler	E. Augeraud-Veron	M. Nishihara
	S. Kindermann	S. Fan	O. Bokanowski	F. Camilli	Y. Tian
BREAK	**9 Khan, Tammer**	**10 Tang, Zhang**	**11 Akian, Vasquez**	**12 Ghilli, Leocata**	**13 Kort**
11:45 - 13:00	F. Tomarelli	K.S. Zhang	Y. Huang	L. Campi	L. Gan
	V.A. Kovtunenko	J. Li	W.M. McEneaney	M. Ricciardi	N.A. Bun
	A. Aspri	K. Du	A. Bulaich Mehamdi	P.J. Graber	
LUNCH BREAK					
Session Chair	**14 Khan, Tammer**	**15 Tang, Zhang**	**16 Akian, Vasquez**	**17 Wrzaczek, Neck**	**18 Kort**
14:00 - 15:15	B. Kaltenbacher	P. Luo	D. Walter	J.E. Márquez-Prado	P.V. Gapeev
	B. Jadamba	Y. Lin	D. Vásquez-Varas	L. Lambertini	X. Wen
	S. Bellavia	D.W. Ngwenya	J. Schröder	A.A. Bondarev	P. Mwangi
COFFEE					
Session Chair	**19 Khan, Tammer**	**20 Hayek, Pickenhain**	**21 Akian, Vasquez**	**22 Wrzaczek, Neck**	**23 Kort**
15:35 - 16:50	E. Hernández	F. Bagagiolo	J.-M. Mirebeau	M. Wrona	M. Castellini
	E. Molho	M.P. Martínez-García	M. Akian	F. Cabo	K. Kogan
	B. Hernández-Jiménez	H. Maurer	V. Coscetti	R. Singh	R. Chenavaz
BREAK					
Session Chair	**24 Khan, Tammer**	**25 Hayek, Pickenhain**	**26 Dambrine**		**29 Kuhn**
17:00 - 18:40	C. Giersbach	J. de Frutos	V. Karnaev		S. Wrzaczek
	L. Seppecher	N. Hayek	H. Harbrecht		M. Haringa
	D.I. Gendin	I. Dikariev	M. Dambrine		M. Kuhn
CLOSE	Y. Yang	J. Blot	Y. Privat		I. Tahri

WEDNESDAY 16th	SAAL 8	SAAL 6	SAAL 4	SAAL 5	SAAL 7	SAAL 3
Session Chair	**30 Khan, Tammer**	**31 Rösch, Pfefferer**	**32 Akian, Vasquez**	**33 Gozzi, Masiero, Zanco**	**34 Prskawetz, Sanchez-Romero**	**35 Dawid**
08:30 - 09:45	C. Tammer	F. Tröltzsch	J. Cui	W. Stannat	G. Feichtinger	H. Dawid
	M.M. Wiecek	H. Meinlschmidt	C. Esteve-Yagüe	C. Bertucci	H. Strulik	F. Lamantia
	J. Vincente-Pérez	J. Schröder	Y. Halevi	S. Rudà	M. Freiberger	L. Colombo
COFFEE	**36 Khan, Tammer**	**37 Rösch, Pfefferer**	**38 Bredies**	**39 Gozzi, Masiero, Zanco**	**40 Deng**	**41 Kort, Kogan**
10:05 - 11:20	K. Liu	B. Vexler	J. Chirinos-Rodriguez	M. Fabrice Djete	B. Zou	F. El Ouardighi
	H.-J. Starkloff	K. Lorenz	M. Gharbi	L. Wessels	T. Ha-Huy	Y.Y Yang
	A. Barbagallo	O. Holroyd	H. Huang	L. Chen	L. Deng	A. Seidl
BREAK	**42 Khan, Tammer**	**43 Rösch, Pfefferer**	**44 Gaubert, Vigeral**	**45 Gozzi, Masiero, Zanco**	**46 El Ouardighi, Feichtinger, Wrzaczek**	**47 Ghilli, Leocata**
11:30 - 12:45	A.A. Khan	D. Leykekhman	I. Portakal	G. Fabbri	L. Lambertini	A. Calvia
		M. Winkler	T. Theobald	F. Bucci	F. Wirl	L. Marzufero
		K. Chrysafinos	E. Tsigaridas	G. Bolli	F.O.O. Wagener	A.R.Mésráros
LUNCH BREAK						
14:00 - 14:50	**48 B. Kaltenbacher**			**49 R. Boucekkine**		
COFFEE						
Session Chair	**50 Mallozzi**	**51 Rösch, Pfefferer**	**52 Gaubert, VIgeral**	**53 Grüne, Worthmann**	**54 Industry session**	**55 Ghilli, Leocata**
15:10 - 16:25	S. Guarino Lo Bianco	C. Lange	S. Sorin	P. Rapisarda	TU Introduction	A. Ocello
	E. Radici	N. Haaf	I. Novikov	G. Pan	C. Mülmann, C. Rieser	D.F. Redaelli
	F. Caruso	W. Wollner	B. Ziliotto	J. Eising	C. Krischanitz	**56 Neck**
						Ç. Sağlam
BREAK						
Session Chair	**57 Mallozzi**	**59 Rösch, Pfefferer**	**60 Gaubert, Vigeral**	**61 Grüne, Worthmann**	**62 Industry session**	**63 Neck**
16:40 - 17:55	G. Zecca	E. Casas	G. Vigeral	M. Sperl	J.F. Ehmke	B. Zou
	F. Angrisani	S.S. Rodrigues	R. Saona	T. Kruse	T. Heinzle	L. Cerboni Baiardi
	58 Baier, Farkhi	M. Stahl	T. Ming Lê	S. Hall	M. Drauch	F. Del Rio
CLOSE	E. Farkhi	J. Marko		R. Denkert		R. Neck
	S. Apostolov					

THURSDAY 17th	SAAL 8	SAAL 6	SAAL 4	SAAL 5	SAAL 7	SAAL 2
Session Chair	**64 Frankowska, Quincampoix**	**65 Rösch, Pfefferer**	**66 Gaubert, Vigeral**	**67 Grüne, Worthmann**	**68 Fabbri, Faggian, Gozzi**	**27 Wrzaczek, Neck**
08:30 - 09:45	J. Li	M. Mateos	L. Marchesini	R. Schurig	G.I. Bischi	D. Machowska
	R. Buckdahn	D. Wachsmuth	S. Gaubert	M. Schaller	S. Bosi	**28 Supply chain**
	M. Quincampoix	D. Walter	A. Aradhye	R. Strässer	T. Upmann	S. Mesrar
COFFEE	**69 Frankowska, Quincampoix**	**70 S.S. Rodrigues**	**71 Sturm, Gangl**	**72 Grüne, Worthmann**	**73 Fabbri, Faggian, Gozzi**	P. Daniele
10:05 - 11:20	R. Capuani	B. Azmi	S. Amstutz	L. Guo	A. Wiszniewska-Matyszkiel	M. Qaisari Hasan Aba
	A. Scagliotti	S. Court	L. Blank	C. Hartmann	A. Martiradonna	**104 Environmental**
	G. Cavagnari	L.M. Moschen	N. Krenn	A. Jain	T. Loch-Temzelides	J. Guo
						T. Palokangas
BREAK	**74 Frankowska, Quincampoix**	**75 S.S. Rodrigues**	**76 Sturm, Gangl**	**77 Semmler**	**78 Fabbri, Faggian, Gozzi**	
11:30-12:45	A. Marigonda	B. Kramer	S. Blauth	W. Semmler	J. Mierzwa	
	C. Pignotti	S.N. Gomes	L. Baeck	L. Grüne	G. Pucci	
	C.Amend	B. Priyasad	M. Winkler	T. Khundadze	M. Leocata	
LUNCH BREAK						
14:00 - 14:50	**79 G. Wachsmuth**			**80 X. Chen**		
COFFEE						
Session Chair	**81 Frankowska, Quincampoix**	**83 Baier, Farkhi**	**84 Health**	**85 Grüne, Worthmann**	**86 Upmann, Gromov**	**FREIHAUS**
15:10 - 16:25	Cardaliaguet	R. Baier	A. Nowakowski	M. Pfefferkorn	T. Lindström	POSTER SESSION
	82 Bot, Malitsky, Daniilidis	W. Achtziger	A. Pérez-Barahona	J. Köhler	F. Gozzi	POSTER SESSION
	Armeniakos	H. Sendov		S. Maslovskaya	S. Faggian	
	G. López					
BREAK						
Session Chair	**87 Bot, Malitsky, Daniilidis**	**88 Baier, Farkhi**	**89 Hayek, Pickhenhain**	**90 Grüne Worthmann**	**91 Upmann, Gromov**	
16:40 - 17:55	M.D.E. Giordano	T. Roubal	M. Nikolova	L. Saluzzi	F. Langelaan	
	T. Moquet	A. Mokhov	N.K. Ribarska	M. Nonhoff	T. Upmann	
	T. Scarinci		M.I. Krastanov	M. Gómez	D. Gromov	
				A. Parra Lafuente		

FRIDAY 18th	SAAL 8	SAAL 6	SAAL 4	SAAL 5	SAAL 7
Session Chair	**92 Bot, Malitsky, Daniilidis**			**93 Calvia, Bandini**	**94 Dawid**
08:30 - 09:45	M. Staudigl			C. Di Girolami	L. Kapera
	C. Schindler			C. Keller	N. Borchard
	Y. Zhang			I. Kharroubi	T. Makarewicz
COFFEE	**95 Bot, Malitsky, Daniilidis**	**96 S.S. Rodrigues**	**97 Sturm, Gangl**	**98 Calvia, Bandini**	**99 Gozzi, Masiero, Zanco**
10:05 - 11:20	P. Georgiev	M. Bailová	S. Zhu	F. Masiero	Aleandri
	A. Yurtsever	N. Schlosser	G. Starke	M. Martini	J.E. Márquez-Prado
	D.A. Hulett	M. Kartmann	A. Laurain	A. De Crescenzo	
BREAK	**100 Bot, Malitsky, Daniilidis**	**101 S.S. Rodrigues**	**102 Sturm, Gangl**	**103 Calvia, Bandini**	**105 Networks**
11:30-12:45	E. Chenchene	W. Al-Khulaifi	P. Baumann	D. Ghili	Y. Tsodikovich
	S. Villa	M. Fritz	P. Beremlijski		F. Herold
	R. Nenov	V. Arzt			**106 Economics of climate change**
					M. Hillebrand
LUNCH BREAK					
Session Chair	**107 Bot, Malitsky, Daniilidis**				
13:45 - 14:35	S. Boshoff				
	L. Jin				

Tuesday, July 15^{th}

1 Opening **08:30 – 09:10**
FH HS 1

2 Semi-plenary **09:10 – 10:00**
***Chair*: S. Gaubert** **FH HS 1**

3 Semi-plenary **09:10 – 10:00**
***Chair*: G. Tragler** **FH Nöbauer 8**

Coffee Break

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4 Uncertainty Quantification in Optimization, Optimal Control, and Identification **10:20 – 11:35**
***Chair*: A. Khan, C. Tammer** **FH Nöbauer 8**

5 Optimal control, backward stochastic differential equations and related topics **10:20 – 11:35**
***Chair*: S. Tang, K. S. Zhang** **FH HS 6**

6 Hamilton-Jacobi equations for optimal control and games: new trends in numerical and analytical aspects **10:20 – 11:35**
***Chair*: M. Akian, D. Vasquez-Varas** **FH HS 4**

7 Mean Field Control and Mean Field Games and Economic Applications **10:20 – 11:35**
***Chair*: D. Ghilli, M. Leocata** **FH HS 5**

8 Dynamics of the firm **10:20 – 11:35**

***Chair*: P. Kort, X. Wen** **FH HS 7**

Break

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9 Uncertainty Quantification in Optimization, Optimal Control, and Identification **11:45 – 13:00**

***Chair*: A. Khan, C. Tammer** **FH Nöbauer 8**

10 Optimal control, backward stochastic differential equations and related topics **11:45 – 13:00**

***Chair*: S. Tang, K. S. Zhang** **FH HS 6**

11 Hamilton-Jacobi equations for optimal control and games: new trends in numerical and analytical aspects **11:45 – 13:00**

***Chair*: M. Akian, D. Vasquez-Varas** **FH HS 4**

12 Mean Field Control and Mean Field Games and Economic Applications **11:45 – 13:00**

***Chair*: D. Ghilli, M. Leocata** **FH HS 5**

13 Dynamics of the firm **11:45 – 13:00**

***Chair*: P. Kort, P. Gapeev** **FH HS 7**

Lunch Break

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14 Uncertainty Quantification in Optimization, Optimal Control, and Identification **14:00 – 15:15**

***Chair*: A. Khan, C. Tammer** **FH Nöbauer 8**

15 Optimal control, backward stochastic differential equations and related topics **14:00 – 15:15**

***Chair*: S. Tang, K. S. Zhang** **FH HS 6**

16 Hamilton-Jacobi equations for optimal control and games: new trends in numerical and analytical aspects **14:00 – 15:15**

***Chair*: M. Akian, D. Vasquez-Varas** **FH HS 4**

17 Dynamic games in economics **14:00 – 15:15**

***Chair*: S. Wrzaczek, R. Neck** **FH HS 5**

18 Dynamics of the firm **14:00 – 15:15**

***Chair*: P. Kort, M. Nishihara** **FH HS 7**

Coffee Break

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19 Uncertainty Quantification in Optimization, Optimal Control, and Identification **15:35 – 16:50**

***Chair*: A. Khan, C. Tammer** **FH Nöbauer 8**

20 Control of dynamic systems and games with applications **15:35 – 16:50**

***Chair*: N. Hayek, S. Pickenhain** **FH HS 6**

21 Hamilton-Jacobi equations for optimal control and games: new trends in numerical and analytical aspects **15:35 – 16:50**

***Chair*: M. Akian, D. Vasquez-Varas** **FH HS 4**

22 Dynamic games in economics **15:35 – 16:50**

***Chair*: S. Wrzaczek, R. Neck** **FH HS 5**

23 Dynamics of the firm **15:35 – 16:50**

***Chair*: P. Kort, El Ouardighi** **FH HS 7**

Break

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24 Uncertainty Quantification in Optimization, Optimal Control, and Identification **17:00 – 18:40**

***Chair*: A. Khan, C. Tammer** **FH Nöbauer 8**

25 Control of dynamic systems and games with applications **17:00 – 18:40**

***Chair*: N. Hayek, S. Pickenhain** **FH HS 6**

26 Advances in shape optimization **17:00 – 18:40**

***Chair*: M. Dambrine** **FH HS 4**

29 Modelling disruptive changes and resilience in economic-environment systems **17:00 – 18:40**

***Chair*: M. Kuhn** **FH HS 7**

Wednesday, July 16th

30 Uncertainty Quantification in Optimization, Optimal Control, and Identification **08:30 – 09:45**

***Chair*: A. Khan, C. Tammer** **FH Nöbauer 8**

31 Recent progress in PDE Constrained optimization **08:30 – 09:45**

***Chair*: P. Rösch, J. Pfefferer** **FH HS 6**

32 Hamilton-Jacobi equations for optimal control and games: new trends in numerical and analytical aspects **08:30 – 09:45**

***Chair*: M. Akian, D. Vasquez-Varas** **FH HS 4**

33 Infinite dimensional optimal control and differential games in Economics **08:30 – 09:45**

***Chair*: F. Gozzi, F. Masiero, A. Zanco** **FH HS 5**

34 Heterogeneity and inequality in population economics **08:30 – 09:45**

***Chair*: A. Prskawetz, M. Sanchez-Romero** **FH HS 7**

35 Industry Dynamics **08:30 – 09:45**

***Chair*: H. Dawid** **FH HS 3**

Coffee Break

36 Uncertainty Quantification in Optimization, Optimal Control, and Identification **10:05 – 11:20**

***Chair*: A. Khan, C. Tammer** **FH Nöbauer 8**

37 Recent progress in PDE Constrained optimization **10:05 – 11:20**

***Chair*: P. Rösch, J. Pfefferer** **FH HS 6**

38 Optimization in inverse problems **10:05 – 11:20**

***Chair*: K. Bredies** **FH HS 4**

39 Infinite dimensional optimal control and differential games in Economics **10:05 – 11:20**

***Chair*: F. Gozzi, F. Masiero, A. Zanco** **FH HS 5**

40 Recent advances in economic dynamics **10:05 – 11:20**

***Chair*: L. Deng** **FH HS 7**

41 Dynamics of the firm **10:05 – 11:20**

***Chair*: P. Kort, K. Kogan** **FH HS 3**

Break

42 Uncertainty Quantification in Optimization, Optimal Control, and Identification **11:30 – 12:45**

***Chair*: A. Khan, C. Tammer** **FH Nöbauer 8**

43 Recent progress in PDE Constrained optimization **11:30 – 12:45**

***Chair*: P. Rösch, J. Pfefferer** **FH HS 6**

44 Operator and Algebraic Methods for Games **11:30 – 12:45**

***Chair*: S. Gaubert, G. Vigeral** **FH HS 4**

45 Infinite dimensional optimal control and differential games in Economics **11:30 – 12:45**

***Chair*: F. Gozzi, F. Masiero, A. Zanco** **FH HS 5**

46 Do nonlinear feedback Nash equilibrium strategies really exist? **11:30 – 12:45**

***Chair*: F. El Ouardighi, G. Feichtinger, S. Wrzaczek** **FH HS 7**

47 Mean Field Control and Mean Field Games and Economic Applications **11:30 – 12:45**

***Chair*: D. Ghilli, M. Leocata** **FH HS 3**

Lunch Break

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48 Semi-plenary **14:00 – 14:50**

***Chair*: M. Dambrine** **FH Nöbauer 8**

49 Semi-plenary **14:00 – 14:50**

***Chair*: A. Fürnkranz-Prskawetz** **FH HS 5**

Coffee Break

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50 Variational Methods and Transportation Problems **15:10 – 16:25**

***Chair*: L. Mallozzi** **FH Nöbauer 8**

51 Recent progress in PDE Constrained optimization **15:10 – 16:25**

***Chair*: P. Rösch, J. Pfefferer** **FH HS 6**

52 Operator and Algebraic Methods for Games **15:10 – 16:25**

***Chair*: S. Gaubert, G. Vigeral** **FH HS 3**

53 Learning methods in optimal and predictive control **15:35 – 16:25**

***Chair*: L. Grüne, K. Worthmann** **FH HS 5**

54 Industry session **15:10 – 16:25**

***Chair*: P. Filzmoser** **FH HS 7**

55 Mean Field Control and Mean Field Games and Economic Applications **15:10 – 16:00**

***Chair*: D. Ghilli, M. Leocata** **FH HS 3**

56 Dynamic Macroeconomics **16:00 – 16:25**

***Chair*: R. Neck** **FH HS 3**

Break

Thursday, July 17^{th}

Coffee Break

69 Control of systems in Wasserstein spaces **10:05 – 11:20**
***Chair*: H. Frankowska, M. Quincampoix** **FH Nöbauer 8**

70 Feedback control and stabilization of nonlinear PDEs **10:05 – 11:20**
***Chair*: S. S. Rodrigues** **FH HS 6**

71 Topology optimization: theory, numerics and applications **10:05 – 11:20**
***Chair*: K. Sturm, P. Gangl** **FH HS 4**

72 Learning methods in optimal and predictive control **10:05 – 11:20**
***Chair*: L. Grüne, K. Worthmann** **FH HS 5**

73 Mathematical Models for Human Impact on the environment **10:05 – 11:20**
***Chair*: G. Fabbri, S. Faggian, F. Gozzi** **FH HS 7**

104 Environmental **10:30 – 11:20**
***Chair*: S. Wrzaczek, R. Neck** **FH HS 2**

Break

74 Control of systems in Wasserstein spaces **11:30 – 12:45**
***Chair*: H. Frankowska, M. Quincampoix** **FH Nöbauer 8**

75 Feedback control and stabilization of nonlinear PDEs **11:30 – 12:45**
***Chair*: S. S. Rodrigues** **FH HS 6**

76 Topology optimization: theory, numerics and applications **11:30 – 12:45**
***Chair*: K. Sturm, P. Gangl** **FH HS 4**

77 Using Model Predictive Control, Reinforcement Learning, and Related Methods for Risk Control **11:30 – 12:45**
***Chair*: W. Semmler** **Saal**

78 Mathematical Models for Human Impact on the environment **11:30 – 12:45**
***Chair*: G. Fabbri, S. Faggian, F. Gozzi** **FH HS 7**

Lunch Break

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79 Semi-plenary **14:00 – 14:50**
***Chair*: M. Quincampoix** **FH Nöbauer 8**

80 Semi-plenary **14:00 – 14:50**
***Chair*: L. Grüne** **FH HS 5**

Coffee Break

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81 Control of systems in Wasserstein spaces **15:10 – 15:35**

***Chair*: H. Frankowska, M. Quincampoix** **FH Nöbauer 8**

82 Continuous Optimization: Theory and Applications **15:35 – 16:25**

***Chair*: R. Bot, Y. Malitsky, A. Daniilidis** **FH Nöbauer 8**

83 Set-valued mappings in approximation, control and optimization **15:35 – 16:25**

***Chair*: R. Baier, E. Farkhi** **FH HS 6**

84 Health **15:10 – 16:00**

***Chair*: M. Freiberger** **FH HS 5**

85 Learning methods in optimal and predictive control **15:35 – 16:25**

***Chair*: L. Grüne, K. Worthmann** **FH HS 5**

86 Dynamic management of natural resources **15:35 – 16:25**

***Chair*: T. Upmann, D. Gromov** **FH HS 7**

Break

87 Continuous Optimization: Theory and Applications **16:40 – 17:55**

***Chair*: R. Bot, Y. Malitsky, A. Daniilidis** **FH Nöbauer 8**

88 Set-valued mappings in approximation, control and optimization **16:40 – 17:55**

***Chair*: R. Baier, E. Farkhi** **FH HS 6**

89 Control of dynamic systems and games with applications **16:40 – 17:55**

***Chair*: N. Hayek, S. Pickenhain** **FH HS 4**

90 Learning methods in optimal and predictive control **16:40 – 17:55**

***Chair*: L. Grüne, K. Worthmann** **FH HS 5**

91 Dynamic management of natural resources **16:40 – 17:55**

***Chair*: T. Upmann, D. Gromov** **FH HS 7**

Friday, July 18th

92 Continuous Optimization: Theory and Applications **08:30 – 09:45**
***Chair*: R. Bot, Y. Malitsky, A. Daniilidis** **FH Nöbauer 8**

93 Stochastic Control and Applications **08:30 – 09:45**
***Chair*: A. Calvia, E. Bandini** **FH HS 5**

94 Industry Dynamics **08:30 – 09:45**
***Chair*: H. Dawid** **FH HS 7**

Coffee Break

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95 Continuous Optimization: Theory and Applications **10:05 – 11:20**
***Chair*: R. Bot, Y. Malitsky, A. Daniilidis** **FH Nöbauer 8**

96 Feedback control and stabilization of nonlinear PDEs **10:30 – 11:20**
***Chair*: S. S. Rodrigues** **FH HS 6**

97 Topology optimization: theory, numerics and applications **10:05 – 11:20**
***Chair*: K. Sturm, P. Gangl** **FH HS 4**

104 Networks **11:30 – 12:15**

***Chair*: S. Wrzaczek, M. Freiberger, M. Kuhn** **FH HS 7**

105 Economics of climate change **12:15 – 12:45**

***Chair*: S. Wrzaczek, M. Freiberger, M. Kuhn** **FH HS 7**

Lunch Break

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106 Continuous Optimization: Theory and Applications **13:45 – 15:00**

***Chair*: R. Bot, Y. Malitsky, A. Daniilidis** **FH Nöbauer 8**

Poster session

Poster Session, Thursday, July 17^{th}

107 Poster session **15:10 – 16:00**

Freihaus, 2^{nd} *Floor*

Abstracts

Tuesday, July 15^{th}

1. Opening — 08:30 – 09:10 — FH HS 1

2. Semi-plenary — 09:10 – 10:00 — FH HS 1

Chair: S. Gaubert

On the optimal control of population dynamics

Pierre Cardaliaguet (1),

(1) Ceremade, Université Paris-Dauphine, France;

In this talk I will present recent results on the optimal control of a large population of devices. This can be for instance the optimal charging of a large number of electric vehicles, or the optimal control of a traffic flow through a junction. When the population is large, one expects the optimization problem to look like the optimal control of a continuous density (a mean field control problem). We will discuss the validity of this heuristic, as well as the optimality conditions for the limit problem and issues related to the associated Hamilton-Jacobi equation.

3. Semi-plenary — 09:10 – 10:00 — FH Nöbauer 8

Chair: G. Tragler

Optimization-based planning and control for adaptive robotic manipulation

Andreas Kugi (1, 2),

(1) Automation and Control Institute (ACIN), TU Wien, Vienna, Austria; (2) AIT Austrian Institute of Technology, Vienna, Austria;

Optimization-based methods are key to enabling robotic systems to operate safely, efficiently, and adaptively in complex and dynamic environments. This talk presents recent advances in real-time path planning and trajectory optimization for robot manipulators, specifically tailored to interact with high-level task specifications from language and vision-language models (LLMs/VLMs).

We introduce a modular planning architecture in which LLMs generate task logic and intermediate goals, while a convex-set-based path planner and a novel model-predictive path-following approach ensure collision-free and kinematically feasible execution. Unlike traditional path-following methods, our approach purposefully deviates from Cartesian reference paths to accommodate kinematic constraints, leading to more robust task execution. The planner is computationally efficient and supports real-time replanning, enabling responsiveness to dynamic changes in the environment and goals.

Beyond manipulation, the talk explores how manufacturing tolerances, process windows, and tool redundancies can be systematically exploited in optimization-based planning. We also discuss strategies for optimizing robot base placement and tool center point positioning in spatially constrained environments.

Extensive experimental results highlight the practical potential of tightly integrated planning and control schemes across a variety of real-world robotic applications.

4. Uncertainty Quantification in Optimization, Optimal Control, and Identification

10:20 – 11:35

Chair: **A. Khan, C. Tammer** **FH Nöbauer 8**

Data driven regularization by projection

Andrea Aspri (1), Martin Hanke (2), Yury Korolev (3), Otmar Scherzer (4),

(1) Department of Mathematics, Università degli Studi di Milano, Via Cesare Saldini 50, 20133, Milano, Italy; (2) Johannes Gutenberg-Universität Mainz, Institut für Mathematik, Staudingerweg 9, 55128 Mainz, Germany; (3) Department of Mathematical Sciences, University of Bath, Bath, BA2 7AY, UK; (4) Department of Mathematics & Christian Doppler Laboratory for Mathematical Modeling and Simulation of Next Generations of Ultrasound Devices (MaMSi), University of Vienna, Oskar-Morgenstern-Platz 1, 1090 Vienna, Austria & Johann Radon Institute for Computational and Applied Mathematics (RICAM), Altenbergerstraße 69, A-4040 Linz, Austria;

We study linear inverse problems under the premise that the forward operator is not at hand but given indirectly through some input-output training pairs. We demonstrate that *regularization by projection* and *variational regularization* can be formulated by using the training data only and without making use of the forward operator. We study convergence and stability of the regularized solutions. Moreover, we show, analytically and numerically, that regularization by projection is indeed capable of *learning* linear operators, such as the Radon transform.

On learning the optimal regularization parameter in inverse problems

Jonathan Chirinos Rodriguez (1), Ernesto De Vito (2), Cesare Molinari (2), Lorenzo Rosasco (3), Silvia Villa (2),

(1) IRIT, Toulouse INP, Toulouse, France; (2) MaLGa, DIMA, Dipartimento di Eccellenza 2023-2027, Università degli Studi di Genova, Genoa, Italy; (3) MaLGa, DIBRIS, Università degli Studi di Genova, Genoa, Italy & Center for Brains, Minds and Machines, MIT, Cambridge, USA & Istituto Italiano di Tecnologia, Genoa, Italy;

Selecting the best regularization parameter in inverse problems is a classical and yet challenging problem. Recently, data-driven approaches based on supervised learning have become popular to tackle this challenge. These approaches are appealing since they do require less a priori knowledge, but their theoretical analysis is limited. In this talk, I propose and study a statistical machine learning approach, based on empirical risk minimization. The main contribution is a theoretical analysis, showing that, provided with enough data, this approach can reach sharp rates while being essentially adaptive to the noise and smoothness of the problem. We show that this analysis can be applied to a large class of examples in inverse problems. Numerical simulations corroborate and illustrate the theoretical findings.

Convexity and Quasiconvexity in Parameter Identification Problems for elliptic PDEs

Stefan Kindermann (1),

(1) Industrial Mathematics Institute, Johannes Kepler University, Linz, Austria;

We consider parameter identification problems for elliptic PDEs from single or multiple and full, incomplete, or boundary measurements of the solution. Besides the fact that these usually are ill-posed problems, an additionally difficulty arises from the fact that they are nonlinear inverse problems. Closely related to this is the complication that the corresponding output-least squares functionals are usually nonconvex. This talk is devoted to the study under what additional conditions on the parameter set such output-least squares functional can lead to locally convex or quasi-convex functionals. Finding such conditions is highly relevant for the numerical computation of regularized solutions by variational regularization or iterative regularization. We extend some known results from the literature, for instance, by using monotonicity conditions, where we in particular consider the case of incomplete or boundary measurements. This talk is mostly based on results from [1].

[1] S. Kindermann. On the tangential cone condition for electrical impedance tomography. *Electron. Trans. Numer. Anal.*, **57**:17–34, 2022.

5. Optimal control, backward stochastic differential equations and related topics

10:20 – 11:35

Chair: **S. Tang, K. S. Zhang** **FH HS 6**

Backward Stochastic Control System with Entropy Regularization

Ziyue Chen (1,2), Qi Zhang (1),

(1) Fudan University, Shanghai, China; (2) University of Edinburgh, Edinburgh, UK;

We study the optimal relaxed control for backward stochastic system, generated by the backward stochastic differential equation and with the entropy regularization in its cost functional. We first establish the stochastic maximum principle by convex variation method. Then we prove sufficient condition for the optimal control and demonstrate the implicit form of optimal control. Finally, the existence and uniqueness of the optimal control for backward linear-quadratic control problem with entropy regularization is proved by decoupling techniques.

Itô-Wentzell-Lions formulae for flows of full and conditional measures on semimartingales

Jisheng Liu (1), Jing Zhang (1),

(1) School of Mathematical Sciences, Fudan University, Shanghai, China;

In this paper, we establish the Itô-Wentzell-Lions formulae for flows of both full and conditional measures on general semimartingales. This generalizes the existing works on flows of measures on Itô processes. The key technical components involve the appropriate approximation of random fields by cylindrical functions. Moreover, we present two specific forms of formulae, including Itô-Wentzell-Lions formulae for time-space-measure-dependent functions and for functions driven by Poisson random measure.

Weighted solutions of random time horizon BSDEs with stochastic monotonicity and general growth generators and related PDEs

Xinying Li (1), Yaqi Zhang (1), Shengjun Fan (1),

(1) School of Mathematics, China University of Mining and Technology, Xuzhou, China;

This study focuses on a multidimensional backward stochastic differential equation (BSDE) with a general random terminal time τ taking values in $[0, +\infty]$. The generator g satisfies a stochastic monotonicity condition in the first unknown variable y and a stochastic Lipschitz continuity condition in the second unknown variable z, and it can have a more general growth in y than the classical one stated in (H5) of [1]. Without imposing any restriction of finite moment on the stochastic coefficients, we establish a general existence and uniqueness result for the weighted solution of such BSDE in a proper weighted L^2-space with a suitable weighted factor, which unifies and strengthens some existing works on BSDEs with stochastic monotonicity generators, stochastic Lipschitz generators, and deterministic Lipschitz/monotonicity generators. We also derive the nonlinear Feynman-Kac formulas for both parabolic and elliptic PDEs in our context.

[1] Ph. Briand, B. Delyon, Y. Hu, E. Pardoux and L. Stoica. L^p solutions of backward stochastic differential equations. *Stochastic Process. Appl.*, **108**:109-129, 2003.

6. Hamilton-Jacobi equations for optimal control and games: new trends in numerical and analytical aspects

10:20 – 11:35

***Chair*: M. Akian, D. Vasquez-Varas**

FH HS 4

Numerical Hopf–Lax formulae for Hamilton–Jacobi equations on unstructured geometries

Simone Cacace (1), Roberto Ferretti (1), Giulia Tatafiore (1),

(1) Dipartimento di Matematica, "Sapienza" Università di Roma, Rome, Italy;

We consider a scheme of Semi-Lagrangian (SL) type for the numerical solution of Hamilton–Jacobi (HJ) equation on unstructured triangular grids. As it is well known, SL schemes are not well suited for unstructured grids, due to the cost of the point location phase; this drawback is augmented by the need for repeated minimization. In this work, we propose a scheme that works only on the basis of node values and connectivity of the grid. In a first version, we obtain a monotone scheme; then, applying a quadratic refinement to the numerical solution, we improve accuracy at the price of some extra computational cost. The scheme can be applied to both time-dependent and stationary HJ equations; in the latter case, we also study the construction of a fast policy iteration solver. We perform a theoretical analysis of the two versions, and validate them with an extensive set of examples, both in the time-dependent and in the stationary case.

[1] S. Cacace, R. Ferretti, and G. Tatafiore. Numerical Hopf–Lax formulae for Hamilton–Jacobi equations on unstructured geometries. `arXiv:2503.13311`, 2025.

Improving Policy Iteration: A Koopman-Based Riccati Analogue for Nonlinear Control Systems

Bernhard Höveler (1),

(1) TU Berlin, Berlin, Germany;

Optimal feedback control for nonlinear systems is a powerful tool with applications in engineering, physics, and many other fields. However, a significant drawback of this approach is that the numerical treatment of the resulting nonlinear first-order partial differential equation—the Hamilton-Jacobi-Bellman (HJB) equation—can be challenging. In this talk, we will show that the HJB equation is linked to a nonlinear operator equation very similar to the Riccati equation. To establish this connection, we define weighted L^p-spaces and develop a theory based on the Koopman operator that generalizes many concepts known from linear quadratic control problems. Moreover, we present a theory rooted in the Koopman operator and weighted L^p-spaces, which extends several well-known ideas from linear quadratic control. We then demonstrate that the HJB equation can be formulated as a minimization problem over a set of nuclear operators, where the solution is characterized by a nonlinear operator equation analogous to the Riccati equation. Furthermore, we show that policy iteration can be interpreted as a specific method for solving this operator equation. However, this method has some unfavorable properties in this context, which we address by introducing a modification. Finally, we present numerical experiments that demonstrate the theoretical properties established in this work and compare the standard policy iteration with the modified version.

Finite difference schemes for first order Hamilton-Jacobi equations in proper CAT(κ) **spaces**

Olivier Bokanowski (1), Othmane Jerhaoui (2), Housnaa Zidani (3),

(1) Laboratoire Jacques-Louis Lions, Université Paris Cité, France.; (2) Univ. Rennes, INSA de Rennes, CNRS, IRMAR-UMR 6625, Rennes, France.; (3) Normandie Univ., INSA Rouen Normandie, Laboratoire LMI, Rouen, France.;

We consider viscosity solutions of first-order Hamilton-Jacobi equations in proper CAT(κ) spaces. The notion of viscosity solutions is defined by taking test functions that are locally Lipschitz, directionally differentiable and can be represented as a difference of two semi-convex functions. We focus on finite difference numerical approximations. Under mild assumptions we prove convergence of numerical schemes as in the Barles-Souganidis framework, with an adapted notion of super-solutions and sub-solutions. Finally, several examples will be discussed to illustrate the efficiency of the numerical schemes.

7. Mean Field Control and Mean Field Games and Economic Applications

10:20 – 11:35

Chair: **D. Ghilli, M. Leocata** **FH HS 5**

A mean field game system and a related deterministic optimal control problem

Ştefana-Lucia Aniţa (1),

(1) Octav Mayer Institute of Mathematics of the Romanian Academy, Iaşi, Romania;

This paper concerns a Mean Field Game (MFG) system related to a Nash type equilibrium for dynamical games associated to large populations. One shows that the MFG system may be viewed as the Euler-Lagrange system for an optimal control problem related to a Fokker-Planck equation with control in the drift. One derives the existence of a weak solution to the MFG system and under more restrictive assumptions one proves a uniqueness result.

A Mean-Field Game Model with Biodiversity as a Welfare Externality

Emmanuelle Augeraud-Véron (1), Emmanuelle Ghili (2), Fausto Gozzi (3), Marta Leocata (4),

(1) BSE, University of Bordeaux, Bordeaux, France; (2) Department of Economics and Management, University of Pavia, Pavia, Italy; (3) Economics and Finance Department, LUISS Guido Carli, Rome, Italy; (4) Economics and Finance Department, LUISS Guido Carli, Rome, Italy;

This paper addresses a fundamental question in environmental economics: how should agents dynamically manage biological resources when biodiversity itself is a global externality that shapes and is shaped by individual actions under uncertainty? Standard models fail to capture the feedback between ecological diversity and individual incentives, especially in heterogeneous and stochastic environments. To overcome these limitations, we develop a novel mean-field framework for biodiversity exploitation, combining stochastic Gompertz dynamics with a biodiversity index—measured via Shannon entropy—that enters both ecological dynamics and preferences. This structure generates a non-trivial aggregate feedback loop between the population distribution and species-level behavior. Importantly, only tools from Mean Field Games (MFG) and Mean Field Control (MFC) theory allow for a tractable yet rigorous treatment of this distributed optimization problem. We characterize the equilibrium in both decentralized (MFG) and planner-led (MFC) regimes, under both logarithmic and isoelastic utility. The model yields closed-form or asymptotic expressions for stationary distributions, optimal effort, and biodiversity. We show that decentralized agents, although partially responsive to ecological feedbacks, underprovide biodiversity compared to the socially optimal benchmark. This inefficiency is amplified under isoelastic utility or when agents ignore stochastic environmental shocks. To restore efficiency, we construct Pigouvian taxes based on species abundance and marginal contributions to biodiversity. These instruments allow decentralization of the MFC optimum and provide a principled basis for conservation policy. Our results highlight the importance of explicitly modeling biodiversity feedbacks, heterogeneity, and environmental risk—dimensions that standard approaches systematically overlook.

Stationary Mean Field Games on networks with sticky transition conditions

Jules Berry (1), Fabio Camilli (2),

(1) INSA, Univ. Rennes, Rennes, France; (2) INGEO, Univ. "G.D'Annunzio" Chieti-Pescara, Pescara, Italy;

I consider stochastic Mean Field Games on networks with sticky transition conditions. In this setting, the diffusion process governing the agent's dynamics can spend finite time both in the interior of the edges and at the vertices. The corresponding generator is subject to limitations concerning second-order derivatives and the invariant measure breaks down into a combination of an absolutely continuous measure within the edges and a sum of Dirac measures positioned at the vertices. Additionally, the value function, solution to the Hamilton-Jacobi-Bellman equation, satisfies generalized Kirchhoff conditions at the vertices.

8. Dynamics of the firm

10:20 – 11:35

Chair: **P. Kort, X. Wen**

FH HS 7

Certainty equivalent and uncertainty premium of time-to-build

Haejun Jeon (1), Michi Nishihara (2),

(1) Department of Management, Tokyo University of Science, Tokyo, Japan; (2) Graduate School of Economics, Osaka University, Osaka, Japan;

Time-to-build of an investment project induces a discrepancy between the timing of investment and that of revenue generation. [1] showed that uncertainty in the time-to-build *always* accelerates investment and enhances pre-investment firm value, regardless of its distribution. This study examines the extent to which the uncertainty advances the timing of investment and improves firm value. Specifically, we show that there *always* exists a unique certainty equivalent of uncertain time-to-build and derive it in an analytic form. This enables us to derive the investment strategy with uncertain time-to-build in the form of the one that would have been adopted in the absence of such uncertainty. Even without full knowledge of the uncertainty, the firm can approximate the optimal investment strategy using only the mean and variance of time-to-build. We also clarify the positive impact of entropic risk measure of time-to-build on investment and derive the dual representation of the certainty equivalent of time-to-build based on relative entropy. Furthermore, we show that there *always* exists an uncertainty equivalent of fixed time-to-build. This implies that the firm can deduce the equivalent risk that its investment strategy, established without considering uncertainty in time-to-build, implicitly assumes. Lastly, we illustrate the practical application of our findings using some representative probability distributions and analyze the effects of the variance of time-to-build. In particular, we contrast the effects of uncertainty in demand with those of uncertainty in time-to-build, deriving the level of variance in time-to-build that offsets the negative impact of increased demand volatility on investment.

[1] H. Jeon. Time-to-build, regulation, and investment. *European Journal of Operational Research*, **319**(3):999-1019, 2024.

Real option to invest in a high-growth market

Michi Nishihara (1),

(1) Graduate School of Economics, The University of Osaka, Osaka, Japan;

This paper examines an investment timing problem in a high-growth market where demand grows rapidly until reaching a threshold, after which the growth rate slows. Three types of analytical solutions are derived, depending on parameter values. Notably, one of these solutions deviates from the conventional threshold strategy. Under certain conditions, the optimal investment timing is independent of the threshold that alters the growth rate. Higher demand volatility can shorten the high-growth period, thereby reducing the option value of investment.

Investment and capital structure decisions with product-market flexibility and financing inflexibility

Pengfei Luo (1), Yuan Tian (2),

(1) Hunan University, Hunan, China; (2) Ryukoku University, Kyoto, Japan;

We analyze the optimal cost-reducing investment and capital structure decisions with product-market flexibility. Our results show that the product-market flexibility has significant influence on firms' optimal investment and capital structure decisions. Specifically, the optimal investment threshold has a U-shaped relation with the product-market flexibility, irrespective of firms' capital structures. Moreover, the relationship between optimal leverage and product-market flexibility exhibits quite differently, depending on whether there is financing inflexibility or not. While the optimal leverage increases with the product-market flexibility without financing constraint, the result is opposite with financing constraint. Our model can reconcile two opposing predictions about the relationship between the optimal leverage and product-market flexibility in the empirical findings. We also demonstrate that the product market flexibility reduces default probabilities and credit spreads, which is consistent with empirical results.

9. Uncertainty Quantification in Optimization, Optimal Control, and Identification

11:45 – 13:00

Chair: **A. Khan, C. Tammer** **FH Nöbauer 8**

High order Symmetrized Fractional Variation for analysis of textured images

Alessandro Lanza (1), Antonio leaci (2), Serena Morigi (1), Franco Tomarelli (3),

(1) Department of Mathematics, University of Bologna, Bologna, Italy; (2) Universitá del Salento, Dipartimento di Matematica e Fisica "Ennio De Giorgi", Lecce, Italy; (3) Politecnico di Milano, Dipartimento di Matematica, Milano, Italy;

We define, study and implement the model SFV: a variational approach to signal analysis exploiting the Riemann-Liouville fractional derivatives of every positive real order higher than zero. The model exploites an L^1 fitting data term together with both right and left fractional derivatives as regularizing terms: this

approach aims to achieve an orientation-independent protocol.
We formulate the model as an energy minimization and introduce the space BV_*^s, a functional framework where existence of minimizers is proved.
To provide evidence of effectiveness for the proposed model we introduce a discretisation based on a second-order consistent Grünwald-Letnikov scheme.
A multi-parameter whiteness criterion is introduced which provides automatic and simultaneous selection of the two free parameters in the model, namely the fractional order of differentiation and the regularization parameter. Numerical experiments on 1-d and 2-d signals are presented which show how the proposed model holds the potential to achieve good quality results for denoising signals corrupted by additive Laplace noise.

Optimal control and shape parameter semidifferentiability with application to identification problem of breaking line in solid

Victor A. Kovtunenko (1,2), Victor A. Kovtunenko (1),

(1) Department of Mathematics and Scientific Computing, University of Graz, Austria; (2) Lavrentyev Institute of Hydrodynamics, Siberian Division of the Russian Academy of Sciences, Novosibirsk, Russia;

A class of shape optimal control and identification problems constrained by variational inequalities is studied with respect to parameter semidifferentiability. A specific problem stemming from fracture mechanics describes elastic bodies with a breaking line subject to contact conditions between its faces. Based on the Lagrange multiplier approach and smooth Lavrentiev penalization, a semi-analytic formula for the shape gradient of the Lagrangian linearized on the solution is proved, which contains both primal and adjoint states. It is used for the descent direction in a gradient algorithm for identification of an optimal shape of the breaking line from boundary measurements. The theoretical result is supported by numerical simulation tests of destructive testing in 2D configuration comparing the problems with and without contact.

[1] F. Cakoni and V. A. Kovtunenko. Topological optimality condition for the identification of the center of an inhomogeneity. *Inverse Probl.*, **34**(3):035009, 2019.

[2] D. Ghilli, K. Kunisch K, and V. A. Kovtunenko. Inverse problem of breaking line identification by shape optimization. *J. Inverse Ill-posed Prob.*, **28**:119-135, 2020.

[3] A. M. Khludnev and V. A. Kovtunenko. *Analysis of Cracks in Solids.* WIT-Press, Southampton, Boston, 2000.

[4] V. A. Kovtunenko. Lagrangian approach and shape gradient for inverse problem of breaking line identification in solid: contact with adhesion. *Inverse Probl.*, **39**(8):084004, 2023.

[5] V. A. Kovtunenko and K. Kunisch. Shape derivative for penalty-constrained nonsmooth-nonconvex optimization: cohesive crack problem. *J. Optim. Theory Appl.*, **194**:597-635, 2022.

[6] V. A. Kovtunenko and K. Kunisch. Directional differentiability for shape optimization with variational inequalities as constraints, *ESAIM: Control Optim. Calc. Var.*, **29**:64, 2023.

Phase-field methods for shape reconstruction in elastic inverse problems

Andrea Aspri (1), Elena Beretta (2), Cecilia Cavaterra (1), Elisabetta Rocca (3), Marco Verani (4),

(1) Departments of Mathematics, Università degli Studi di Milano, Milan, Italy; (2) Departments of Mathematics, NYU Abu Dhabi, Abu Dhabi, United Arab Emirates; (3) Departments of Mathematics, Università degli Studi di Pavia, Pavia, Italy; (4) MOX, Departments of Mathematics, Politecnico di Milano, Milan, Italy;

In this talk, I will present recent advances in elastic inverse problems, focusing on the shape reconstruction of cavities and inclusions in a bounded linear isotropic medium using boundary measurements. We approach these inverse problems from an optimal control perspective, formulating them as minimization problems where the objective is to minimize a misfit boundary functional or an energy-based functional within the class of Lipschitz domains. To enhance stability and well-posedness, we introduce a regularization term that penalizes the perimeter of the cavity or inclusion. The optimization problem is tackled using a phase-field approach, where the perimeter functional is approximated through a Modica-Mortola relaxation.

10. Optimal control, backward stochastic differential equations and related topics

11:45 – 13:00

Chair: **S. Tang, K. S. Zhang** **FH HS 6**

Characterization of the optimal solutions to the monopolist problem in the plane

Robert J. McCann (1), Cale Rankin (2), Kelvin Shuangjian Zhang (3),

(1) University of Toronto, Toronto, Canada; (2) Monash University, Melbourne, Australia; (3) Fudan University, Shanghai, China;

The principal-agent problem is one of the central problems in microeconomics with many applications. Existence, uniqueness, convexity/concavity, regularity, and characterization of the solutions have been widely studied after Mirrlees and Spence in the 1970s. For multidimensional spaces of agents and products, Rochet and Choné [1] reformulated this problem to a concave maximization over the set of convex functions, by assuming agent preferences combine bilinearity in the product and agent parameters with a quasilinear sensitivity to prices. We characterize solutions to this problem by identifying a dual minimization problem. This duality allows us to reduce the solution of the square example of Rochet-Choné to a novel free boundary problem, giving the first analytical description of an overlooked market segment, where the regularity built by Caffarelli-Lions plays a crucial role —— an extension of their regularity work to the quasilinear case is also recently studied. The results profoundly connect with the Optimal Transport theory, a powerful tool with potential applications in many areas.

[**1**] J.-C. Rochet and P. Choné. Ironing, sweeping and multidimensional screening. *Econometrica*, **66**:783–826, 1998.

Propagation of chaos for mean-field reflected BSDEs with jumps

Yiqing Lin (1), Kun Xu (1),

(1) School of Mathematical Sciences, Shanghai Jiao Tong University, Shanghai, China;

In this talk, we present our results on the study of mean-field reflected backward stochastic differential equations (MF-RBSDEs) driven by a marked point process and MF-RBSDEs driven by a Poisson process. Based on a g-expectation representation lemma, we establish the existence and uniqueness of the particle system of MF-RBSDEs driven by a marked point process under Lipschitz generator conditions and obtain a convergence result of this system. In the Poisson setting, we obtain furthermore the convergence rate of the corresponding particle system toward the solution to the MF-RBSDEs driven by a Poisson process under bounded terminals and bounded obstacle conditions.

Optimal control of SDEs with measurable drift and cost

Kai Du (2), Qingmeng Wei (1),

(1) Fudan University, Shanghai, China; (2) Northeast Normal University, Changchun, China;

In this work, we study an optimal control problem for a stochastic differential equation whose drift coefficient and cost function are merely measurable with respect to the state variable. We show that the value function of this problem coincides with the solution to the associated Hamilton–Jacobi–Bellman (HJB) equation and exploit this equivalence to establish the dynamic programming principle. Furthermore, when the control variable takes values in a countable set, we prove that the value functions obtained from smooth approximations converge to the original value function.

11. Hamilton-Jacobi equations for optimal control and games: new trends in numerical and analytical aspects

11:45 – 13:00

Chair: **M. Akian, D. Vasquez-Varas**

FH HS 4

High-dimensional Hamilton-Jacobi-Bellman PDEs for global optimization

Yuyang Huang (1), Michael Herty (2), Dante Kalise (1), Nikolas Kantas (1),

(1) Department of Mathematics, Imperial College London, UK; (2) IGPM, RWTH Aachen University, Aachen, Germany;

This talk introduces a novel approach to global optimization leveraging solutions of Hamilton-Jacobi-Bellman (HJB) equations, with an application to accelerating consensus-based optimization (CBO) algorithms. We begin by reformulating the global optimization problem as an infinite-horizon optimal control problem. The solution to the associated HJB equation provides a value function that approximates the objective function's landscape. By extracting gradient information from this value function, we obtain a control signal that guides the search towards the global optimum without requiring explicit derivatives of the original objective function. We then demonstrate how this HJB-derived control can be integrated into the CBO framework to significantly enhance its performance. The resulting controlled CBO method exhibits faster convergence rates and improved robustness compared to standard CBO, especially in challenging scenarios with limited particles or poor initialization.

Stochastic and deterministic representations for solutions of Schrödinger initial value problems and associated numerical methods

William M. McEneaney (1), Hidehiro Kaise (2), Yifei Zheng (1),

(1) Mech. and Aero. Eng., Univ. California San Diego, La Jolla, USA; (2) Mathematics, Univ. of Kumamoto, Kumamoto, Japan;

The lack of an ability to computationally solve problems where the systems are driven by Schrödinger-equation dynamics is a barrier to further development. The machinery indicated here demonstrates exceptional progress in that direction. First, a diffusion-process representation for the Maslov dequantization of the solution of the Schrödinger initial-value problem is obtained. The representation employs a complex-valued diffusion process and stationarity of an action functional. The use of stationarity removes the need for convexity of the action, and hence there is no restriction on the duration of the problem. The Coulomb potential is addressed via staticization (similar to extremization) over an additional controller, resulting in a polynomial expression, albeit at the complexity cost of the additional controller. Next, through application of complex-case extensions of existing iterated-staticization results, a reordering of operations is obtained, yielding a form where the inner problem takes the classical linear-quadratic-Gaussian form. The original Schrödinger-equation solution is obtained from [re-]quantization of these representations.

Duality between polyhedral approximation of value functions and optimal quantization of measures

Abdellah Bulaich Mehamdi (1,2,3), Wim van Ackooij (1), Luce Brotcorne (3), Stéphane Gaubert (2, 3), Quentin Jacquet (1),

(1) EDF Lab Paris-Saclay, Palaiseau, France; (2) CMAP, École polytechnique, Institut Polytechnique de Paris, Palaiseau, France; (3) INRIA;

We establish a connection between polyhedral approximation of functions and optimal quantization of measures. Building on recent stability results in optimal transport, by Delalande and Mérigot, we deduce that the polyhedral approximation of a convex function is equivalent to the quantization of the Monge-Ampère measure of its Legendre-Fenchel dual. This duality motivates a simple greedy method for computing a parsimonious approximation of a polyhedral convex function, by clustering the vertices of a Newton polytope. We evaluate our algorithm on a high-dimensional optimal control problem (quantum gate synthesis), leveraging McEneaney's max-plus-based curse-of-dimensionality attenuation method.

12. Mean Field Control and Mean Field Games and Economic Applications 11:45 – 13:00

Chair: **D. Ghilli, M. Leocata** **FH HS 5**

Coarse correlated equilibria in mean field games

Luciano Campi (1),

(1) University of Milan, Italy;

We will give an overview of coarse correlated equilibria in mean field games (MFGs) in continuous time, focusing on a specific example (where Nash MFG solution do not exist) and on the linear-quadratic case. If time allows, we will also discuss how to construct approximate equilibria in the corresponding N-player games and some future developments, such as the computation via learning algorithms of optimal coarse correlated MFG solutions from the point of view of a moderator. This talk will be based on the joint works [1,2] with F. Cannerozzi, F. Cartellier, and M. Fischer, and on a work in progress with F. Cannerozzi and I. Tzouanas.

[1] L. Campi, F. Cannerozzi, F. Cartellier. Coarse correlated equilibria in linear quadratic mean field games and application to an emission abatement game. In *Applied Mathematics and Optimization*, Vol. 91, article number 8, 2025.

[2] L: Campi, F. Cannerozzi, M. Fischer. Coarse correlated equilibria for mean field games in open loop strategies. In *Electronic Journal of Probability* 29, 1-56, 2024.

Time dependent first-order Mean Field Games with Neumann boundary conditions

Diogo Gomes (1), Michele Ricciardi (2),

(1) KAUST University, Thuwal, Saudi Arabia; (2) LUISS University, Rome, Italy;

The primary objective of this talk is to understand first-order, time-dependent mean-field games with Neumann boundary conditions, a question that remains under-explored in the literature. This matter is particularly relevant given the importance of boundary conditions in crowd models. In our model, the Neumann conditions result from players entering the domain Ω according to a prescribed current j, for instance, in a crowd entry scenario into an open-air concert or stadium. We formulate the model as a standard mean-field game coupling a Hamilton-Jacobi equation with a Fokker-Planck equation. Then, we introduce a relaxed variational problem and use Fenchel-Rockafellar duality to study the relation between these problems. Finally, we prove the existence and uniqueness of solutions for the system using variational methods.

Some new results on mean field games of controls

P. Jameson Graber (1), Elizabeth Matter (1), Kyle Rosengartner (1),

(1) Baylor University, Waco, TX USA;

Mean field games of controls are used to model large-scale strategic interactions between players, where the phrase "of controls" refers to the fact that they use the statistical distribution of players' states *and controls* to coordinate behavior. Here is the formulation as a system of nonlinear PDE system:

$$\begin{aligned} -\partial_t u + Au + H(x, Du, \mu) &= 0, \\ \partial_t m + Am - \operatorname{div}_x(D_p H(x, Du, \mu) m) &= 0, \\ \mu = (I, -D_p H(\cdot, Du, \mu))_\sharp m, \\ m(0) = m_0, \quad u(T) &= g(m(T)) \end{aligned}$$

where H is a Hamiltonian and A is a diffusion operator. Compared to the vast literature that now exists on mean field games, there are relatively few results on mean field games of controls. Nevertheless, when applying mean field games to economics, it is natural to coordinate behavior through the statistical distribution of controls, e.g. the price. In this talk, I will present some of our latest findings on the existence and uniqueness of solutions to these PDE systems under new technical conditions on the data. In particular, I will discuss some new methods for proving uniqueness, which may be applicable to certain economics models.

13. Dynamics of the firm

11:45 – 13:00

Chair: **P. Kort, P. Gapeev**

FH HS 7

Capacity expansion investment under financing constraint

Luoyuan Gan (1), Xingang Wen (1), Herbert Dawid (1),

(1) Faculty of Business Administration and Economics, Bielefeld University, Germany;

This research studies a monopoly firm's timing and size decision to invest in a partially substitute product under demand uncertainty. We assume the investment cost is financed by the instantaneous profit of the old product. Therefore, the financing constraint is "binding" in case the investment size is bounded from above. We find that 1) The optimal investment size increases (decreases) with the old product output, q_o, when the constraint is binding (non-binding). For relatively high levels of demand uncertainty, the optimal investment threshold is U-shaped with respect to the q_o, i.e., a trade-off outcome between the revenue and the cannibalization effects of the new investment that generates additional revenues but reduces revenues of the old product. 2) For a given q_o, the financing constraint is only binding for intermediate levels of demand uncertainty. Otherwise, the constraint is not binding because the optimal investment is either early with small costs (under low uncertainty), or so late that the instantaneous profit from the old product is sufficiently high (under high uncertainty).

Investment under Uncertainty: Learn Local to Grow Global

Nathan A. Bun (1), Kuno J.M. Huisman (1), Peter M. Kort (1),

(1) EOR, Tilburg University, Tilburg, the Netherlands;

Recently, Chinese firms have started to expand into Western markets after learning to manufacture their products cheaply in the Chinese market, resulting in fierce competition with incumbents. Inspired by these dynamics, this paper applies a real options model while considering two markets: a smaller local and a larger global market, where the firm starts learning as it invests. We find that, due to considering consumer surplus locally, a social planner first invests in the local market, and learns, before it invests in the global market. This contrasts with a for-profit firm that invests first in the global market. More significant correlation between local and global markets pursues entering the local market with a later and larger investment.

14. Uncertainty Quantification in Optimization, Optimal Control, and Identification

14:00 – 15:15

Chair: **A. Khan, C. Tammer** **FH Nöbauer 8**

On Bayesian inference and optimal experimental design in photoacoustic tomography with fractional attenuation

Phuoc Truong Huynh (1), Barbara Kaltenbacher (1), Anna Posch(Schlintl) (1,2),

(1) Department of Mathematics, University of Klagenfurt, Austria; (2) Infineon Technologies, Villach, Austria;

In this talk we study the identification of the absorption density in photoacoustic tomography, formulated as the inverse problem of reconstructing either an initial condition or a space dependent source term in the wave equation. As relevant in ultrasound propagation, we incorporate power law frequency dependent attenuation by means of a time fractional damping term. We solve the inverse problem in a Bayesian framework using a Maximum A Posteriori (MAP) estimate, using an adjoint approach for gradient computation. On top of this, we consider optimization of the choice of the laser excitation function, which is the time-dependent part of the source in this model, to enhance the reconstruction result.

[1] B. Kaltenbacher and W. Rundell. Some inverse problems for wave equations with fractional derivative attenuation. *Inverse Problems*, **37**(4):045002, 2021.

[2] B. Kaltenbacher and A. Schlintl. Fractional time stepping and adjoint based gradient computation in an inverse problem for a fractionally damped wave equation. *Journal of Computational Physics*, **449**:110789, 2022.

[3] P.-T. Huynh and B. Kaltenbacher. On the optimal choice of the illumination function in photoacoustic tomography. submitted

Computational framework for estimation of tissue elasticity in biomechanical imaging

Rachel Hawks (1), Basca Jadamba (1),

(1) School of Mathematics and Statistics, Rochester Institute of Technology, U.S.A;

The primary objective of this study is to address the computational challenges associated with parameter estimation in a linear elasticity model. The underlying motivation is the characterization of soft tissue mechanical properties using measured displacement data. A finite element based computational framework is developed and its performance is discussed.

Randomised Levenberg-Marquardt methods for nonlinear least-squares

Stefania Bellavia (1), Greta Malaspina (1), Benedetta Morini (1),

(1) DIEF, Università di Firenze, Italia;

In recent years, randomized linear algebra [1] has emerged as a powerful tool for solving problems with high computational and memory demands. Randomized sampling and randomized embeddings are the core of a variety of optimization methods with random models that are suitable for solving many applications, including machine learning. We focus on nonlinear least-squares problems and present stochastic Levenberg-Marquardt/Gauss-Newton methods combined with a line-search strategy. We discuss randomised dimensionality reduction either in the variable dimension through subsampling or in the dimension of the observations through random embedding, referred to as sketching. In this latter case, the computation of the trial step is restricted to a random subspace of reduced dimension. The algorithms proposed have per-iteration computational complexity lower than classical deterministic methods. We discuss the construction of the random models and the iteration complexity results to drive the gradient below a prescribed accuracy, then we present results from our computational experience.

[1] P.G. Martinsson, J. A. Tropp, Randomized numerical linear algebra: Foundations and algorithms, *Acta Numerica*, **29**:403–572, 2020.

15. Optimal control, backward stochastic differential equations and related topics

14:00 – 15:15

***Chair*: S. Tang, K. S. Zhang** **FH HS 6**

Linear-quadratic extended mean field games with common noises

Tianjiao Hua (1), Peng Luo (1),

(1) School of Mathematical Sciences, Shanghai Jiao Tong University, China;

In this paper, we consider a class of linear quadratic extended mean field games (MFGs) with common noises where the state coefficients and the cost functional vary with the mean field term in a nonlinear way. Based on stochastic maximum principle, solving the mean field game is transformed into solving a conditional mean field forward-backward stochastic differential equation (FBSDE). We first establish

solvability for a type of (more general) conditional mean field FBSDEs under monotonicity conditions. We further provide some regularity results which lead to classical solutions for the associated master equations. In particular, the linear quadratic extended mean field game is solved and classical solution for (extended mean field game) master equation is obtained.

Propagation of chaos for mean-field reflected BSDEs with jumps

Yiqing Lin (1), Kun Xu (1),

(1) School of Mathematical Sciences, Shanghai Jiao Tong University, Shanghai, China;

In this talk, we present our results on the study of mean-field reflected backward stochastic differential equations (MF-RBSDEs) driven by a marked point process and MF-RBSDEs driven by a Poisson process. Based on a g-expectation representation lemma, we establish the existence and uniqueness of the particle system of MF-RBSDEs driven by a marked point process under Lipschitz generator conditions and obtain a convergence result of this system. In the Poisson setting, we obtain furthermore the convergence rate of the corresponding particle system toward the solution to the MF-RBSDEs driven by a Poisson process under bounded terminals and bounded obstacle conditions.

TCP congestion control as a nonlinear optimal control problem

Dumisa Wellington Ngwenya (1,2), Mduduzi Comfort Hlophe (1), Bodhaswar T. Maharaj (1),

(1) Department of Electrical, Electronic and Computer Engineering, University of Pretoria, Pretoria 0002, South Africa; (2) Research and Innovation Department, SENTECH SOC Limited, Johannesburg 2040, South Africa;

Most existing congestion control mechanisms on the Internet rely on static optimization, which fails to capture the system's temporal dynamics. In our work, we propose a dynamic optimization framework rooted in a differential form of Little's Law [1], which serves as the foundation for deriving an optimality condition for congestion control, as developed in our recent study [2]. The key relationship in [2] is given by:

$$\frac{dL(t)}{dt} = R(t) \cdot \frac{d\lambda(t)}{dt}, \tag{1}$$

where L represents the inflight data (or average system occupancy), λ is the data flow rate, and R is the round-trip time (RTT). Since λ can be approximated by W/R, where W denotes the congestion window (cwnd), Equation (1) defines a state equation involving the state variables L, R, and W. The control input is defined as $u = \frac{dW}{dt}$, which manipulates W to influence the network's behavior. The control objective is twofold: (1) to regulate the RTT $R(t)$ toward a desired target R_{target} to avoid excessive queuing delays, and (2) to minimize control effort. These objectives are captured in the following cost function:

$$J = \int_0^T \left[q_R \left(R(t) - R_{\text{target}} \right)^2 + r_u u(t)^2 \right] dt,$$

where q_R and r_u are positive weights that penalize RTT deviation and control actuation effort, respectively. This talk presents our proposed approach and demonstrates how the resulting nonlinear optimal control problem is solved using gradient descent in combination with Runge-Kutta integration. Simulation results will be shown to compare the proposed method with existing congestion control algorithms.

[1] J.D.C. Little, "Little's Law as Viewed on Its 50th Anniversary," *Operations Research*, vol. 59, no. 3, pp. 536–549, 2011.

[**2**] D.W. Ngwenya, M.C. Hlophe, and B.T. Maharaj, "TCP Congestion Control Algorithm Using Queueing Theory-Based Optimality Equation," *Electronics*, vol. 14, p. 263, 2025.

16. Hamilton-Jacobi equations for optimal control and games: new trends in numerical and analytical aspects 14:00 – 15:15

Chair: **M. Akian, D. Vasquez-Varas** **FH HS 4**

Localized actuators for parabolic systems: stabilizability and learning of optimal setups

Sergio S. Rodrigues (1), Karl Kunisch (1,2), Daniel Walter (3),

(1) RICAM Linz; (2) Karl-Franzens-Universität Graz; (3) Humboldt-Universität zu Berlin;

The present talk is split in two parts: First, we show that a broad class of nonautonomous, linear parabolic systems is stabilizable by a finite number of localized actuators each acting on a single point of the spatial domain. Stabilizing feedback controllers, both, explicit as well as optimal ones, are constructed. The second part of the talk revolves around improving the actuator setup by optimizing the number of employed actuators as well as their positions. We model this task as a nonsmooth minimization problem including a Riccati equation as a constraint. After proving its well-posedness and deriving first-order optimality conditions, a greedy minimization algorithm is employed for its solution. The talk is concluded by extensive numerical experiments confirming the presented theoretical results.

Machine-learning-based approximation of semi-concave functions for the synthesis of optimal feedback laws

Karl Kunisch (1), Donato Vásquez-Varas (2),

(1) RICAM, Austrian Academy of sciences, Linz, Austria; (2) Institute of Mathematics and Scientific Computing, University of Graz, Graz, Austria;

The synthesis of optimal feedback laws is a topic that has been extensively studied. Classical methods suffer from the curse of dimensionality, that is, their computational cost increases exponentially with the dimension. To mitigate this issue, approaches based on machine learning techniques have been proposed. In [1], performance guarantees are provided by controlling the semi-concavity of the approximation. In light of this, we propose a machine-learning-based approximation method that preserves the semi-concavity. Furthermore, under appropriate regularity assumptions, we provide theoretical guarantees on the optimality of the generated feedback law. Notably, these regularity conditions do not imply that the value function is smooth. We illustrate the proposed method through numerical experiments, and the results highlight the capability of the approach to efficiently approximate semi-concave functions and synthesize optimal feedback laws for non-smooth value functions.

[**1**] K. Kunisch and D. Vásquez-Varas. Consistent smooth approximation of feedback laws for infinite horizon control problems with non-smooth value functions. *Journal of Differential Equations*, **411**:438-477, 2024.

Numerical realization of the minimum energy state estimator via a local value function approximation

Tobias Breiten (1), Karl Kunisch (2,3), Jesper Schröder (2),

(1) Modeling, Simulation, and Optimization of Real Processes, Technische Universität Berlin, Berlin, Germany; (2) Optimization and Optimal Control, Johann Radon Institute for Computational and Applied Mathematics, Linz, Austria; (3) Department of Mathematics and Scientific Computing, University of Graz, Graz, Austria;

We consider the problem of reconstructing the state of a disturbed finite-dimensional nonlinear dynamical system based on incomplete and disturbed measurements. Following Mortensen's approach the disturbances are modeled as unknown deterministic functions and the estimation problem is recast as an optimal control problem of tracking type running backward in time. Consequently the analytical well-posedness as well as the numerical realization of the resulting state estimator critically depend on the associated value function and Hamilton-Jacobi-Bellman equation. Inspired by recent developments in value function approximation we propose a data-driven numerical scheme to realize the presented state estimator. A data set is generated via pointwise evaluation of the value function and its spatial derivatives in carefully chosen sampling points. We obtain a highly local and specialized approximation of the value function by fitting a polynomial to the constructed data. Said approximation is utilized for the realization of the state estimator in various numerical examples up to dimension 40.

17. Dynamic games in economics — 14:00 – 15:15

Chair: **S. Wrzaczek, R. Neck** — **FH HS 5**

A class of deterministic and stochastic differential games with myopic Nash equilibria

Onésimo Hernández-Lerma (1), José E. Márquez-Prado (1), Héctor Jasso-Fuentes (1),

(1) Department of Mathematics, Cinvestav, Mexico City, Mexico;

We give conditions for a class of deterministic and stochastic differential games to have myopic Nash equilibria, that is, Nash equilibria obtained by solving static games independent of the state trajectory. We show how these conditions are the same for the deterministic and stochastic cases, which means that the certainty equivalence principle is satisfied. Moreover, for the infinite-horizon time-homogeneous case, we will see that Nash equilibria are obtained by solving a single static game; hence, Nash equilibria are constant functions.

[1] J. E. Márquez–Prado, O. Hernández–Lerma. Linear–State Control Problems and Differential Games: Deterministic and Stochastic Systems. *Journal of Optimization Theory and Applications.* 205:41 (2025).

[2] J. E. Márquez–Prado, O. Hernández–Lerma, H. Jasso-Fuentes. Myopic optimal strategies for a class of continuous-time deterministic and stochastic control problems. *Systems & Control Letters.* 196, 106016 (2025).

On the feedback solutions of differential games where objective functions are parabolic cylinders

Flavio Delbono (1), Luca Lambertini (1,2),

(1) Department of Economics and Alma Climate Centre, University of Bologna, Bologna, Italy;
(2) International Institute for Applied Systems Analysis (IIASA), Laxenburg, Austria;

We analyse the feedback solution of a class of state-linear linear-quadratic differential games in which the individual objective function is a parabolic cylinder defined in the domain of controls only, as in [1]. The model accounts for the presence of a generic number of players and encompasses the cases of a single state variable common to all of them and that where the number of states is the same as the players'. In both versions of the games, we show that (i) irrespective of the number of states, there exists a unique pair of linear feedback strategies; (ii) the open-loop is obviously one of them, although not necessarily stable; and (iii) since instantaneous best replies are orthogonal, any Stackelberg equilibrium is strongly time consistent and replicates the simultaneous Nash equilibrium, as in [2]. The structures of the games we treat, and their solutions are clearly related with [3]. Indeed, we also illustrate that property (i) is a direct consequence of the model failing to meet the requirement, identified in [3], for the arising of multiple linear feedback equilibria . Applications to economics and politics are also illustrated.

[1] F. Delbono, and L. Lambertini. Folk theorems in a class of additively separable games. *Mathematical Social Sciences*, **92**:10-15, 2018.

[2] S. Rubio. On the coincidence of feedback Nash equilibria and Stackelberg equilibria in economic applications of differential games. *Journal of Optimization Theory and Applications*, **128**(1):203–221, 2006.

[3] F. Grandits, R. M. Kovacevic, and V. M. Veliov. On the non-uniqueness of linear Markov perfect equilibria in linear-quadratic differential games: a geometric approach. *Economic Theory*, https://doi.org/10.1007/s00199-024-01606-4.

On crossing limit cycles in linear-quadratic hybrid differential games with applications to sticky prices

Anton Bondarev (1),

(1) IBSS, Xi'an Jiaotong-Liverpool University, Suzhou, P. R. China;

In this talk I am presenting one general result and it's application ot a particular model. First, I consider a simple linear-quadratic differential game (LQG) with two players and one state in the spirit of discounted hybrid optimal control problems (DHOCP) with players maximizing discounted payoffs of quadratic type subject to common linear stock constraint of piecewise-smooth type with one state-driven switch. It has been shown previously that such games *may* possess crossing limit cycles (both hybrid and not), but general necessary and sufficient conditions for that are still lacking. LQG allows for complete explicit solution and thus the exact form of such conditions may be easily shown. I next discuss uniqueness and optimality (in OLNE sense) of such cycles depending on parameters of the game. Next, to stress the applied importance of this result I consider classical sticky prices game, which is of LQG type, and apply the PWS structure of the state (price level) to it. I show that this model naturally and *generically* possesses hybrid crossing limit cycles which might constitute the Nash equilibrium in addition to conventional ones.

18. Dynamics of the firm

14:00 – 15:15

Chair: **P. Kort, M. Nishihara**

FH HS 7

Perpetual American standard and lookback options in models with progressively enlarged filtrations

Pavel V. Gapeev (1), Libo Li (2),

(1) Department of Mathematics, London School of Economics, London, United Kingdom; (2) School of Mathematics and Statistics, University of New South Wales, Sydney, Australia;

We derive closed-form solutions to optimal stopping problems related to the pricing of perpetual American standard and lookback put and call options in extensions of the Black-Merton-Scholes model under progressively enlarged filtrations. It is assumed that the information available from the market is modelled by Brownian filtrations progressively enlarged with the random times at which the underlying process attains its global maximum or minimum, that is, the last hitting times for the underlying risky asset price of its running maximum or minimum over the infinite time interval, which are supposed to be progressively observed by the holders of the contracts. We show that the optimal exercise times are the first times at which the asset price process reaches certain lower or upper stochastic boundaries depending on the current values of its running maximum or minimum depending on the occurrence of the random times of the global maximum or minimum of the risky asset price process. The proof is based on the reduction of the original necessarily three-dimensional optimal stopping problems to the associated free-boundary problems and their solutions by means of the smooth-fit and either normal-reflection or normal-entrance conditions for the value functions at the optimal exercise boundaries and the edges of the state spaces of the processes, respectively. This work is the continuation of [1] and [2].

[1] P.V. Gapeev, L. Li. Optimal stopping problems for maxima and minima in models with asymmetric information. *Stochastics: An International Journal of Probability and Stochastic Processes*, **94**(4):602–628, 2022.

[2] P.V. Gapeev, L. Li. Perpetual American defaultable standard and lookback options in models with incomplete information. *SIAM Journal on Financial Mathematics*, **13**(3):773–801, 2022.

Investment under uncertainty in a durable good market

Herbert Dawid (1), Peter M. Kort (2), Xingang Wen (1),

(1) Faculty of Business Administration and Economics, Bielefeld University, Bielefeld, Germany; (2) Department of Econometrics & Operations Research, Tilburg University, Tilburg, The Netherlands;

We consider a monopolistic firm that decides on the timing and production capacity for introducing a durable good into a market characterized by consumer heterogeneity and demand uncertainty. We show that when consumers are less heterogeneous, the firm should invest later, i.e. wait for demand to grow to a sufficiently high level, in a large production capacity. In case consumers are very heterogeneous, the firm should invest early in a small production capacity. In the latter case, selling the durable good in small quantities enables the firm to price discriminate over time, generating a high payoff. An increase in consumer heterogeneity can have a positive impact on the value of the firm's investment opportunity if demand uncertainty is small, where under large uncertainty the value decreases with consumer heterogeneity.

This is a major difference from a nondurable goods market, where even under small demand uncertainty an increase in consumer heterogeneity typically has a negative effect on the firm's value.

Capacity Investment and Market-Driven Price Caps in Electricity Markets

Jacco Thijssen (1), Peter Mwangi (1),

(1) Department of Mathematics, University of York, United Kingdom.;

We build a continuous-time model of incremental capacity investment in electricity generation by a representative firm with stochastically evolving electricity prices and time-to-build. The model results in a "market-driven price cap" on electricity prices at pledged capacity. This cap is increasing in the cost of investment, time-to-build, volatility of electricity prices, and the market price of electricity price risk. The cap is decreasing the elasticity of demand. It is shown that investment only takes place when demand is elastic and that electricity prices can still increase substantially, even after new capacity has been pledged, due to construction delays. This opens up the possibility of welfare-increasing policy intervention.

19. Uncertainty Quantification in Optimization, Optimal Control, and Identification

15:35 – 16:50

Chair: **A. Khan, C. Tammer** **FH Nöbauer 8**

Set-valued dynamics: a unified approach to multifunctions in discrete-time systems

Elvira Hernández (1), Juan Perán (1),

(1) Department of applied mathematics, UNED, Madrid, Spain;

We investigate the asymptotic behavior of discrete-time set-valued dynamical systems governed by difference inclusions. Key concepts such as limit sets, invariant sets, and attractors are rigorously defined using set-valued analysis, and connections with various notions of stability (in the Lyapunov sense) are established. This work develops a theoretical framework for multifunctions in dynamical systems, emphasizing stability analysis and invariance principles. By leveraging tools from set-valued maps and relations, we provide foundational results that bridge dynamical systems theory and set-valued analysis, along with illustrative applications in stability and control.

Stochastic approximation in convex multiobjective optimization

Carlo Alberto De Bernardi (1), Elena Molho (2), Enrico Miglierina (1), Jacopo Somaglia (3),

(1) DiMSEFA, Università Cattolica del Sacro Cuore, Milano, Italy ; (2) DSEA, Università di Pavia, Pavia, Italy; (3) Dipartimento di Matematica, Politecnico di Milano, Milano, Italy;

Given a strictly convex multiobjective optimization problem with objective functions $f_1, \ldots, f_N$, let us denote by x_0 its solution, obtained as minimum point of the linear scalarized problem, where the objective function is the convex combination of $f_1, \ldots, f_N$ with weights $t_1, \ldots, t_N$. The main result of this work gives an estimation of the averaged error that we make if we approximate x_0 with the minimum point of the

convex combinations of n functions, chosen among $f_1, \ldots, f_N$, with probabilities $t_1, \ldots, t_N$, respectively, and weighted with the same coefficient $\frac{1}{n}$. In particular, we prove that the averaged error considered above converges to 0 as n goes to ∞, uniformly w.r.t. the weights $t_1, \ldots, t_N$. The key tool in the proof of our stochastic approximation theorem is a geometrical property, called by us small diameter property, ensuring that the minimum point of a convex combination of the function $f_1, \ldots, f_N$ continuously depends on the coefficients of the convex combination.

Gradient descent algorithm for interval-valued functions

Beatriz Hernández-Jiménez (1), Antonio Beato-Moreno (2), Rafaela Osuna-Gómez (3), Antonio Pascual-Acosta (4),

(1) Universidad Pablo de Olavide, Sevilla, Spain; (2) (3); (4)Universidad de Sevilla, Sevilla, Spain

The Interval Analysis was introduced by Ramon Moore [1] to manage the imprecision or lack accurate information that appear naturally in mathematical models in order to solve problems closet to human reality. In this paper with the idea in mind that the gH-differentiability notion is the more appropriate and basing on Osuna-Gómez et al.'s works [2] and [3], we present an unconstrained problem where the objective function is an interval-valued function of several variables. We characterize all the descent directions and define those that would be efficient directions and use one of them explicitly calculated to describe the algorithm.

[1] R.E. Moore. Automatic Error Analysis in Digital Computation. Technical Report. LMSD-48421, Palo Alto, CA, Lockheed Missiles and Space Co, 1959.

[2] R. Osuna-Gómez, T.M. da Costa, Y. Chalco-Cano and B. Hernández-Jiménez. Quasilinear aproximation for interval-valued functions via generalized Hukuhara differentiability. *Comp. Appl. Math.* **41** 149, 2022.

[3] R. Osuna-Gómez, T.M. da Costa, B.Hernández-Jiménez and G. Ruíz-Garzón. Necessary and sufficient conditions for interval-valued differentiability. *Math Meth. Appl. Sci.* **46**:2319–2333, 2023.

20. Control of dynamic systems and games with applications 15:35 – 16:50

Chair: **N. Hayek, S. Pickenhain** **FH HS 6**

Viscosity solutions on Hilbert spaces for centralized constrained optimal control and differential games of evolutionary masses

Fabio Bagagiolo (1),

(1) University of Trento, Trento, Italy;

In the articles [1] and [2] a differential game between a single player and a mass as well as between two masses are studied. A corresponding Hamilton-Jacobi-Isaacs equation is derived on an invariant set of a Hilbert space and studied in the framework of the viscosity solutions theory. More recently, similar techniques are applied in [3] to the case of the optimal control for a single mass under a constraint on its support. In all cases, the evolution of the masses is given by a first order partial differential equation in $\mathbb{R}^n$ (the continuity equation) which is controlled in a centralized manner by choosing the velocity field. A suitable spatial regularity of the admissible controls allows to restrict to the case where the mass has a density, given by a function on $\mathbb{R}^n$, and hence to cast the problem into a Hilbert setting. This is a joint research program with Rossana Capuani, Luciano Marzufero and Ivan Romanò.

[1] F. Bagagiolo, R. Capuani and L. Marzufero. A single player and a mass of agents: a pursuit evasion-like game. *ESAIM COCV Control Optimisation and Calculus of Variations*, **30**:17, 2024.

[2] F. Bagagiolo, R. Capuani and L. Marzufero. A zero-sum differential game for two opponent masses. *Minimax Theory and its Applications.* To appear. Preprint 2024, Arxiv: https://arxiv.org/abs/2408.03860

[3] F. Bagagiolo and I. Romanò. State-constrained optimal control for a mass. In preparation.

Environmental policy and productivity in an economic geography model

Mar'ıa Pilar Martínez-García (1), José R Morales (1),

(1) University of Murcia, Murcia, Spain;

This paper develops a New Economic Geography model of two regions with polluting firms subject to regional abatement policies. Pollution reduces the welfare of the population. Firms exhibit different levels of productivity, with more productive firms emitting fewer emissions per unit of output. Only the more productive firms can afford the fixed costs of entering the export market, while the less productive ones produce exclusively for domestic consumers. Environmental policy has two opposing welfare effects: it reduces nominal wages and improves environmental quality. It also raises production costs, which induces some firms to stop exporting and pushes less productive firms out of the market. Our model incorporates all the effects of Krugman's model but also introduces a pollution effect that works against firm agglomeration. Greater heterogeneity in firms' productivity weakens the agglomerative forces and increases the propensity to export. Firms' heterogeneity fosters the dispersion of the industry. This paper investigates the effects of environmental policy on the geolocation of the most productive firms and its welfare consequences.

Switch time algorithm for bang-singular controls: second-order sufficient conditions, sensitivity analysis and approximation

Helmut Maurer (1), Georg Vossen (2),

(1) Universität Münster, Institute für Analysis und Numerik, Münster, Germany;
(2) Hochschule Niederrhein, University of Applied Sciences, Fachbereich Maschinenbau und Verfahrenstechnik, Krefeld, Germany;

We consider optimal control problems with control appearing linearly. It follows from Pontryagin's Maximum Principle that the optimal control is a concatenation of bang-bang and singular arcs. In many practical applications, the singular control can be determined in feedback form. Assuming that there exists only *finitely many switching times* between bang-bang and singular arcs one can set up a finite-dimensional optimization problem w.r.t. the switching times which is called the induced optimization problem (IOP). Some applications may be found in the older and newer papers [1], [2], [3]. Second-order sufficient optimality conditions (SSC) for the bang-singular control problem have been given in [5] in a very abstract form which has been verified so far only on simple examples. In this paper, we study (SSC) for the finite-dimensional (IOP). In case that (SSC) hold it follows from the sensitivity analysis in [4] that the optimal switching times are locally differentiable functions w.r.t. parameters in the control problem. This allows to perform a numerical sensitivity analysis w.r.t. parameter changes. We give numerical results for the Goddard Problem in [1],[2], for the problem of combination therapies of cancer [6] and the Rayleigh problem in electrical engineering.

[1] H. Maurer. Numerical solution of singular control problems using multiple shooting techniques, *Journal of Optimization Theory and Applications*, **18**(2): 235–257, 1976.

[2] G. Vossen. Switching time optimization for bang-bang and singular controls, *Journal of Optimization Theory and Applications*, **144**: 409—429, 2010.

[3] M. Aghaee and W. W. Hager. The switch point algorithm. *SIAM J. Control and Optimization*, **59**(4): 2570–2593, 2021.

[4] A. Y. Fiacco. Introduction to Sensitivity and Stability Analysis. Academic Press, New York, London, 1983.

[5] M. S. Aronna, J. F. Bonnans, A. Dmitruk and P. Lotito. Quadratic order conditions for bang-singular extremals. *Numerical Algebra, Control and Optimization*, **2** (3): 511–546, 2012.

[6] U. Ledzewicz, H. Maurer and H. Schättler. Optimal and suboptimal protocols for a mathematical model for tumor anti-angiogenesis in combination with chemotherapy. *Mathematical Biosciences and Engineering*, **8**: 307–323, 2011.

21. Hamilton-Jacobi equations for optimal control and games: new trends in numerical and analytical aspects — 15:35 – 16:50

Chair: **M. Akian, D. Vasquez-Varas** — **FH HS 4**

Solving non-causal schemes for anisotropic eikonal equations, with quasi-linear complexity

Jean-Marie Mirebeau (1),

(1) Centre Borelli, ENS Paris-Saclay, University Paris-Saclay, CNRS, Gif-sur-Yvette, France;

The eikonal equation is a first order non-linear static Hamilton-Jacobi-Bellman PDE, characterizing the first arrival time of a front, whose speed depends on the front position and possibly on some anisotropic

velocity profile. Discretizations schemes for the eikonal equation are typically *monotonous*, and may or may not obey an additional property referred to as *causality*. The latter property is related with the geometry of the scheme stencil, and is non-trivial to enforce in (strongly) anisotropic settings. If causality is satisfied, then the scheme can be solved efficiently using the fast marching method, with quasi-linear complexity $\mathcal{O}(N \ln N)$ w.r.t. the number of discretization points. If it is not, then various alternative methods are available, such as fast sweeping, but their numerical complexity has only been empirically estimated and can be superlinear $\mathcal{O}(N^{1+1/d})$ in unfavorable cases. In this talk, we present a numerical solver of anisotropic eikonal equations, which does not rely on the causality property, and whose complexity $\mathcal{O}(N \ln(N/\varepsilon)^2)$ is quasi-linear, where $\varepsilon > 0$ denotes the tolerance for the solution error.

[1] J.-M. Mirebeau, R. Mansour, Solving non-causal schemes for anisotropic eikonal equations, with quasi-linear complexity, *Preprint*, 2025.

Multi-level fast-marching method and semi-Lagrangian scheme for the minimum time problem

Marianne Akian (1), Stéphane Gaubert (1), Shanqing Liu (2),

(1) Inria and CMAP, École polytechnique, CNRS, Institut Polytechnique de Paris; (2) Division of Applied Mathematics, Brown University;

We will present the multi-level fast marching method introduced in [1] which rely on several grid approximations of the stationary Hamilton-Jacobi partial differential equations arising from a minimum time optimal control problem. It consists in reducing the domain to neighborhoods of approximate optimal trajectories constructed from coarse grid approximations. The convergence of the method and its complexity follow from error estimates of the discretization scheme for a state constrained Hamilton-Jacobi equation. We then consider as in [2] a semi-Lagrangian scheme with state and time steps of same order, and prove order 1 estimates under some concavity and convexity assumptions. We shall also extend our convergence results to problems with particular state constraints. These results allow us to analyze the convergence rate and computational complexity of the fast-marching method, and also of the multi-level fast-marching method.

[1] M. Akian, S. Gaubert, and S. Liu. *A multilevel fast marching method for the minimum time problem*, SIAM Journal on Control and Optimization, 62 (2024), pp. 2963–2991.

[2] M. Akian, and S. Liu. *Convergence and Error Estimates of A Semi-Lagrangian scheme for the Minimum Time Problem*, Preprint arXiv:2407.06969, 2024.

Numerical approximation for the critical value of eikonal Hamilton-Jacobi equations on networks

Valentina Coscetti (1), Marco Pozza (2),

(1) Department of Mathematics, Sapienza University of Rome, Rome, Italy; (2) Link Campus University, Rome, Italy;

The critical value of an eikonal Hamilton–Jacobi equation is the unique parameter for which the equation admits solutions. A numerical algorithm for the critical value approximation of eikonal equations

posed on networks is presented. The proposed method is based on the long time behavior of the solutions to the corresponding time-dependent Hamilton-Jacobi equations, which are numerically solved with a semi-Lagrangian scheme. An error estimate is provided and some numerical tests in the case of networks in $\mathbb{R}^2$ are shown, to illustrate the convergence of the algorithm.

22. Dynamic games in economics

15:35 – 16:50

Chair: **S. Wrzaczek, R. Neck**

FH HS 5

Optimal cartel strategy if facing a myopic fringe in a non-renewable resource economy

Agnieszka Wiszniewska-Matyszkiel (1), Maciej Wrona (2),

(1) Institute of Applied Mathematics and Mechanics, University of Warsaw, Warsaw, Poland; (2) Faculty of Economic Sciences, University of Warsaw, Warsaw, Poland;

We consider a cartel-versus-fringe model of a crude oil market, with many fringe members exploiting a common deposit and having constraints on pumping. From the game-theoretic point of view, cartel-versus-fringe equilibria are Stackelberg equilibria, as the cartel maximizes payoff provided that each of the fringe members best responds to its strategy and the strategies of the remaining fringe members. However, if we consider a cartel-versus-fringe interaction with the fringe treated as one decision maker, then its behavior may seem inconsistent: the oil price has to equal the inverse demand, which depends also on the fringe decision. To this end, we model the fringe as a continuum of rational players and we prove that this implies that in the case of a common deposit, the fringe equilibrium decisions are equivalent to myopic Nash equilibrium given the strategy of the cartel. We fully characterize this equilibrium. First, the equilibrium price does not have to be continuous at depletion time. Second, depending on parameters, it may turn out that the obvious case when the cartel shares the market with the fringe before the fringe members deplete their deposit is not optimal. It also might be optimal to wait until the fringe members deplete their deposit, or to set a price excluding the fringe from extraction by selling crude oil at the marginal cost of extraction of the fringe. After conducting sensitivity analysis, we obtain that the former scenario is optimal when the interest rate is low and the constraint high, whereas the latter scenario when the cartel's marginal extraction cost is substantially lower than the marginal cost of the fringe.

Environmental regulation and green reputation: tax evasion versus greenwashing

Francisco Cabo (1), Guiomar Martín-Herrán (1), Laís Ramos (1),

(1) IMUVa, Universidad de Valladolid, Valladolid, Spain;

The paper analyzes the dynamic interaction between a regulator and polluting firms as a differential game played *à la Stackelberg*, where the regulator settles an emission tax, acting as the leader. Correspondingly, the firms choose the reported (taxed) emissions and the actual emissions. The firms maximize profits net of taxes and fines from evasion, defined as the gap between actual and reported emissions. Environmentally concerned consumers value the green reputation of the firms. This reputation is measured by a stock variable which decays with the size of firms' evasion, detected at a rate proportional to the regulator's monitoring effort. Moreover, low reported emissions can be regarded by naïve consumers as signal of firm's greenness, which boots green reputation. This effect can be interpreted as firms greenwashing, which intensity is inversely proportional to the monitoring effort. The regulator maximizes social welfare.

We characterize the feedback strategies as well as the consumers' surplus, the environmental damage and the social welfare as a function of the consumers credibility on the firms emission disclosure. The dynamic results are confronted against the results in a static game.

Market share battles in a competitive advertising clutter

Dominika Machowska (1), Agnieszka Wiszniewska-Matyszkiel (1), Rajani Singh (2),

(1) University of Warsaw, Poland; (2) Copenhagen Business School, Denmark;

In today's media landscape, consumers are overwhelmed by a vast array of advertising messages across multiple channels, contributing to what is commonly known as advertising clutter. This phenomenon is defined as "the level of advertising and other non-programming material within a medium" (see [1]). In this context, we address the research question: *How does advertising efficiency influence market share dynamics and long-term profitability?* To explore this, we study a dynamic game model of brand advertising in continuous time with a finite time horizon. The total payoff or profit of the players (or firms) is discounted over time. The rate at which firms attract customers from their rivals—referred to as the attraction rate—is adopted from the economic theory of contests. A key novelty of our model lies in the formulation of the attraction rate: it depends not only on a firm's own advertising effort but also on the advertising efforts of its rivals. This reflects the reality that each firm's campaign must "break through the noise" generated by competing advertisements. We look for both closed-loop Nash equilibria and Pareto optima, proving nonexistence of Pareto optima and examining economic properties of the unique Nash equilibrium.

[1] Hammer, Peter, Erica Riebe, and Rachel Kennedy. "How clutter affects advertising effectiveness." Journal of Advertising Research 49.2 (2009): 159-163.

23. Dynamics of the firm

15:35 – 16:50

***Chair*: P. Kort, El Ouardighi**

FH HS 7

Optimal investment in an energy storage system

Marta Castellini (1,2), Chiara D'Alpaos (3), Fulvio Fontini (4,5), Michele Moretto (1),

(1) Department of Economics and Management Marco Fanno, University of Padova, Italy.; (2) Fondazione Eni Enrico Mattei (FEEM), Milano, Italy.; (3) Department of Civil, Environmental and Architectural Engineering, University of Padova, Italy.; (4) Department of Law Studies, University of Salento, Italy.; (5) Climate Economics Chair, University Paris Dauphine, France.;

Renewable energy production plays a crucial role in the energy transition. However, many renewable energy sources (RES) are intermittent, and there is often a mismatch between energy production and consumption, which can be partially solved by storage.
In this paper, we investigate the investment decision in a photovoltaic (PV) power plant coupled with a Battery Energy Storage System (BESS), namely an Energy Storage System (ESS). We aim to investigate the relationship between the net present value (NPV) of the investment and the technical implications related to the maximum amount of energy to be stored while also accounting for the impact of energy prices. In our setting, the BESS is connected to the national power grid and the PV plant. Energy can

be produced, purchased from the grid, stored, self-consumed, and fed into the grid. PV production and energy consumption loads evolve stochastically over time. In addition, as BESS are costly, energy stored has an opportunity cost, which depends on the prices of energy purchased from the grid and energy fed in and sold to the grid, respectively. However, BESS can significantly contribute to increase ESS managerial flexibility and, in turn, ESS value. In detail, we investigate the optimal BESS size that minimizes ESS net operating costs. We also provide insights on ESS optimal management strategy. Our results show that ESS net operating costs are relatively small. They reduce for increasing selling prices of energy, whereas they increase for increasing volatility of the stock of energy stored in the battery.

Duopoly competition with asymmetric technology: flexible vs. dedicated production capacity and the dynamics of conversion

Konstantin Kogan (1), Alon Tsukerman (1),

(1) Department of Management, Bar-Ilan University, Ramat Gan, Israel;

We examine a dynamic Cournot duopoly competition under asymmetric initial technology conditions: one firm operates with flexible production capacity, while the other relies on dedicated capacity. The dedicated capacity produces at a constant rate, whereas the flexible capacity allows output adjustments. The dedicated capacity, however, can be converted into flexible capacity over time.

Our model is a continuous-time differential game in which firms simultaneously adjust their production rates within flexible capacity constraints. Additionally, the firm with dedicated capacity may invest in converting it, either partially or fully. Given that market prices include a stochastic component, making them difficult to track, and the product life is finite, we adopt open-loop Nash strategies over a finite planning horizon. The objective is to analyze how flexible capacity investments influence continuous-time output competition between the two firms and the resulting pricing dynamics. We characterize equilibrium trajectories, identifying conditions under which: (i) the flexible firm is satisfied with a smaller market share than the dedicated firm, (ii) the dedicated firm refrains from conversion and becomes a monopoly, and (iii) both firms attain equal market shares as the dedicated capacity is either partially or fully converted at a maximum rate, with flexible capacities being utilized either partially or fully.

Capacity Investment and Market-Driven Price Caps in Electricity Markets

Jacco Thijssen (1), Peter Mwangi (1),

(1) Department of Mathematics, University of York, United Kingdom.;

We build a continuous-time model of incremental capacity investment in electricity generation by a representative firm with stochastically evolving electricity prices and time-to-build. The model results in a "market-driven price cap" on electricity prices at pledged capacity. This cap is increasing in the cost of investment, time-to-build, volatility of electricity prices, and the market price of electricity price risk. The cap is decreasing the elasticity of demand. It is shown that investment only takes place when demand is elastic and that electricity prices can still increase substantially, even after new capacity has been pledged, due to construction delays. This opens up the possibility of welfare-increasing policy intervention.

24. Uncertainty Quantification in Optimization, Optimal Control, and Identification

17:00 – 18:40

Chair: **A. Khan, C. Tammer** **FH Nöbauer 8**

Sample-based consistency in infinite-dimensional conic-constrained stochastic optimization

Caroline Geiersbach (1), Johannes Milz (2),

(1) University of Hamburg, Germany; (2) Georgia Institute of Technology, USA;

This talk is concerned with a class of risk-neutral stochastic optimization problems defined on a Banach space with almost sure conic-type constraints. These types of problems have applications in physics-based models where a system described by a partial differential equation (PDE) with random inputs or parameters should be optimized, and where additional constraints on the PDE's solution are imposed. For this class of problems, we investigate the consistency of optimal values and solutions corresponding to sample average approximation (SAA) as the sample size is taken to infinity. Additionally, the consistency of SAA Karush–Kuhn–Tucker conditions with Moreau–Yosida-type regularization is shown under mild conditions. This work provides theoretical justification for the numerical computation of solutions frequently used in the literature and in experimentation. Examples from PDE-constrained optimization under uncertainty are analyzed, demonstrating how the framework can be used in practice.

Interferometric optimization for complex linear inverse problems with large uncertainties

Laurent Seppecher (1),

(1) École Centrale de Lyon, France;

Classical approaches to solve a linear inverse problem $Ax = b$ from noisy data consists of minimizing a least sqaures cost functional of the form $J(x) = \|Ax-b\|_2^2 + R(x)$, where $R(x)$ is a regularization term. For instance the LASSO technic uses a ℓ^1-regularization $R(x) := \varepsilon\|x\|_1$ in order to promote sparse solutions. In the case of highly noisy data b, this type of approach may fail as there is no reason for the linear misfit $Ax - b$ to be small.

In the case of complex linear systems (such as the inversion of the Fourier transform or the wave source identification problem), the interferometric optimization appears to be a strong alternative. It consists of considering some cross-correlations data $b_i\bar{b}_j$ instead of the data vector b. The associated interferometric misfit functional reads

$$J_E^{\text{int}}(x) := \sum_{i,j\in E} |(Ax)_i\overline{(Ax)}_j - b_i\bar{b}_j|^2$$

where E is a set of index pairs that must be chosen carefully. Minimizing this functional is challenging as it is a non-convex fourth order polynomial. Its interest comes from the fact that some cross-products $b_i\bar{b}_j$ are very stable in the case of strong correlated phase errors on the data.

I will present some recent results regarding the global minimization of this functional as well as the proof of convergence of a descent algorithm. Moreover, I will propose some numerical applications that show the ability of this approach to recover sparse wave sources (or scatters) in highly uncertain media.

Uncertainty and information in reconstructions of elastic nonlinearity

Daniel I. Gendin (1), Paul E. Barbone (2),

(1) University at Buffalo, Buffalo, NY, US; (2) Boston University, Boston, MA, US;

We are interested in recovering linear and non-linear elastic parameters for the non-invasive differential diagnosis of breast masses. This is done through the solution of an inverse problem that is formulated as a constrained optimization problem. By considering our problem in the framework of Bayesian statistics, we are able to consider uncertainty and compute quantities such as point-wise variance and information gain. Measurement sensitivity to the value of elastic nonlinearity is much higher for large (finite) strains than for smaller strains. Due to this, large strains tend to be used for such measurements. Measurements of larger deformations, however, tend to exhibit greater levels of noise. A key finding of this work is that, when evaluating elastic nonlinearity, information gain can be used to characterize a trade-off between larger strains with higher noise levels and smaller strains with lower noise levels. These results can be used to inform experimental design. We consider several formulations of the elastic inverse problem and interpret the results from both a deterministic and Bayesean point of view. A key finding here is that accurate characterization of the joint posterior probability distribution over both linear and nonlinear elastic parameters requires that the estimates be performed jointly rather than sequentially. We apply our methods to simulations, phantom data, and *in vivo* measurements.

Stochastic Optimization Methods for Scalar PDE Inverse Problem

Yidan Yang (1), Basca Jadamba (2),

(1) School of Mathematical Sciences, Xiamen University, Xiamen, China; (2) School of Mathematics and Statistics, Rochester Institute of Technology, Rochester NY, USA;

We present stochastic variants of optimization method when solving the inverse problem (coefficient identification problem) in scalar partial differential equation (PDE). In many PDE inverse problems, the goal is to leverage observational measurements of the state variable in the estimation of the model parameter(s). The inverse problem we are exploring in this talk is estimating the two-dimensional elliptic PDE system coefficient $q(x)$ by knowing the measurement data of state variable $u(x)$ and source term $f(x)$ with form of where boundary conditions are well-defined. We implemented various optimization approach including gradient descent with Landweber iteration, batch gradient, etc. Meanwhile, we investigate the effect of computational parameters in distinct optimization methods such as step size, regularization parameter, and finite element mesh size in the quality of the recovered parameter in scalar PDEs. We conduct numerical experiments on both analytical problem with synthetic data and an application of groundwater flow with the simulation of flow through an aquifer with water channels, in which we recovered the system coefficient (hydraulic conductivity in groundwater case) with the knowledge of measurement data of system state variable (hydraulic head in groundwater case). We also extended our PDE inverse problem computational framework to stochastic PDE.

25. Control of dynamic systems and games with applications 17:00 – 18:40

***Chair*: N. Hayek, S. Pickenhain** **FH HS 6**

POD approximations of infinite horizon optimal control problems based on time derivatives

Javier de Frutos (1), Bosco García-Archilla (2), Julia Novo (3),

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We consider the numerical approximation of infinite horizon optimal control problems via the dynamic programming approach. As it is well known, if the Hamilton-Jacobi-Bellman (HJB) equation is approximated by a fully discrete method the so called curse of dimensionality can become a serious issue in order to get meaningful approximate optimal policies. To mitigate this issue we apply a method of order reduction by means of a proper orthogonal decomposition (POD) method based on time derivatives. We carry out the error analysis of the method using recently proved optimal bounds for the fully discrete approximations [1]. Moreover, the use of snapshots based on time derivatives allow us to bound some terms of the error that could not be bounded in a standard POD approach. Some numerical experiments show the good performance of the method in practice. This talk is based in the paper [2]

[1] J. de Frutos and J. Novo. Optimal bounds for numerical approximations of infinite horizon problems based on dynamic programming approach. *SIAM. J. Control Optim.*, **61**:415-433, 2023.

[2] J. de Frutos, B. García-Archilla and J. Novo. Optimal bounds for POD approximations of infinite horizon control problems based on time derivatives. *J. Sci. Comput.*, **103**:19, 2025.

Necessary conditions of optimality for difference games with asymptotical constraints.

Naila Hayek (1),

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This paper studies necessary conditions for difference games in presence of asymptotical constraints. The dynamics are governed by difference equations. Tools of optimization in Banach spaces are used to provide Pontryagin maximum principles in the strong form and in the weak form as necessary conditions for Nash equilibrium.

[1] J. Blot and T.N. Ngo, *Pontryagin principles in infinite horizon in the presence of asymptotical constraints.* Vietnam J. Math. 45, 541559 (2017).
https://doi.org/10.1007/s10013-016-0205-z

[2] N. Hayek *A note on multiobjective optimal control in presence of asymptotical constraints.* Communications in Optimization Theory, Vol. 2025 (2025), Article ID 18, pp. 1-12.
https://doi.org/10.23952/cot.2025.18

Applications of existence results for relaxed problems of optimal control with infinite horizon and unbounded controls

Sabine Pickenhain (1), Ilya Dikariev (1),

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We consider an SIT model [1] and a Lotka-Volterra model with an infinite time horizon, non-linear dynamics, unbounded controls and non-convex cost functions. For the considered relaxation methods of the problem, [2], [3], equivalence is shown under appropriate conditions. For the relaxed problem with Young measures [2], the existence of the solution can be shown. We use Weighted Sobolev spaces as state spaces, which allows weaker growth assumptions compared to the literature [4, 5]. The existence theorems can be applied to the discussed applications.

[1] P.-A. Bliman, D. Cardona, Y. Dumont, and O. Vasilieva. Optimal Control Approach for Implementation of Sterile Insect Techniques. *Journal of Mathematical Sciences*, **279**(5):607–622, 2024.

[2] R.V. Gamkrelidze. *Principles of Optimal Control Theory.* Plenum Press, 1978.

[3] D.A. Carlson. Nonconvex and relaxed infinite – horizon optimal control problems. *Journal of optimization theory and applications*, **78**(3):465–491, 1993.

[4] E.J. Balder. An existence result for optimal economic growth problems. *Journal of mathematical analysis and applications*, **95**(1):195–213, 1983.

[5] S.M. Aseev. An Existence Result for Infinite-Horizon Optimal Control Problem with Unbounded Set of Control Constraints. *IFAC-PapersOnLine*, **51**(32):281–285, 2018.

New Necessary Conditions of Optimality for problems of Calculus Of Variations under an infinity of inequality constrains.

Joël Blot (1),

(1) Laboratoire SAMM UR 4543 and FR 2036 CNRS, Université Paris 1 Panthéon-Sorbonne, Paris, France;

We present new Euler-Lagrange equation and new Fritz John multipliers for problems of Calculus of Variations under an infinity of inequality constraints.
We use Stepanov Lipschitzean functions to obtain the Gateaux differentiability of functionals under integral form, and we use recents works on Static Optimization in infinite dimension (obtained with Hilmaz and Bachir).

These results are based on recent works with Mohammed Bachir.

26. Advances in shape optimization

17:00 – 18:40

***Chair*: M. Dambrine**

FH HS 4

Optimization of cuts configuration for skin grafting

Helmut Harbrecht (1), Viacheslav Karnaev (1),

(1) DMI, University of Basel, Basel, Switzerland;

The subject of this work is the problem of optimizing the configuration of cuts for skin grafting in order to improve the efficiency of the procedure. We consider cuts located in periodic cells, which fill the entire body Ω. The configuration of the N cuts is then uniquely determined by the array of slope angles $\boldsymbol{\alpha} := (\alpha_1, \ldots, \alpha_N) \in [0, 2\pi)^N$ We consider the optimization problem in the framework of a linear elasticity

model. We choose three mechanical measures that define optimality via related objective functionals: the compliance, the L^p-norm of the von Mises stress, and the volume of the stretched skin:

$$\mathcal{C}(\Omega_{\boldsymbol{\alpha}}) := \int_{\Omega_{\boldsymbol{\alpha}}} \sigma(\boldsymbol{u}_{\boldsymbol{\alpha}}) : \varepsilon(\boldsymbol{u}_{\boldsymbol{\alpha}}) \, \mathrm{d}\boldsymbol{x},$$

$$\mathcal{VM}(\Omega_{\boldsymbol{\alpha}}) := \int_{\Omega_{\boldsymbol{\alpha}}} \sigma_d(\boldsymbol{u}_{\boldsymbol{\alpha}}) : \sigma_d(\boldsymbol{u}_{\boldsymbol{\alpha}}) \, \mathrm{d}\boldsymbol{x},$$

$$\mathcal{VM}(\Omega_{\boldsymbol{\alpha}}) := \int_{\Omega_{\boldsymbol{\alpha}}} \det(\boldsymbol{I} + \boldsymbol{\nabla} \boldsymbol{u}_{\boldsymbol{\alpha}}) \, \mathrm{d}\boldsymbol{x}.$$

We provide a proof of the existence of solutions for each problem, but we cannot claim uniqueness. We compute the gradient of the objectives with respect to the cut configuration using shape calculus concepts. To solve the problem numerically, we use the gradient descent method, which performs well under uniaxial stretching. However, in more complex cases, such as multidirectional stretching, its effectiveness is limited due to the low sensitivity of the functionals. To avoid this difficulty, we use a combination of the genetic algorithm and the gradient descent method, which leads to a significant improvement in the results.

Shape optimization of a thermoelastic body under thermal uncertainties

Marc Dambrine (1), Giulio Gargantini (1), Helmut Harbrecht (2), Viacheslav Karnaev (2),

(1) Université de Pau et des Pays de l'Adour, E2S UPPA, CNRS, LMAP, Pau, France; (2) Departement Mathematik & Informatik, Universität Basel, Basel, Switzerland;

We consider a shape optimization problem in the framework of the thermoelasticity model under uncertainty. The uncertainty is supposed to be located in the Robin boundary condition of the heat equation. The purpose of considering this model is to account for thermal residual stresses or thermal deformations, which may hinder the mechanical properties of the final design in case of a high environmental temperature. In this situation, the presence of uncertainty in the external temperature must be taken into account to ensure the correct manufacturing and performance of the device of interest. The objective functional under consideration is based on volume minimization in the presence of an inequality constraint for a quadratic shape functional. Exemplarily, we consider the L^2-norm of the von Mises stress and demonstrate that the robust constraint and its derivative are completely determined by low order moments of the random input, thus computable by means of low-rank approximation. The resulting shape optimization problem is discretized by using the finite element method for the underlying partial differential equations and the level-set method to represent the sought domain. Numerical results for a model case in structural optimization are given.

[1] M. Dambrine, G. Gargantini, H. Harbrecht, and V. Karnaev. Shape optimization of a thermoelastic body under thermal uncertainties. *Journal of Computational Physics*, **527**:113794, 2025.

Shape optimization with Ventcel transmission condition: application to the design of a heat exchanger

Marc Dambrine (1), Fabien Caubet (1), Rodrigo Zelada (1),

(1) LMAP, Université de Pau et des Pays de l'Adour, Pau, France;

Shape optimization is a valuable tool in industrial contexts, with applications ranging from design to production. The problems considered frequently involve multiphysics and complex geometries, which

can present significant challenges. Numerical resolution of these problems can be costly and limit the application of shape optimization. Consequently, reducing the cost of optimization is paramount, and one approach is to consider asymptotic models that take into account small physical or geometric parameters. This work represents a progress in this direction: it consists in optimizing the geometry of a tube in a heat exchanger, taking advantage of the property that the wall separating a heat transfer fluid from a fluid to be heated is thin. The flow of two coupled fluids with different temperatures must also be considered. To this end, we will employ an approximate model derived from asymptotic analysis with respect to the small parameter represented by this thickness. optimize the shape of a fluid-to-fluid heat exchanger in order to maximize heat exchange under constraints of energy dissipation and volume. The novelty consists in taking into account the thin layer separating the two fluids by using Ventcel-type second-order transmission conditions. The physical model is then a weakly coupled problem between the steady-state Navier-Stokes equations for the dynamics of the two fluids dynamics and the convection-diffusion equation for the heat. One of the original features of this work lies in the optimization of a surface where the quantity of interest, the temperature, is discontinuous. Non-standard transmission conditions are satisfied on this surface at the end of the asymptotic analysis. This problem is original and poses significant technical challenges, particularly in justifying the sensitivity analysis and implementing an optimization method. Our contributions are twofold: first, a theoretical sensitivity analysis in relation to the transmission surface, and second, a numerical analysis with the implementation of a method of optimization of this surface that is both efficient and robust in relation to this small parameter.

Optimal Long-Time Observability of the Heat Equation and Optimal Sensor Placement

Idriss Mazari (1), Yannick Privat (2), Emmanuel Trélat (3),

(1) CEREMADE, University Paris-Dauphine; (2) Lorraine University, Institut Élie Cartan de Lorraine.; (3) Sorbonne University, Laboratoire Jacques-Louis Lions;

It is well known that reconstructing the initial data associated with a parabolic equation from internal measurements of its solution over a time interval T within a domain ω (referred to as the observation domain) is equivalent to the question of observability. More precisely, it relates to the positivity of the so-called observability constant associated with ω. This constant depends not only on the observation domain ω but also, crucially, on the time horizon T.

In this talk, we focus on the optimal placement of thermal sensors. A natural way to model this problem is to seek extremal domains (when they exist) that maximize the observability constant. To ensure physical relevance, we impose a constraint on the measure of the observed domain.

After introducing a convex relaxation of the shape optimization problem, we analyze the asymptotic behavior of the maximizers as $T \to +\infty$. Using a quantitative bathtub principle as a key tool, we establish the strong convergence of the maximizers towards the characteristic function of a measurable set, which we characterize explicitly. Moreover, we show that this convergence occurs at an exponential rate.

This work is a collaboration with Idriss Mazari (Université Paris Dauphine) and Emmanuel Trélat (Sorbonne Université).

[1] I. Mazari, Y. Privat, and E. Trélat. Large-time optimal observation domain for linear parabolic systems To appear in *Ann. Inst. H. Poincaré Anal. Non Linéaire.*

29. Modelling disruptive changes and resilience in economic-environment systems

17:00 – 18:40

Chair: **M. Kuhn** **FH HS 7**

Resilience in optimal control models of environmental economics

Stefan Wrzaczek (1,2), Michael Kuhn (1,2),

(1) International Institute for Applied Systems Analysis, Laxenburg, Austria; (2) Wittgenstein Centre for Demography and Global Human Capital (IIASA, VID/OeAW, University of Vienna), Vienna, Austria;

We study a simple economic-ecological framework of resource extraction that explicilty incorporates (i) a regime changing shock and (ii) the possibility a resistence threshold/Skiba-point to lead to divergent behaviours towards either systemic recovery or system collaps following the shock. This structure allows us to model resilience (in the sense of regime-changing shocks either being avoided or systemic recovery being possible and optimal) and its behavioural implications in a meaningful way. Specifically, we propose a model-based measure of resilience that upon proper calibration of the model can be employed in numerical assessments of the implications for resilience of different extraction policies. We also study a simple example and show that anticipation of shocks is leading to precautionary behaviour that boosts systemic resilience.

Resilient livestock and fire management of savannas facing woody encroachment

Marit Haringa (1), Florian Wagener (1), Frank van Langevelde (2),

(1) CeNDEF, University of Amsterdam, Amsterdam, Netherlands; (2) WEC, Wageningen University and Research, Wageningen, Netherlands;

Savanna ecosystems cover an ever-increasing proportion of the Earth's surface and house a rapidly growing share of the world's population and livestock. An often overlooked but increasingly urgent threat to savannas is posed by the process of woody encroachment, by which woodland encroaches upon grasslands used for livestock production, the principal means of subsistence in these areas. In this paper, as a benchmark for subsequent optimal control analysis, we look at the impact of non-varying livestock and fire management on the resilience of savannas against bush encroachment in a well-known dynamic savanna model with tipping points taken from the ecology literature [1][2]. We contribute to the existing literature by studying the sensitivity of savanna resilience to economic, rather than purely physical, parameters. This is done by employing numerical bifurcation analysis techniques. We also provide a definition of ecological resilience for the case of a three-dimensional system. We find that livestock extraction, while increasing the resilience of the savanna equilibrium without grazers, decreases the resilience of the savanna equilibrium with grazers for all biophysical conditions. Increased grazing intensity, in turn, buffers against this reduction in resilience to an extent, but has an adverse effect at very high levels of grazing intensity. Welfare analysis follows.

[1] F. van Langevelde, C. A. D. M. van de Vijver, L. Kumar, J. van de Koppel, N. de Ridder, J. van Andel and M. Rietkerk. Effects of fire and herbivory on the stability of savanna ecosystems. *Ecology*, **82**(2):337-350, 2003.

[2] F. van Langevelde, C. A. D. M. van de Vijver, H. H. T. Prins and A. T. Groen. Effects of Grazing and Browsing on Tropical Savanna Vegetation In *The Ecology of Browsing and Grazing II* L. J. Gordon & H. H. T. Prins, Eds. Ecological Studies, vol. 239, 2019.

Studying the mitigation and adaptation nexus under the risk of climate tipping

Maddalena Muttoni (1), Michael Freiberger (2), Michael Kuhn (2), Stefan Wrzaczek (2),

(1) University of Padova, Padova, Italy; (2) International Institute for Applied Systems Analysis, Laxenburg, Austria;

There is increasing evidence that with advancing climate change, tipping points may be crossed that lead to regime-shifts in climate patterns with consequences for the scope to counteract climate change by mitigation or responding to climate change by adaptation. In contrast to mere "shocks", such regime changes lead to a reconfiguration of climate or economic dynamics, changes in the relevance and interaction of systems states, and possible changes in the objective function or constraints. They are typically considered to be irreversible or very difficult to reverse. Examples include sea-ice or ice-shield meltdown, reversal of the Gulf Stream, or switching from carbon sinks to carbon sources as in case of the demise of rain forests or swampland, the thawing of permafrost soil [1]. The study of such regime switches is technically involved, in particular the representation of adaptive actions after a switch as a function of both the time of switch and the duration since the switch, as well as the linkage between pre-switch actions (mitigation or build-up of adaptation capacity) and post-switch behaviour. We study the role of random regime switches, depending on an endogenous climate-state dependent hazard rate of tipping, within a DICE-style model in which a planner trades off emission abatement, saving and consumption to maximise intertemporal welfare. Specifically, we build the analytical framework on a recent approach by [2] that employs an age-structured optimal control approach for the solution and numerical representation of optimal control problems with stochastic regime-switch. We study mitigation and savings patterns under the expectation of tipping (ice-shield meltdown, capital destruction, and others) and compare them against naive settings, in which they are (partially) ignored. We also derive the social cost of carbon for the different scenarios and decompose it into its components relating to tipping risks but also the scope to adjust to them.

[1] T. M. Lenton, H. Held, E. Kriegler, J. W. Hall, W. Lucht, S. Rahmstorf, and H. J. Schellnhuber. Tipping elements in the Earth's climate system. *Proceedings of the national Academy of Sciences*, **105**(6):1786-1793, 2008.

[2] S. Wrzaczek, M. Kuhn, and I. Frankovic. Using age structure for a multi-stage optimal control model with random switching time. *Journal of Optimization Theory and Applications*, **184**(3):1065-1082, 2020.

Resilience of the green transition

Michael Kuhn (1), Stefan Wrzaczek (1), Ibrahim Tahri (1),

(1) International Institute for Applied System Analysis, Laxenburg, Austria;

The green transition's resilience is tested by energy-intensive technologies and political inertia that risk fossil fuel lock-in. This paper develops a dynamic two-sector model of green (clean) and dirty (brown)

capital accumulation, where the green capital share $x_t = \frac{K_{g,t}}{K_t}$ (with total capital $K_t = K_{g,t} + K_{d,t}$), to propose a resilience assessment framework. The baseline model exhibits two stable equilibria at $x = 0$ (dirty dominance) and $x = 1$ (green dominance), separated by Skiba thresholds (x_c). Building on model-based metrics [1], we formalize three dimensions: *resistance* (thresholds for sustaining green trajectories), *recovery* (adjustment dynamics post-shock), and *robustness* (persistence under shocks). Using stability analysis of the canonical system, we derive thresholds x_c that theoretically separate basins of attraction for these equilibria. Analytical explorations suggest: (i) resistance diminishes nonlinearly near x_c, (ii) recovery paths depend asymmetrically on sectoral adjustment costs (θ_g, θ_d), and (iii) robustness correlates inversely with proximity to x_c. We further extend the model to investigate temperature feedbacks, where dirty capital emissions may accelerate climate hazards. This framework aims to bridge dynamic optimization and resilience theory, offering policymakers a structured approach to evaluate trade-offs between short-term stability and long-term decarbonization. Ongoing work explores how subsidies and carbon taxes reshape resilience thresholds, with preliminary insights highlighting the critical role of green capital buffers ($x_t \gg x_c$) in avoiding lock-in.

[1] M. Kuhn and S. Wrzaczek. A model-based measure for resilience under disruption risk. *IIASA Working Paper* WP-24-009, 2024.

Wednesday, July 16th

30. Uncertainty Quantification in Optimization, Optimal Control, and Identification

08:30 – 09:45

Chair: **A. Khan, C. Tammer** **FH Nöbauer 8**

Characterizations of robustness in vector optimization w.r.t. variable domination structures under uncertainty via image space analysis

Cong Fan (1), Christiane Tammer (2), Qilin Wang (1), Jen-Chih Yao (3),

(1) College of Mathematics and Statistics, Chongqing Jiaotong University, Chongqing, China; (2) Institute of Mathematics, Martin-Luther-University Halle-Wittenberg, Halle (Saale), Germany; (3) Center for General Education, China Medical University, Taichung, Taiwan;

It is well known that many real-world optimization problems are contaminated with uncertain data, and even a small amount of uncertainty can make the nominal optimal solution to the problem completely meaningless from a practical standpoint. So, it is essential to deal with optimization problems under uncertainty, where uncertainties are involved in the objective function and/or the constraint. In 1973, Soyster [1] first proposed the robust counterpart problem to linear programming problems under uncertainty and gave a deterministic optimization problem based on an uncertain optimization problem in the worst-case robust counterpart model. For an overview of robustness concepts, see [2], [3] and references therein. In this talk, we investigate a unified method to characterize robustness for uncertain vector optimization problems with variable domination structures in the framework of linear topological image spaces. We introduce new concepts of robustness for uncertain vector optimization problems with variable domination structure where the domination structure is given by a general set-valued map. By exploiting linear and nonlinear scalarization approaches for convex and nonconvex domination structures, respectively, as well as

image space analysis (see [4]), a series of robust optimality conditions in the context of variable domination structures are obtained under weak assumptions concerning the domination structure.

[1] A. Soyster. Convex programming with set-inclusive constraints and applications to inexact linear programming. *Oper. Res.* **21**(5):1154-1157, 1973.

[2] A. Ben-Tal, L.E. Ghaoui, A. Nemirovski. Robust optimization. *Princeton University Press*, Princeton and Oxford, 2009.

[3] K. Klamroth, E. Köbis, A. Schöbel, C. Tammer. A unified approach to uncertain optimization. *European J. Oper. Res.* **260**(2):403-420, 2017.

[4] F. Giannessi. Constrained Optimization and Image Space Analysis, Separation of Sets and Optimality Conditions, Vol. 1. *Springer*, 2005.

On adaptive robust multiobjective linear programs

Rakhi Goswami (1), Hervé Kerivin (2,1), Margaret M. Wiecek (1),

(1) School of Mathematical and Statistical Sciences, Clemson University, Clemson, SC, USA; (2) LIMOS, University Clermont Auvergne, Clermont-Ferrand, France;

Adaptive multiobjective optimization addresses decision problems under conflict and uncertainty allowing for making decisions in stages to suit changing environments or currently available information. In a multistage process, strategic decisions are made first, while tactical or operational decisions are made at later time after more information about the decision problem becomes available. Two-stage robust multiobjective linear programs (TSRMOLPs) model two-stage decision processes under uncertainty and having conflicting objectives at every stage. The goal is to compute the first-stage feasible solutions that are efficient with respect to the first and second-stage objectives and account for the worst-case uncertainty scenario. The assumptions on discrete or continuous uncertainty, the number of second-stage objectives, and the weighted-sum scalarization transform the TSRMOLP into single-objective optimization problems (SOPs) of an increasing level of difficulty. The SOPs' optimal solutions provide exact or approximate efficient (Pareto) solutions to the TSRMOLP. A solution approach using a combination of Benders' decomposition and parametric linear and nonlinear programmig is presented. Biobjective examples are included.

Conic relaxations for classes of parametric robust convex polynomial programs

Thai Doan Chuong (1), José Vicente-Pérez (2),

(1) Department of Mathematics, Brunel University of London, London, UK;
(2) Department of Mathematics, University of Alicante, Alicante, Spain;

In this talk, we examine stable exact relaxations for classes of parametric robust convex polynomial optimization problems under affinely parameterized data uncertainty in the constraints. More precisely, we show that a parametric robust convex polynomial problem with convex compact uncertainty sets enjoys stable exact conic relaxations under the validation of a characteristic cone constraint qualification. Such relaxations become: stable exact semidefinite programming relaxations for a parametric robust SOS-convex polynomial problem, where the uncertainty sets are assumed to be bounded spectrahedral; and stable exact second-order cone programming relaxations for some classes of parametric robust convex quadratic programs under ellipsoidal uncertainty sets.

Furthermore, we establish, under a suitable regularity condition, an exact conic relaxation for conic minimax convex polynomial optimization problems. We also consider a general conic minimax ρ-convex polynomial optimization problem, showing that a KKT condition at a global minimizer is necessary and sufficient for ensuring an exact relaxation with attainment of the conic programming relaxation. The obtained results are applied to SOS-convex polynomial programs and to other special settings including conic robust SOS-convex polynomial problems and difference of SOS-convex polynomial programs.

[1] T.D. Chuong, and J. Vicente-Pérez. Conic relaxations with stable exactness conditions for parametric robust convex polynomial problems. *Journal of Optimization Theory and Applications*, **197**:387–410, 2023.

[2] T.D. Chuong, and J. Vicente-Pérez. Conic relaxations for conic minimax convex polynomial programs with extensions and applications. *Journal of Global Optimization*, **91**:743–763, 2025.

31. Recent progress in PDE Constrained optimization

08:30 – 09:45

Chair: **P. Rösch, J. Pfefferer**

FH HS 6

Second order analysis for the optimal selection of time delays

Karl Kunisch (1), Fredi Tröltzsch (2),

(1) Institute of Mathematics, University of Graz, Graz, Austria; (2) Institute of Mathematics, TU Berlin, Berlin, Germany;

For a nonlinear ordinary differential equation with time delay, the differentiation of the solution with respect to the delay is investigated. Special emphasis is laid on the second-order derivative. The results are applied to an associated optimization problem for the time delay. A first- and second-order sensitivity analysis is performed including an adjoint calculus that avoids the second derivative of the state with respect to the delay.

[1] K. Kunisch and F. Tröltzsch. Second order analysis for the optimal selection of time delays. *Accepted 2024 by Mathematical Control and Related Fields.*

Optimal control of quasilinear parabolic PDEs with gradient constraints

Fabian Hoppe (1), Hannes Meinlschmidt (2), Ira Neitzel (3),

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We consider optimal control problems with control- and state constraints $u \in U_{\mathrm{ad}}$ and $y \in Y_{\mathrm{ad}}$, governed by abstract quasilinear parabolic type equation of the form

$$\partial_t y + \mathcal{A}(y)y = Bu + \mathcal{F}(y) \quad \text{in } X, \quad y(0) = y_0,$$

on a finite time interval $(0,T)$, posed in some Banach space X. The abstract formulation allows to consider mixed boundary conditions with inhomogeneous Neumann boundary data for the PDE, and we only assume the underlying spatial domain $\Omega \subseteq \mathbb{R}^d$ to be a Lipschitz manifold compatible with mixed boundary conditions. While the quasilinear operator $\mathcal{A}(y)y$ is of second-order elliptic type $-\operatorname{div}(\xi(y)\mu\nabla y)$ with the nonlinearity acting on y directly, we also allow the nonlinearity $\mathcal{F}$ to act on spatial derivatives of y, the motivating and notoriously hard prototype example being $\mathcal{F} = |\nabla y|^2$. We combine this setup with Y_{ad} modeling constraints on the gradient on the state. To deal with the state equation, we analyze the state equation in a scale of Banach spaces (Bessel dual scale) and obtain a flexible wellposedness theory in dependence on precise polynomial growth conditions for $\mathcal{F}$. We show further that if the gradient constraints in Y_{ad} combine well with the growth conditions on $\mathcal{F}$, then one obtains a well-posed optimal control problem and first-order optimality conditions. In fact, with only the minimal assumptions needed for state equation analysis, we can deal with the squared gradient nonlinearity $\mathcal{F}(y) = |\nabla y|^2$ by constraining the $L^q(\Omega)$ norm of $\nabla y(t)$ for any $q > d$ *pointwise* for every t. For more smooth data, we can also handle (t,x)-pointwise constraints on ∇y and $\mathcal{F}$ of arbitrary polynomial growth.

The minimum energy estimator for a cubic wave equation

Tobias Breiten (1), Karl Kunisch (2,3), Jesper Schröder (2),

(1) Modeling, Simulation, and Optimization of Real Processes, Technische Universität Berlin, Berlin, Germany; (2) Optimization and Optimal Control, Johann Radon Institute for Computational and Applied Mathematics, Linz, Austria; (3) Department of Mathematics and Scientific Computing, University of Graz, Graz, Austria;

The minimum energy estimator – also called Mortensen observer – was originally designed for the reconstruction of the state of nonlinear finite-dimensional dynamical systems subject to deterministic disturbances based on partial and flawed measurements. In this presentation we propose a generalization to systems governed by hyperbolic PDEs. Using the example of a nonlinear defocusing wave equation we formulate the underlying optimal control problem and formally derive the associated observer. After discussing theoretical results on well-posedness we introduce a spatial discretization of the wave equation inspired by spectral methods. This allows the numerical realization of the observer based on a polynomial approximation of the value function. We conclude with a comparison of the obtained state reconstruction to the well-known extended Kalman filter.

32. Hamilton-Jacobi equations for optimal control and games: new trends in numerical and analytical aspects 08:30 – 09:45

Chair: M. Akian, D. Vasquez-Varas **FH HS 4**

A supervised learning scheme for computing Hamilton-Jacobi equation via density coupling

Jianbo Cui (1), Shu Liu (2), Haomin Zhou (3),

(1) Department of Applied Mathematics, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong; (2) Department of Mathematics, University of California, Los Angeles, USA.; (3) School of Mathematics, Georgia Tech, Atlanta, USA;

We propose a supervised learning scheme for the first order Hamilton–Jacobi PDEs in high dimensions. The scheme is designed by using the geometric structure of Wasserstein Hamiltonian flows via a density coupling strategy. It is equivalently posed as a regression problem using the Bregman divergence, which provides the loss function in learning while the data is generated through the particle formulation of Wasserstein Hamiltonian flow. We prove a posterior estimate on L^1 residual of the proposed scheme based on the coupling density. Furthermore, the proposed scheme can be used to describe the behaviors of Hamilton–Jacobi PDEs beyond the singularity formations on the support of coupling density. Several numerical examples with different Hamiltonians are provided to support our findings.

Finite-difference least square method for solving Hamilton-Jacobi equations using neural networks

Carlos Esteve-Yagüe (1), Richard Tsai (2), Alex Massucco (3),

(1) University of Alicante, Department of Mathematics, Spain; (2) University of Texas at Austin, Oden Institute for Computational Engineering and Sciences, USA; (3) University of Cambridge, Department of Applied Mathematics and Theoretical Physics, United Kingdom.;

In recent years, advancements in deep learning and new optimisation algorithms have motivated the use of artificial neural networks to solve non-linear problems in high-dimensional setups. One of the crucial steps during the implementation of any deep learning method is the choice of the loss functional, which is used to train the neural network parameters, typically through a gradient-based method. In this talk, I will consider the approximation of the viscosity solution for Hamilton-Jacobi equations by means of an artificial neural network. I will discuss the choice of the loss functional, which should be such that any critical point approximates the viscosity solution. I will present some recent results concerning loss functionals involving a consistent and monotone numerical Hamiltonian of Lax-Friedrichs type. Using the numerical diffusion built in the numerical Hamiltonian, we are able to prove that any critical point solves the associated finite-difference problem and, therefore, approximates the viscosity solution. I will also present a method in which the numerical diffusion of the numerical scheme is decreased during the training, allowing for approximations with less numerical diffusion. This presentation is based on my recent publication [1], in collaboration with Richard Tsai and Alex Massucco.

[1] C. Esteve-Yagüe, Richard Tsai, and Alex Massucco. Finite-difference least square methods for solving Hamilton-Jacobi equations using neural networks. *Journal of Computational Physics*, **524**, 113721.

Bilevel optimization of redundant manipulators path

Yoram Halevi (1),

(1) Faculty of Mechanical Engineering Technion, Haifa, Israel;

Redundant robots are mechanical systems with more degrees of freedom (DOF) than required for their task. The redundancy is used in this work to minimize energy consumption for a required path which is partially given. It consists of several effective tasks, where the end-effector should move along a predetermined geometric shape at a given velocity, yet with free order of execution and possibly unspecified start and end points. Because of the redundancy there is room for optimization even in those tasks. In between effective tasks the motion is free.
In previous studies the problem was formulated as optimal control (OC) and was solved via subspace decompositions applied to the states and inputs. They yield components that guarantee perfect tracking, i.e. fully defined, and others which are free for optimization. The results had some limitations. Unnecessary conservatism in dealing with the constraints, having to define a full path and most importantly the well-known computation problems associated with OC.
The approach applied in this work combines an evolutionary method, specifically genetic algorithm (GA), with OC and creates a bi-level algorithm for the solution. The method decomposes the overall task into several small sub-tasks with a set of design parameters that define the interface between adjacent tasks. This allows the formulation of a bi-level optimization problem. The higher level is optimizing the design parameters and is solved using GA. Having that, the lower level is a continuous OC problem. The result is an iterative procedure that is monotonically decreasing. The work considers a variety of approximation strategies aimed at reducing the computation time. That includes using time domain Finite Element (FE) schemes to replace the general functions used by optimal control and adding a final refinement step to compensate for the inaccuracies of the approximations.

33. Infinite dimensional optimal control and differential games in Economics 08:30 – 09:45

Chair: **F. Gozzi, F. Masiero, A. Zanco** **FH HS 5**

The Lions Derivative in Infinite Dimensions - Applications to Mean-Field SPDEs

Johan Spille (1), Wilhelm Stannat (1), Alexander Vogler (1),

(1) Institut für Mathematik, TU Berlin, Berlin, Germany;

We present a new interpretation of the Lions derivative as the Radon-Nikodym derivative of a vector measure, which provides a canonical extension of the Lions derivative for functions taking values in infinite dimensional Banach spaces. This is of particular relevance for the analysis of Hilbert space valued Mean-Field equations. As our main application we establish a stochastic maximum principle for optimal control problems for Mean-Field stochastic partial differential equations (SPDEs). Another application is a mild Ito-formula for Mean-Field SPDEs, which provides the basis for a higher order Taylor expansion and higher order numerical schemes.

The talk is based on joint work with J. Spille and A. Vogler.

An approximation of the squared Wasserstein distance and an application to Hamilton-Jacobi equations

Charles Bertucci (1), Pierre-Louis Lions (2),

(1) CNRS, CMAP, École polytechnique, Palaiseau, France; (2) Collège de France, Paris, France;

We provide a simple $C^{1,1}$ approximation of the squared Wasserstein distance on $\mathbb{R}^d$ when one of the two measures is fixed. This approximation converges locally uniformly. More importantly, at points where the differential of the squared Wasserstein distance exists, it attracts the differentials of the approximations at nearby points. Our method relies on the Hilbertian lifting of PL Lions and on the regularization in Hilbert spaces of Lasry and Lions. We then provide an application of this result by using it to establish a comparison principle for an Hamilton-Jacobi equation on the set of probability measures.

Infinite horizon optimal control of McKean-Vlasov SDEs via HJB equation

Silvia Rudà (1),

(1) University of Milan, Italy;

We will study optimal control of McKean-Vlasov equations with infinite time horizon via the associated Hamilton-Jacobi-Bellman equation. We consider a system whose dynamics on the time interval $[0, +\infty)$ is described by a controlled McKean-Vlasov SDE. An optimization problem is defined by means of a discounted reward function, which can also depend explicitly on the law of the state process. Inspired by [2], analysing the finite horizon counterpart of our problem, we want to rewrite the value function V as a map v on the Wasserstein space of order 2, $\mathcal{P}_2(\mathbb{R}^d)$. We first prove that V is law invariant, i.e. that it only depends on the initial condition through its probability distribution. Moreover, we provide a time invariance property, stating that the value function is in fact independent of the initial time of the dynamics. This crucial result follows from the fact that, under suitable assumptions, the class of

admissible controls can be restricted, "forgetting" the past trajectory of the Brownian motion, without modifying the value. We subsequently prove that the value function v satisfies a Dynamic Programming Principle, from which we derive an elliptic Hamilton-Jacobi-Bellman equation on the Wasserstein space $\mathcal{P}_2(\mathbb{R}^d)$: even though the state process evolves in a finite dimensional space $\mathbb{R}^d$, the value is in fact fully characterised by a PDE on the infinite dimensional space $\mathcal{P}_2(\mathbb{R}^d)$. As a matter of fact, we prove that v is the unique solution to the equation in a suitable viscosity sense, relying on an analogous comparison principle for solutions to parabolic PDEs on $\mathcal{P}_2(\mathbb{R}^d)$ (see e.g. [1]).

[1] E. Bayraktar, H. Cheung, I. Ekren, J. Qiu, H. Tai, and X. Zhang. Viscosity solutions of fully second-order HJB equations in the Wasserstein space. *arXiv preprint arXiv:2501.01612*, 2025.

[2] A. Cosso, F. Gozzi, I. Kharroubi, H. Pham, and M. Rosestolato. Optimal control of path-dependent McKean-Vlasov SDEs in infinite-dimension. *Annals of Applied Probability*, 33(4):2863–2918, 2023.

[3] S. Rudà. Infinite Time Horizon Optimal Control of McKean-Vlasov SDEs. *arXiv preprint arXiv:2503.20572*, 2025.

34. Heterogeneity and inequality in population economics 08:30 – 09:45

Chair: **A. Prskawetz, M. Sanchez-Romero** **FH HS 7**

Optimization for age-structured sub-replacement human populations

Gustav Feichtinger (1), Thomas Fent (2), Stefan Wrzaczek (3), Andreas Novak (4),

(1) VADOR, TU Wien, Vienna, Austria; (2) Austrian Academy of Sciences, OeAW, Austria; (3) IIASA, Austria; (4) Universität Wien, Vienna, Austria;

While half a century ago, excessive population growth was a dominating topic in demography, nowadays the shrinking of populations, even depopulation, is in the focus of demographic analysis. Conceiving either fertility or migration as control variables, this contribution presents some age-structured population models optimizing various objectives. Among them are the efficient trade-off between the decline of a population and the fluctuation of its age-structure as well as the maximization of the support ratio. While in the first case, a variant of Pontryagin's maximum principle provides a valuable tool, in the second, ordinary calculus delivers interesting insights. In particular, the support ratio of a stationary-through immigration population shows a two-peaked maximum with respect to the immigration-age. Which one of the two local maxima turns out to be the global one depends on the level and timing of age-specific fertility and survival rates. This delivers a new and objective threshold for low fertility regimes.

Je ne regrette rien? Time-inconsistent health behavior and retirement planning

Holger Strulik (1), Katharina Werner (1),

(1) University of Goettingen, Goettingen, Germany;

In this paper, we examine the health and retirement outcomes of present-biased individuals who engage in time-inconsistent behavior and fail to follow through on plans to save more, invest more in their health, and reduce unhealthy consumption. For that purpose, we set up a health deficit model with hyperbolic discounting in which longevity and the disutility from work depends on health and the retirement decision

depends on health, wealth, and the public pension system. We calibrate the model for Germany and show that inconsistent savings behavior tends to push the retirement age upward, while inconsistent health-related decisions lead to a downward adjustment. Which effect dominates is highly context specific. For example, rich persons and persons with little preference for unhealthy goods are predicted to retire later than planned, poor persons and persons with a strong preference for unhealthy goods retire earlier than planned. However, all individuals die earlier than expected and the loss of life due to inconsistent behavior depends on the strength of present-bias. We explore how the pension system affects the retirement decision and whether it could be designed to nudge individuals to adhere to their retirement plans.

Modelling land-use decisions under price uncertainty for heterogeneous farmers

Michael Freiberger (1), Michael Kuhn (1),

(1) Economic Frontiers, International Institute for Applied Systems Analysis, Laxenburg, Austria;

The literature investigating the spillover impacts from financial markets on commodity and food prices has increased in the recent past. The Covid-19 pandemic further highlighted the global importance of the issue resulting from globally interconnected supply-chains. Level or volatility shocks in food prices have been widely linked to a multitude of adverse social and ecological impacts, such as increased deforestation or land-use change. These changes in agricultural production are simultaneously long-term as well as difficult to reverse through e.g. reforestation or land-use reclaiming efforts. Climate change, furthermore, plays a dual role. First, the productivity of agricultural production in the future can be substantially hampered through climate change with the available land-use options being restricted as a result of decisions made in the present. Second, changes in land-use (such as deforestation) can reduce available carbon sinks and slow down efforts combating climate change. In this paper we propose a framework describing the land-use decision process by farmers. These farmers take expectations on stochastic food prices into account and compete on a limited markets for agricultural land. We distinguish between different types of farmers including (i) small subsistence farmers with no access to financial markets and related borrowing constraints, and (ii) large farming conglomerates, which are able to access external financing options. Following the optimal decision rules of all types of farmers, we use Monte-Carlo simulations to determine the long-term impact of instantaneous price-shocks as well as long-run climate developments. Finally, we evaluate different policy intervention and their success in mitigating the identified negative effects.

35. Industry Dynamics

08:30 – 09:45

Chair: **H. Dawid**

FH HS 3

Navigating uncertainty in the presence of negative word of mouth

Herbert Dawid (1), Dirk Kohlweyer (1), Melina Schleef (1), Christian Stummer (1), Frederik Tolkmitt (1),

(1) Faculty of Business Administration and Economics, Bielefeld University;

We study how different ways in which consumers deal with signals about product characteristics influence the adoption dynamics of a product which is characterized by large (ex-ante) uncertainty about its quality. Based on survey data we consider three different types of consumers representing different approaches for processing word of mouth information about the product and for making purchasing decisions. In particular, the approaches differ with respect to the way consumers deal with contradicting

signals about the product quality. We show that there are substantial differences in the diffusion dynamics between the three approaches with respect to the speed and shape of the diffusion as well as with respect to the long run number of adopters of the product. Furthermore, we demonstrate that the navigation of uncertainty by consumers might give rise to double-S diffusion curves which have been empirically observed in consumer-goods markets.

Strategic interaction between green firms, greenwashers, and brown firms: a dynamic analysis

Fabio Lamantia (1), Davide Radi (2),

(1) Department of Economics and Business, University of Catania, Italy; (2) DiMSEFA, Catholic University of Sacred Heart, Milan, Italy;

The growing demand for green products has led firms to become green but has also given rise to the phenomenon of greenwashing, where firms twist their sustainability efforts. This work proposes a new partial equilibrium framework for a multi-sector economy where firms might engage in observable and unobservable green activities, creating incentives for greenwashing. Moreover, firms can internalize the environmental damage into their objective function (environmentally concerned firms), or they can operate as standard profit maximizers. Either non-point source pollution taxes or taxes on individual emissions are introduced in the model. The presence of transitional risk may lead some industries to significantly reduce or completely cease production plans if the expected damage from environmental degradation is substantial, thus leading to discontinuous production dynamics. The corresponding dynamics of environmental degradation is analyzed through a discrete-time discontinuous map, whose study can be carried out through the methods of global analysis.

Technology choice and industry dynamics

Luca V.A. Colombo (1), Herbert Dawid (2),

(1) Università Cattolica del Sacro Cuore, Milano, Italy; (2) Bielefeld University, Bielefeld, Germany;

Technological change often requires firms to choose which technologies to develop within a set of competing alternatives. The selection of such technologies – among the opportunities identified by fundamental research – is subject to substantial uncertainty on their merits and potential to challenge existing technologies. While it is natural to think that only a limited number of technologies will become prevalent in the future, it is far from obvious how to identify them ex ante. In the early stages, firms are likely to explore different technologies to better assess their potential. At the same time, the development of any technology may benefit from coordinating firms' efforts in terms of scale effects, standardization, reduced uncertainty for investors, etc. Policy making can play a key role in shaping firms' incentives, but it has to face the fundamental trade-off between fostering coordination on a single technology (or a small subset of technologies) and providing incentives for a wide exploration of alternative technologies. In this paper, we develop a model to address these issues along the lines of the recent literature focusing on the interplay between economic growth and innovation. How productive each technology is depends both on the properties of the technology itself and on the fraction of firms investing in that technology. Although for each firm the timing of an innovation is stochastic, firms expect a larger productivity increase upon innovation the larger is the number of firms that are developing the technology. We explore the model through the analytical characterization of the equilibria at each point in time, and industry simulations. We investigate

under which circumstances there emerges co-existence of different technologies in the long-run, and under which conditions lock-in into inefficient technologies may occur.

36. Uncertainty Quantification in Optimization, Optimal Control, and Identification

10:05 – 11:20

Chair: **A. Khan, C. Tammer** **FH Nöbauer 8**

Mean field optimization: stability results and Lagrangian discretization

Kang LIU (1), Laurent Pfeiffer (2),

(1) IMB, CNRS, Université Bourgogne Europe, Dijon, France; (2) L2S, Inria, CentraleSupélec, CNRS, Université Paris-Saclay, Gif-sur-Yvette, France;

We introduce and study a convex optimization problem over a set of probability measures μ with a prescribed marginal m, which we term the *Mean Field Optimization (MFO)* problem. The cost function depends on an aggregate quantity, defined as the expectation of μ with respect to a given contribution function. This framework arises naturally in various settings, including machine learning training, social cooperation models, and the Lagrangian potential mean field games. We establish first-order optimality conditions and prove strong duality for the MFO problem. Furthermore, we investigate its stability properties with respect to perturbations of the prescribed marginal, analyzing both primal and dual formulations. As part of our stability analysis, we develop a method to recover an approximate solution to an MFO problem by leveraging an approximate solution corresponding to a different marginal m, typically an empirical distribution. Finally, we integrate this approach with the stochastic Frank-Wolfe algorithm proposed in [1] to derive a complete solution method.

[1] J.F. Bonnans, K. Liu, N. Oudjane, L. Pfeiffer, and C. Wan. Large-scale nonconvex optimization: randomization, gap estimation, and numerical resolution. In *SIAM Journal on Optimization*, **33**(4), 3083-3113, 2023.

On stochastic aspects of stochastic elliptic inverse problems

Hans-Jörg Starkloff (1),

(1) Institute of Stochastics, Technische Universität Bergakademie Freiberg, Freiberg, Germany;

Stochastic elliptic problems arise mainly by substituting deterministic parameters in elliptic problems by certain random parameters. Then one issue in the consideration of random equations is the measurability of desired solutions. There exist different measurability concepts, whereby the most important ones are the Borel, weak and strong measurability. It is important to use the appropriate measurability concept for each problem. In elliptic problems this is related to the fact that here also non-separable Banach spaces play a certain role and in such spaces the measurability concepts mentioned above do not coincide necessarily. Based on these findings measurability properties of solutions of elliptic problems are investigated. Furthermore it will be shown exemplarily, which stochastic elliptic inverse problems can be treated as abstract elliptic inverse problems and which such stochastic inverse problems require a specific stochastic investigation.

Levitin-Polyak well-posedness of stochastic variational inequalities and applications to a random equilibrium problem

Annamaria Barbagallo (1),

(1) Department of Mathematics and Applications “R. Caccioppoli", University of Naples Federico II, Naples, Italy;

The aim of the paper is to study the Levitin–Polyak well-posedness (shortly, LP well-posedness) of stochastic variational inequalities. A characterization of the LP well-posedness is obtained considering the size of LP approximating solution sets. The equivalence between the LP well-posedness of a stochastic variational inequality with the existence and uniqueness of the solution is proved. In addition, the LP wellposedness in the generalized sense is characterized. Finally, the theoretical results are applied to the random spatial price equilibrium problem in the price formulation.

[1] A. Barbagallo. Levitin-Polyak well-posedness of stochastic variational inequalities and applications to a random spatial price equilibrium problem. *Submitted.*

37. Recent progress in PDE Constrained optimization 10:05 – 11:20

Chair: **P. Rösch, J. Pfefferer** **FH HS 6**

Optimal Control of the Navier-Stokes equations via Pressure Boundary Conditions

Boris Vexler (1),
(1) Technische Universität München, München, Deutschland;

In this talk we discuss an optimal control problem subject to the instationary Navier-Stokes equations, where the control enters via an inhomogeneous Neumann/Do-Nothing boundary condition. Despite the Navier-Stokes equations with these boundary conditions not being well-posed for large times and/or data, we obtain wellposedness of the optimal control problem by choosing a proper tracking type term. In order to discuss the regularity of the optimal control, state and adjoint state, we present new regularity results of solutions to a Stokes problem with mixed inhomogeneous boundary conditions.

[1] Boris Vexler, Jakob Wagner *Optimal Control of the Navier-Stokes equations via Pressure Boundary Conditions, arXiv, 2501.04548, 2025*

Numerical analysis of the Stokes Problem with non-homogeneous Dirichlet boundary conditions

Thomas Apel (1), Katharina Lorenz (1), Johannes Pfefferer (1),

(1) Institute for Mathematics and Computer-Based Simulation, University of the Bundeswehr Munich, Munich, Germany;

This presentation deals with the finite element solution of the Stokes problem

$$\begin{aligned} -\Delta y + \nabla p &= 0 \quad \text{in } \Omega, \\ \nabla \cdot y &= 0 \quad \text{in } \Omega, \\ y &= u \quad \text{on } \Gamma \end{aligned}$$

on a polygonal or polyhedral domain $\Omega \subset \mathbb{R}^d$, $d = 2, 3$. We focus on the case where the Dirichlet boundary data u is not sufficiently regular, specifically when $u \in H^t(\Gamma)^d$ with $t \in [-\frac{1}{2}, \frac{1}{2})$, such that a weak solution cannot be expected. We show how to derive a very weak formulation using the method of transposition. Existence, uniqueness and regularity results are presented. For the finite element discretization, a regularization method is employed, and the boundary datum is treated using an L^2-projection. Error estimates that show the influence of both the maximal interior angle of the domain and the regularity of the datum are obtained. Numerical experiments are provided to validate the theoretical results.

Feedback control of thin liquid films falling down inclined planes

Oscar Holroyd (1), Radu Cimpeanu (1), Susana N. Gomes (1),

(1) Warwick Mathematics Insitute, University of Warwick, Coventry, UK;

We outline methods to control a thin liquid film falling down an inclined plane towards an unstable flat solution by injecting or removing fluid from the base. The two-phase Navier-Stokes equations that govern the dynamics of a falling liquid film pose a challenging control problem: it is an infinite-dimensional, nonlinear system with complex boundary conditions, and we are limited to a finite-dimensional boundary control. By using a hierarchy of successively simplifying assumptions, we show that a linear quadratic regulator (LQR) control can be used to stabilize the otherwise unstable flat (or Nusselt) solution. We demonstrate that applying the LQR controls to the Navier-Stokes problem is successful well outside the parameter regime that the simplified models are designed for, and also in cases where observations of the system are restricted.

38. Optimization in inverse problems

10:05 – 11:20

Chair: **K. Bredies**

FH HS 4

Learning firmly nonexpansive operators

Kristian Bredies (1), Jonathan Chirinos-Rodriguez (2), Emanuele Naldi (3),

(1) Department of Mathematics and Scientific Computing, University of Graz, Graz, Austria; (2) IRIT, Toulouse INP, Toulouse, France; (3) MaLGa, DIMA, University of Genova, Genova, Italy;

In this talk, we propose a data-driven approach for constructing (firmly) nonexpansive operators. We demonstrate its applicability in Plug-and-Play (PnP) methods, where classical algorithms such as Forward-Backward splitting, Chambolle–Pock primal-dual iteration, Douglas–Rachford iteration or alternating directions method of multipliers (ADMM), are modified by replacing one proximal map by a learned firmly nonexpansive operator. We provide sound mathematical background to the problem of learning such an operator via expected and empirical risk minimization. We prove that, as the number of training points increases, the empirical risk minimization problem converges (in the sense of Gamma-convergence) to the expected risk minimization problem. Further, we derive a solution strategy that ensures firmly nonexpansive and piecewise affine operators within the convex envelope of the training set. We show that this operator converges to the best empirical solution as the number of points in the envelope increases in an appropriate way. Finally, the experimental section details practical implementations of the method and presents an application in image denoising, where we consider a novel, interpretable PnP Chambolle–Pock primal-dual iteration.

Implicit monotone operator splitting-based neural network for inverse problems in imaging

Kristian Bredies (1), Mouna Gharbi (1),

(1) Department of Mathematics and Scientific computing, University of Graz, Graz, Austria;

Inverse problems arising in signal and image processing applications, including medical imaging, have traditionally been addressed using either model-based optimization algorithms or deep learning methods. Recent efforts have aimed to unify both through hybrid approaches, notably unrolling and plug-and-play methods. These approaches face various challenges related to memory consumption during training, convergence guarantees, and generalization capacity. In this paper, we propose a novel neural network architecture for solving inverse problems based on monotone operator splitting. We formulate the inverse problem as a monotone inclusion problem and parametrize the involved monotone operators via designed learnable blocks. The zero of their sum corresponds to the solution of the original problem and the fixed point of our provably convergent, learnable splitting scheme. Training is performed using implicit differentiation, enabling efficient gradient computation and reduced memory usage. We validate the proposed method on the task of MRI reconstruction, demonstrating competitive performance compared to both traditional iterative algorithms and learning-based baselines, including unrolled and plug-and-play approaches.

A particle consensus approach for constrained optimization

Jonas Beddrich (1), Enis Chenchene (2), Massimo Fornasier (1), Hui Huang (3), Barbara Wohlmuth (1),

(1) Department of Mathematics, Technical University of Munich, Germany; (2) Faculty of Mathematics, University of Vienna, Austria; (3) Department of Mathematics and Scientific Computing, University of Graz, Austria;

Consensus-Based Optimization (CBO) is a versatile, zeroth-order multi-particle method for tackling nonconvex and nonsmooth global optimization problems in high-dimensional spaces. In this talk, we extend CBO to constrained optimization problems on both compact and unbounded domains with boundaries by incorporating reflective boundary conditions. We establish a global convergence proof in the many-particle regime, including explicit convergence rates. Additionally, we enhance CBO's efficiency by introducing an adaptive region control mechanism and geometry-specific random noise. In particular, by integrating a hierarchical noise structure with a multigrid finite element method, we achieve significant improvements in convergence and complexity. As a key application, we compute global minimizers for a constrained p-Allen–Cahn problem with obstacles. This presentation is based on joint work appearing in

[1] J. Beddrich, E. Chenchene, M. Fornasier, H. Huang, B. Wohlmuth. Constrained consensus-based optimization and numerical heuristics for the few particle regime. *arXiv preprint arXiv:2410.10361*, 2024.

39. Infinite dimensional optimal control and differential games in Economics

10:05 – 11:20

***Chair*: F. Gozzi, F. Masiero, A. Zanco** **FH HS 5**

A notion of BSDE on the Wasserstein space and its applications to control problems and PDEs

Mao Fabrice Djete (1), (1) Ecole Polytechnique, Paris, France;

Many phenomena in science, economics, and engineering involve systems where the dynamics depend not only on individual states but also on their distribution—leading to so-called mean-field models. In this talk, I will present a new class of backward stochastic differential equations (BSDEs) formulated directly on the Wasserstein space of probability measures, providing a natural framework to analyze such systems.

Finite dimensional projections of HJB equations in the Wasserstein space

Andrzej Święch (1), Lukas Wessels (1),

(1) School of Mathematics, Georgia Institute of Technology, Atlanta, GA, USA;

In this talk, we consider the optimal control of particle systems with mean-field interaction and common noise, and their limit as the number of particles tends to infinity. First, we prove the convergence of the value functions u_n corresponding to control problems of n particles to the value function V corresponding to an appropriately defined infinite dimensional control problem. Then, we prove, under certain additional assumptions, $C^{1,1}$ regularity of V in the spatial variable. In the second part of the talk, we discuss conditions under which the value function V projects precisely onto the value functions u_n. Using this projection property, we show that optimal controls of the finite dimensional problem correspond to optimal controls of the infinite dimensional problem and vice versa.

[1] A. Święch and L. Wessels. Finite Dimensional Projections of HJB Equations in the Wasserstein Space. *Preprint*, https://arxiv.org/abs/2404.05185 (2024).

Sensitivity relations and verification theorem for infinite dimensional stochastic control systems

Liangying Chen (1), Qi Lü (2),

(1) CHEN, TU Berlin and FU Berlin, Berlin, Germany; (2) LU, Sichuan University, Chengdu, China;

In this talk, we focus on the necessary and sufficient optimality conditions for the infinite dimensional stochastic control systems, with the control variables entering into both the drift and the diffusion terms. In particular, we investigate the Pontryagin-type maximum principle (PMP) and dynamic programming principle (DPP). Using the transpostion solution method, we establish the sensitivity relations that reveal the relationships between PMP and DPP, and then establish the verification theorem, which serves as a sufficient optimality condition for the control problems under consideration.

[1] L. Chen and Q. Lü. Relationships between the Maximum Principle and Dynamic Programming for infinite dimensional stochastic control systems. In *J. Differential Equations.* **358**: 103–146, 2023.

[2] L. Chen and Q. Lü. Stochastic Verification Theorem for Semilinear Infinite Dimensional Stochastic Control Systems. Preprint 2024.

40. Recent advances in economic dynamics

10:05 – 11:20

Chair: **L. Deng**

FH HS 7

Strategic Consideration and Optimal Timing of Critical Mineral's Depletion, Recycling, and Substitution

Yiwen Chen (1), Nora Paulus (2), Weihua Ruan (3), Benteng Zou (4),

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The global transition to renewable energy technologies has significantly increased the demand for critical minerals, underscoring the need for strategies to mitigate supply chain vulnerabilities. This paper examines the dynamics between exporting and importing countries in the critical minerals market, with a focus on recycling and substitution as alternative solutions. We develop a dynamic game model to analyze the investments of the importing country in recycling and the development of backstop substitutions, while considering the optimal supply strategies of the exporting country. Our study simultaneously investigates the optimal rates of recycling and substitution within a Markovian subgame-perfect Nash equilibrium framework. This paper explores regime-switching conditions across four distinct modes: exclusive reliance on non-renewable resources, recycling readiness, substitution readiness, and the coexistence of recycling and substitution. Key contributions include deriving regime-switching conditions through impulse control, determining optimal supply strategies under different market modes, characterizing explicit mode-switching criteria, and identifying the emergence of multiple Nash equilibria in the presence of recycling. These findings offer valuable insights into reducing dependence on critical minerals, promoting sustainable resource management, and fostering resilient supply chains.

Endogeneous discounting and economic dynamics

Kirill Borissov (1), Stefano Bosi (2), Thai Ha-Huy (2), Van Quy Nguyen (3), Mikhail Pakhnin (2),

(1) Non-government Educational Institution European University at Saint-Petersburg;
(2) CEPS, University of Evry Paris-Saclay;
(3) University of the Balearic Islands, Palma, Spain;

We consider a Ramsey model where, in the spirit of Chambers and Echenique (2018), intertemporal utility is a weighted sum of period utilities (the sum of discount factors equals one). More precisely, the discount factor can depend: positively on current capital or consumption in the spirit of Fisher (1930) (richer people are more patient); negatively on future capital. We address the problem in different contexts and study the evolution of the economy, the differentiability of value function, the existence of a poverty trap due to a low discount factor even when the productivity of a very low level of capital intensity is very high. Some examples are provided to prove that such a phenomena is not rare in economic dynamics when with endogenous discount factor.

[1] Chambers, C. and F. Echenique (2018): On multiple discount rates. *Econometrica* 86, 1325-1346.

On optimal choice of technique in a miniature growth model without discounting

Xinyang Wang (1), Liuchun Deng (2), Minako Fujio (3), M. Ali Khan (4), Metin Uyanik (5),

(1) Department of Economics, Instituto Tecnológico Autónomo de México; (2) Department of Economics, National University of Singapore and Social Sciences Division, Yale-NUS College; (3) Faculty of International Social Sciences, Yokohama National University; (4) Department of Economics, Johns Hopkins University; (5) School of Economics, University of Queensland;

We study the choice of technique in a miniature discrete-time model of optimal growth without discounting, originally formulated by Robinson, Solow, and Srinivasan. In the case of linear felicity, our characterization of optimal policy (i) provides an algebraic proof for the results on the two-sector special case of the model which have been established geometrically, and (ii) sheds light on the choice of technique when the marginal rate of transformation of the "efficient" technique is relatively high, a problem left open by Khan and Mitra (2005). Our results also enable a discussion about how the economy optimally allocates resources in transition when a new technique arrives.

41. Dynamics of the firm

10:05 – 11:20

Chair: **P. Kort, K. Kogan**

FH HS 3

How to best supervise a research (in theory, at least)?

Sara Mesrar (1), Fouad El Ouardighi (1), Suresh P. Sethi (2),

(1) ESSEC Business School, 95021, Cergy Pontoise, France; (2) The University of Texas at Dallas, Richardson, Texas 75080-3021, USA;

We consider a student that seeks to prepare a doctoral research in a given academic institution. The duration of the preparation of the research is finite and the quality of the thesis is a state variable. The completion of the research by the student requires that a minimum requirement is fulfilled. The preparation of the research combines the student's research effort and the academic institution's research competency, that is being built up by the academic institution at cost over time. Once completed, the research provides a reward to the student. Also, the quality level of the completed research may enhance the reputation of the research competency of the academic institution. Two options are considered: either there is no direct scientific involvement of the research supervisor in the student's efforts, or the converse. The first option reflects a non-cooperative mode of play, with open-loop, closed-loop or feedback equilibrium strategies, while the second option is a cooperative mode of play where the supervisor has a direct share, either as a Stackelberg leader or follower, in the student's research efforts.

Worker-optimal stability for many-to-many matching with contracts

Yi-You Yang (1),

(1) Department of Applied Mathematics, Chung Yuan Christian University, Taoyuan City, Taiwan;

We introduce the notion of weak firm-quasi-stability in the setting of many-to-many matching with contracts, where workers' preferences are substitutable and firms' preferences are unilaterally substitutable. We show that a relaxation of substitutability, termed substitutability across workers, is equivalent to

unilateral substitutability. Using this result, we prove that a worker-optimal weakly firm-quasi-stable allocation exists and is stable. Furthermore, with the law of aggregate demand, we show that the existence of a worker-optimal stable allocation implies the rural hospitals theorem.

Learning vs. earning: an optimal control approach

Andrea Seidl (1), Peter M. Kort (2), Richard F. Hartl (3),

(1) Department of Information Systems and Operations Management, WU Vienna, Vienna, Austria;
(2) Department of Econometrics and Operations Research, Tilburg University, Tilburg, The Netherlands;
(3) Department of Business Decisions and Analytics, University of Vienna, Vienna, Austria;

Learning is an investment into the knowledge stock. As such, it not only plays a crucial role in individual career development as knowledge faciliates building up reputation, which (in line with the Matthew effect) can become self-enforcing. In the context of firms, knowledge can enhance both effectiveness and efficiency, particularly when output generation relies on skilled employees. However, learning requires time—time that could otherwise be spent on activities with immediate financial returns. In the present talk, we will present an optimal control model dealing with the trade-off between learning and revenue generating activities. We study the occurrence and implications of history-dependent solutions within this framework. We will discuss implications for teaching and for universities and academia in general. Furthermore, we talk about consequences for change management in firms considering the introduction of a new technology.

42. Uncertainty Quantification in Optimization, Optimal Control, and Identification

11:30 – 12:45

Chair: **A. Khan, C. Tammer**

FH Nöbauer 8

Stochastic Auxiliary Problem Principle Extended to Stochastic Variational Inequalities. Convergence, Regularization, and Applications

Akhtar Khan (1),

(1) School of Mathematics and Statistics, Rochester Institute of Technology, Rochester, New York, 14623, USA.;

This talk pursues two primary objectives. First, we present a stochastic auxiliary problem principle to address stochastic variational inequalities. We establish the almost sure convergence of the proposed iterative scheme, derived using the stochastic auxiliary problem principle, under the assumptions of strong monotonicity and a growth condition on the involved mapping. Our results demonstrate convergence under highly general conditions on the random noise. While the step lengths α_n are assumed to be diminishing, we also provide an alternative result that does not require the step lengths to converge to zero. The second objective is the development of an iteratively regularized stochastic auxiliary problem principle, which allows us to relax the strong monotonicity assumption. The practical relevance and effectiveness of the proposed framework are illustrated through its application to the stochastic inverse problem of estimating coefficients in stochastic partial differential equations. To be precise, we estimate the diffusion coefficient in the stochastic diffusion equation, the flexural rigidity coefficient in the stochastic fourth-order model, and

the Lamé parameters in the stochastic linear elasticity. This is achieved by leveraging both a nonconvex output least-squares functional and a convex energy least-squares functional.

43. Recent progress in PDE Constrained optimization 11:30 – 12:45

Chair: **P. Rösch, J. Pfefferer** **FH HS 6**

Numerical analysis of optimal control problem along curves in three dimensions

Dmitriy Leykekhman (1), Boris Vexler (2),

(1) University of Connecticut, Storrs, USA; (2) Technical University of Munich, Garching b. München, Germany;

In the recent years the problem of finite element approximation and analysis of elliptic problems where the forcing function is supported on a lower dimensional manifold, like a point or a curve in three dimensions Euclidian space have received a certain attention. There are many interesting and insightful results in this direction. However, the known results are not sufficient for obtaining optimal error estimates for optimal control problem with controls acting on curves in three dimensions. To obtain optimal (or nearly optimal) error estimates, we establish new weighted error estimates for the elliptic problems and applied them to the optimal control problems. In some sense, the established results generalized the technique of pointwise finite element error estimates.

Optimal Dirichlet control problems with uncertain data

Max Winkler (1), Hamdullah Yücel (2),

(1) TU Chemnitz, Germany; (2) METU Ankara, Turkey;

Considered are optimal Dirichlet boundary control problems governed by partial differential equations with random inputs, in particular, the diffusion and source term may be uncertain. We investigate existence of solutions, optimality conditions and the regularity of the solution. Furthermore, we propose a numerical scheme using standard finite elements for the spatial discretization and a stochastic Galerkin discretization in the stochastic space to obtain a fully-discrete scheme. We also provide error estimates for that approximations and confirm the validity of these results in numerical experiments. As the resulting linear systems are huge and expensive to solve sophisticated preconditioning techniques are unavoidable. We present a block-diagonal preconditioner and show the robustness with respect to regularization and discretization parameters.

Stability properties of the Allen-Cahn equation and applications to optimal control

Konstantinos Chrysafinos (1),

(1) Department of Mathematics, School of Applied Mathematics and Physical Sciences, National Technical University of Athens, Athens, Greece;

A stability result for the Allen-Cahn equation is presented. In particular, the Lipschitz continuity of the data to solution mapping, in the natural energy norm, is established with Lipschitz constant that depend upon $1/\epsilon$, where $\epsilon << 1$ is the parameter that models the thickness of the transition layer. Such estimates

are critical in the context of optimal control problems and their numerical approximation (see e.g.[1]). In particular, we present results regarding the Lipschitz continuity of the control to state, and adjoint-state mappings that are valid without imposing strict assumptions regarding the size of the difference between the corresponding data. The proposed approach is based on suitable duality and "boot-strap" arguments. Finally, a stability result with respect to pertubations of initial data is discussed based on results of [2].

[1] K. Chrysafinos and D. Plaka. Analysis and approximations of an optimal control problem for the Allen-Cahn equations. *Numer. Math.*, **155**(1-2): 35-82, 2023.

[2] E. Casas and F. Tröltzsch. Stability for Semilinear Parabolic Optimal Control Problems with Respect to Initial Data., *Appl. Math. Optim.*, **86**(16), 2022.

44. Operator and Algebraic Methods for Games

11:30 – 12:45

***Chair*: S. Gaubert, G. Vigeral**

FH HS 4

A vector bundle approach to Nash equilibria

Hirotachi Abo (1), Irem Portakal (2), Luca Sodomaco (2),

(1) Department of Mathematics, University of Idaho, United States of America; (2) Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany;

Using vector bundle techniques, we study the locus of totally mixed Nash equilibria of an n-player game in normal form. When the payoff tensor format is balanced, we define and study the Nash discriminant variety, that is the algebraic variety of n-player games whose Nash equilibria scheme is either non-reduced or has a positive dimensional component. We verify that its one-codimensional part is irreducible and we compute its degree. At a boundary format, we prove that the Nash discriminant variety also contains a two-codimensional component, which is in particular not irreducible. The vector bundle approach reveals interesting similarities and differences among totally mixed Nash equilibria, eigenvectors, and singular tuples of higher-order tensors. In contrast to singular tuples of tensors, a generic n-player game with an unbalanced payoff tensor format does not admit totally mixed Nash equilibria. We define the Nash resultant variety as the proper subvariety of games admitting a positive number of totally mixed Nash equilibria. We prove that the Nash resultant variety is irreducible and determine its codimension and degree. We explicitly describe equations for Nash discriminant and resultant varieties for specific payoff tensor formats. Our results are supported by various explicit computational examples.

A stable-set bound and maximal numbers of Nash equilibria in bimatrix games

Constantin Ickstadt (1), Thorsten Theobald (1), Bernhard von Stengel (2),

(1) FB 12 – Institut für Mathematik, Goethe-Universität, Frankfurt am Main, Germany; (2) Department of Mathematics, London School of Economics, London, United Kingdom;

For a finite N-person game, the set of Nash equilibria in mixed strategies is a semialgebraic set and already for bimatrix games determining the maximal number of the Nash equilibria of non-degenerate $n \times n$ games is an open problem. Quint and Shubik (1997) conjectured that a non-degenerate $n \times n$ game has at most $2^n - 1$ Nash equilibria in mixed strategies. The conjecture is true for $n \leq 4$ but false for $n \geq 6$. We answer it positively for the remaining case $n = 5$, which had been open since 1999. The problem can

be translated to a question about the vertices of a pair of simple n-polytopes with $2n$ facets. Based on algebraic and geometric techniques, such as the index of an equilibrium, we introduce a novel obstruction, which states that equilibrium vertices belong to two equal-sized disjoint stable sets of the graph of the polytope. This bound is verified directly using the known classification of the 159,375 combinatorial types of dual neighborly polytopes in dimension 5 with 10 facets. Non-neighborly polytopes are analyzed with additional techniques where the bound is used for their disjoint facets.

Semidefinite games

Constantin Ickstadt (1), Thorsten Theobald (1), Elias Tsigaridas (2), Antonios Varvitsiotis (3),

(1) Goethe-Universität, FB 12 – Institut für Mathematik, Germany; (2) Inria Paris and Sorbonne Université, France; (3) Singapore University of Technology and Design, Singapore;

Semidefinite games generalize bimatrix games and finite N-person games by replacing the simplex of the mixed strategies for each player by a slice of the positive semidefinite cone. We demonstrate the equivalence of semidefinite two-player zero-sum games to semidefinite programming, generalizing Dantzig's result on the almost equivalence of bimatrix games and linear programming. We also consider network semidefinite games where players are situated at the nodes of a graph and their payoffs depend on the actions taken by their neighbors. These games provide a simplified framework for representing quantum strategic interactions.

[1] C. Ickstadt, T. Theobald, and E. Tsigaridas. Semidefinite games. *International Journal of Game Theory*, 1-31, 2024.

[2] C. Ickstadt, T. Theobald, E. Tsigaridas, and A. Varvitsiotis. Semidefinite network games: multiplayer minimax and complementarity problem. *arXiv:2310.20333, 2024.*

45. Infinite dimensional optimal control and differential games in Economics

11:30 – 12:45

Chair: **F. Gozzi, F. Masiero, A. Zanco** **FH HS 5**

Intra-Industry Reallocation: Emergent Macro Properties through Gradient Flow

Giorgio Fabbri (1), Davide Fiaschi (2), Cristiano Ricci (3),

(1) Univ. Grenoble Alpes, CNRS, INRA, Grenoble INP, GAEL, Grenoble, France;
(2) Universtiy of Pisa, Dipartimento di Economia e Management, Pisa, Italy;
(3) Universtiy of Pisa, Dipartimento di Economia e Management, Pisa, Italy;

This paper develops a dynamic industry model within a general equilibrium framework to examine the effects of intra-industry reallocation on aggregate behaviour. The analysis focuses on scenarios where, in the short run, sectoral factor returns are not necessarily equal due to the presence of temporarily immobile factors. Specifically, the paper adapts the results of Sonnenschein (1982) to an economy composed of short-sighted competitive firms and workers who move across sectors in response to differential factor returns under quadratic movement costs. We demonstrate that the macrodynamics of the system can be represented as a gradient flow. Consequently, the dynamics of firm sectoral distribution—i.e., the sequence of

short-run general equilibria—can be interpreted as a sequence of instantaneous aggregate optimizations. In the continuous-time limit, this process is characterized by a partial differential equation. We establish the existence, global stability, and efficiency of the long-run equilibrium under varying conditions of consumer preferences, technology, and labor mobility. Furthermore, we analyze the properties of the convergence path, which reveals specific dynamic aggregate efficiency, even when it arises from the uncoordinated actions of individual firms. When only quadratic movement costs are present, the economy converges to a unique long-run equilibrium, regardless of the initial distribution of firms. In this equilibrium, sectoral profit rates are equalized, and the distribution of firms across sectors reflects sectoral productivity (and sectoral labor allocation, in cases of labor immobility). Conversely, when fixed movement costs are introduced, an infinite number of equilibria can emerge, with the specific outcome heavily influenced by initial conditions, sectoral profits, and productivity levels. Additionally, the presence of increasing returns driven by intra-sectoral positive externalities amplifies initial productivity differentials, whereas congestion effects or negative pecuniary externalities counteract these disparities.

Optimal synthesis for the linear quadratic control of partial differential equations: recent advances for hyperbolic systems and equations with memory

Francesca Bucci (1),

(1) Università degli Studi di Firenze, Italy;

The question of attaining a full synthesis of the optimal solution in the finite time horizon optimal control problem with quadratic functionals for important classes of linear partial differential equations (PDE) subject to boundary actions has been extensively studied in the last forty-five years or so. Given that the unique (*open-loop*) minimizer does exist, the actual sought-after goal is to find a representation of the optimal control in *closed-loop* form, first of all, and then to prove that a certain operator that occurs in the feedback formula solves uniquely a Riccati equation. Investigating the well-posedness of appropriate Riccati equations is thus a crucial step in the study of the linear quadratic (LQ) problem for evolutionary PDE. Distinct theoretical frameworks that cover significant parabolic/parabolic-like PDE, hyperbolic PDE – under certain restrictions on the allowed functionals to be minimized –, and also coupled PDE systems comprising both parabolic and hyperbolic components (such as e.g. some which describe thermoelastic systems, acoustic- or fluid-structure interactions) have been devised in the literature and applied effectively. This talk aims at providing an overview of recent progress which concerns (i) the optimal boundary control for the Maxwell system (in the absence of smoothing observations), and (ii) the LQ problem for a control system in Hilbert spaces, whose evolution depends also on past values of the control actions. In the latter case, the adopted variational and Riccati-based approach proves successful, with findings which extend and subsume the ones pertaining to memoryless dynamics.
(The talk is based on past and ongoing joint work with Paolo Acquistapace (Univ. di Pisa, Ret.), and in part on joint work with Matthias Eller (Georgetown Univ., Washington DC, USA).)

A class of infinite horizon control problems on separable Hilbert spaces

Gabriele Bolli (1),

(1) Sapienza University, Department of Mathematics G. Castelnuovo, Rome, Italy.;

We study a class of stochastic optimal control problems with infinite horizon and unbounded control operator using the dynamic programming approach. Our analysis is related to the regularizing properties of the Ornstein-Uhlenbeck semigroup in separable Hilbert spaces. By applying a partial smoothing result,

we prove the existence and uniqueness of solutions to the associated Hamilton-Jacobi-Bellman equations. These results can be applied to the analysis of boundary control problems and stochastic delay equations. Such problems are relevant in economics, for instance, in studying spatial growth dynamics, optimal advertising and optimal growth models with vintage capital.

46. Do nonlinear feedback Nash equilibrium strategies really exist? 11:30 – 12:45

Chair: **F. El Ouardighi, G. Feichtinger, S. Wrzaczek** **FH HS 7**

Nonlinear feedback strategies: a matter of existence or rather selection ?

Luca Lambertini (1,2),

(1) Department of Economics and Alma Climate Centre, University of Bologna, Bologna, Italy; (2) International Institute for Applied Systems Analysis (IIASA), Laxenburg, Austria;

This discussion outlines a defense of the solution in nonlinear strategies as properly identifying a continuum of Nash equilibria emerging from first order conditions. Parallel to this, however, there arises an issue relative to equilibrium selection, not dissimilar from that affecting supergames.

Asymmetry in dynamic economic games

Franz Wirl (1),

(1) University of Vienna;

The central point of this note is that the symmetric setup of a dynamic game played by few, in most papers only two, players cannot be considered a generic model for many economic situations, typically, characterized by non-negativity constraints. The reason is that dropping the implausible assumption of perfect symmetry can affect the equilibrium strategies in a substantial way. Using a familiar tragedy of the commons game, one player switches (in finite time) to the boundary solution that leads to discontinuous strategies or eliminates the entire game if one player prefers the boundary strategy from the outset.

Differential games of public investment

Niko Jaakkola (1), Florian Wagener (2),

(1) Università di Bologna, Bologna, Italy; (2) CeNDEF, Universiteit van Amsterdam, Amsterdam, Netherlands;

We define a differential game of dynamic public investment with a discontinuous Markovian strategy space. The best response correspondence for the game is well-behaved: best responses exist and uniquely map almost all profiles of opponents' strategies back to the strategy space. Our chosen strategy space thus makes the differential game well-formed, resolving a long-standing open problem and allowing the analysis of a wider class of differential games and Markov-perfect equilibria. We provide necessary and sufficient conditions for constructing the best response, and demonstrate its use with a canonical model of non-cooperative mitigation of climate change. Our approach provides novel, economically important

results: we obtain the entire set of symmetric Markov-perfect Nash equilibria, and demonstrate that the best equilibria can yield a substantial welfare improvement over the equilibrium which previous literature has focused on. Our methods do not require specific functional forms.

47. Mean Field Control and Mean Field Games and Economic Applications 11:30 – 12:45

Chair: **D. Ghilli, M. Leocata** **FH HS 3**

A mean-field game of optimal investment.

Alessandro Calvia (1), Salvatore Federico (2), Giorgio Ferrari (3), Fausto Gozzi (4),

(1) Department of Mathematics, Politecnico di Milano, Milan, Italy; (2) Department of Mathematics, University of Bologna, Bologna, Italy; (3) Center for Mathematical Economics (IMW), Bielefeld University, Bielefeld, Germany; (4) Department of AI, Data and Decision Sciences, LUISS University, Rome, Italy;

In this talk, I will introduce a continuous-time mean-field game of optimal investment for a population of firms in a given economy. I will consider the case where the representative company can increase its production capacity via costly investments and aims at optimizing its discounted operating profit, which at equilibrium depends on average production capacity of all the firms in the economy. I will discuss the economic foundation of this model and I will formulate both its finite and infinite time horizon versions. Then, I will analyze a specific linear-quadratic case, where it is possible to prove existence and uniqueness of an equilibrium.

A mean-field game network model for urban planning

Fabio Camilli (1), Adriano Festa (2), Luciano Marzufero (3),

(1) Università degli Studi "G.d'Annunzio" Chieti-Pescara, Pescara, Italy; (2) Polytechnic University of Turin, Turin, Italy; (3) Free University of Bozen-Bolzano, Bolzano, Italy;

We study a mathematical model to describe the evolution of a city, which is determined by the interaction of two large populations of agents, workers and firms. The map of the city is represented by a network with the edges representing at the same time residential areas and communication routes. We obtain a two population Mean-Field Game system coupled withan Optimal Transport problem defined on the network. We prove existence and uniqueness of the solution and several numerical simulations are also provided.

Mean field games and master equations within the regime of displacement monotonicity

Alpár R. Mészáros (1),

(1) University of Durham, United Kingdom;

In this talk we will survey on several results on mean field games and the associated master equations under the framework of *displacement monotonicity*. This monotonicity condition on the data stems from the notion of displacement convexity arising in the theory of optimal mass transportation. This is a

sufficient condition –in dichotomy with the *Lasry–Lions monotonicity condition*– which results in the global in time well-posedness of a large class of mean field games and master equations. This framework allows the treatment of different (potentially degenerate) models, including common noise, jump diffusion and a general class of non-separable Hamiltonians.

48. Semi-plenary — 14:00 – 14:50

Chair: **M. Dambrine** — **FH Nöbauer 8**

Inverse problems for fractional PDEs

Barbara Kaltenbacher (1), William Rundell (2),

(1) Department of Mathematics, University of Klagenfurt, Austria; (2) Texas A&M University, College Station, USA;

This talk is based on recent joint work with Bill Rundell, [1,2,3]. In particular, we plan to focus on two prototypical problems in which the order of the fractional time derivative in the governing PDE has a crucial influence on the degree of ill-posedness of the inverse problem under consideration. In the first one, this effect will be exploited for designing a regularization method with the time fractional differentiation order as a regularization parameter. In the second one - quantitative photoacoustic imaging - the order of the fractional derivative is not at our disposal but imposed by the physical model. Modelling with fractional derivatives, in particular in ultrasonics, will be therefore be topic of this talk as well. If time permits, some selected further inverse problems for fractional PDEs will be highlighted.

[1] B. Kaltenbacher and W. Rundell. Regularization of a backwards parabolic equation by fractional operators. *Inverse Problems and Imaging*, **13**(2):401–430, 2019.

[2] B. Kaltenbacher and W. Rundell. Some inverse problems for wave equations with fractional derivative attenuation. *Inverse Problems*, **37**(4):045002, 2021.

[3] B. Kaltenbacher and W. Rundell. Inverse Problems for Fractional Partial Differential Equations. Graduate Texts in Mathematics. AMS, 2023

49. Semi-plenary — 14:00 – 14:50

Chair: **A. Fürnkranz-Prskawetz** — **FH HS 5**

When Lions meets Krugman: A mean-field game framework for the New Economic Geography

Mohamed Bahlali (1), Raouf Boucekkine (1), Quentin Petit (2),

(1) AMSE, Aix-Marseille University and CNRS, Marseille, France; (2) EDF, Paris, France;

The New Economic Geography literature typically consider discrete space models (such like the core and periphery models popularized by Krugman, 1991). Continuous space models are very scarce. When such a space setting is adopted, it usually comes at the cost of assuming adaptive expectations, Mossay (2013) being a notable exception. We extend in this work the 1996 Krugman's trade model, originally

in continuous space, to allow for forward-expectations and stochastic idiosyncratic shocks. In the torus $\mathcal{W} = \mathbb{R}^2/(R\mathbb{Z} \times R\mathbb{Z})$, the density of workers/consumers is given at any location, x. Each location produces a number of varieties of the good. A representative agent residing at any location x of the domain has Dixit-Stiglitz preferences over the continuum of varieties produced in all the locations of the domain. Under monopolistic competition à la Krugman, (Iceberg) trade costs and for given spatial profiles of wages, we define a static equilibrium equation as a fixed-point problem in terms of this spatial wage profile, and we prove existence of such an equilibrium. We next add individual migration behavior where forward-looking workers decide to move across space maximizing intertemporal utility (built on the static optimization problem mentioned above) subject to a law of motion including a control (velocity) and an idiosyncratic shock, with a quadratic mobility cost. We define the corresponding dynamic equilibrium and show that it can be interpreted as a mean-field game with the corresponding HJB and Fokker-Planck equations. We prove existence of a solution to the mean-field game. We next propose a local analysis of the agglomeration vs dispersion effect in the special case of the racetrack economy, where the geography is specifically a one-dimensional circle of radius R $>$ 0. We finally compare the outcomes of Krugman's model with the counterpart with perfect competition (Armington, 1969).

50. Variational Methods and Transportation Problems

15:10 – 16:25

Chair: **L. Mallozzi**

FH Nöbauer 8

Aspects of total variation and connections with image processing

Luigi Greco (1), Serena Guarino Lo Bianco (2), Roberta Schiattarella (1),

(1) Dipartimento di Matematica e applicazioni, Università degli studi di Napoli Federico II, Napoli, Italy;
(2) Dipartimento di scienze FIM, Università degli studi di Modena e Reggio Emilia, Modena, Italy;

We present some results concerning the approximation of the total variation of a function by a family of BMO-type functionals (see [1], [3]). The mode of convergence is rather delicate as for a general BV function the pointwise limit does not always exist. De Giorgi's concept of Γ-convergence save the situation. We also deal with a minimization problem coming from applications in image processing based on the total variation image denoising model of Rudin, Osher, and Fatemi. Based on joint works with L. Greco and R. Schiattarella.

[1] L. Ambrosio, J. Bourgain, H. Brezis, A. Figalli, BMO-type norms related to the perimeter of sets. *Comm. Pure Appl. Math.*, **69**: 1062–1086, 2016.

[2] F.Farroni , N. Fusco, S. Guarino Lo Bianco, R. Sschiattarella, An anisotropic formula for the total variation of SBV functions. *J. Functional Analysis*, **278**(9), 2020.

[3] N. Fusco, G. Moscariello, C. Sbordone, A formula for the total variation of SBV functions. *J. Funct. Anal.* . **270**(1), 419–446, 2016.

[4] S. Guarino Lo Bianco, R. Schiattarella, BMO-type functionals related to the total variation and connections to denoising models, *Journal of Convex Analysis.* **31**(2), 671–688, 2024.

Entropy solutions of nonlocal scalar conservation laws with congestion

Simone Di Marino (1), Lorenzo Portinale (2), Emanuela Radici (3),

(1) DIMA, University of Genova, Genova, Italia; (2) Federigo Enriques, University of Milano, Milano, Italia; (3) DISIM, University of L'Aquila, L'Aquila, Italia;

We consider a class of scalar nonlinear models describing crowd dynamics. The congestion term appears in the transport equation in the form of a compactly supported nonlinear mobility function, thus making standard weak-type compactness arguments and uniqueness of weak solutions fail. We introduce two different approaches to the problem and discuss their connections with the wellposedness of entropy solutions of the target pde in the sense of Kružkov. A deterministic particle approach relying on suitable generalisations of the Follow-the-leader scheme, which can be interpreted as the Lagrangian discretisations of the problem; and a variational approach in the spirit of a minimising movement scheme exploiting the gradient flow structure of the evolution in a suitable metric framework.

Lower Stackelberg equilibria

Francesco Caruso (1), Maria Carmela Ceparano (1,2), Jacqueline Morgan (2),

(1) Department of Economics and Statistics, University of Naples Federico II, Naples, Italy; (2) Centre for Studies in Economics and Finance (CSEF), Naples, Italy;

Bilevel optimization problems (BOPs) may be not stable under perturbation when the lower-level problem has not a unique solution. In this presentation, we investigate the notion of lower Stackelberg equilibrium, an equilibrium concept originally appearing as a limit point in the analysis of the behavior of pessimistic and of optimistic BOPs under perturbation. First, existence, closure and stability results are discussed, together with connections with pessimistic equilibria and optimistic equilibria. Then, moving to the game theory viewpoint, the relation between the set of lower Stackelberg equilibria and the set of subgame perfect Nash equilibria of the associated Stackelberg game is examined.

[1] F. Caruso, M. B. Lignola, and J. Morgan. Regularization and approximation methods in Stackelberg games and bilevel optimization. In *Bilevel optimization: Advances and next challenges.* S. Dempe, A. Zemkoho, Eds. Springer, 2020.

[2] F. Caruso, M. C. Ceparano, and J. Morgan. Asymptotic behavior of subgame perfect Nash equilibria in Stackelberg games. *Annals of Operations Research*, 2024.

[3] F. Caruso, M. C. Ceparano, and J. Morgan. Lower Stackelberg equilibria: from bilevel optimization to Stackelberg games. *Optimization*, 2024.

51. Recent progress in PDE Constrained optimization 15:10 – 16:25

Chair: **P. Rösch, J. Pfefferer** **FH HS 6**

Optimal control of a Fokker-Planck/transport equation with BV-drift using renormalized solutions

Karl Kunisch (1), Christian Lange (2), Hannes Meinlschmidt (2),

(1) RICAM, Linz, Austria; (2) Chair for Dynamics, Control, Numerics and machine learning, FAU, Erlangen, Germany;

This talk focuses on the bilinear optimal control of a Fokker-Planck equation, also considering the degenerate case of a transport equation. We consider a drift field with very low regularity (specifically, BV-regularity in space), which necessitates the use of renormalized solutions, a technique developed by Ronald DiPerna and Pierre-Louis Lions [1]. This foundational theory has been significantly extended by Luigi Ambrosio to encompass cases with BV-regular drift fields [2]. The theory of renormalized solutions is essential not only for establishing the uniqueness of solutions to the controlled PDE but also serves as a method for defining these solutions. Our findings indicate that this approach is particularly natural in the context of optimal control. I am going to outline the PDE-constrained optimization problem we have considered and present our functional-analytic framework concerning the existence of optimal controls and the corresponding optimality criteria.

[1] R. J. DiPerna, P. L. Lions, Ordinary differential equations, transport theory and Sobolev spaces. *Inventiones Mathematicae*, **98**:511-547, 1989.

[2] L. Ambrosio. Transport equation and Cauchy problem for BV vector fields. *Inventiones Mathematicae* vol. 158, Issue 2, p227, 2004.

New results on optimal control problems with total variaton penalty

Nico Haaf (1),

(1) IWR, Heidelberg University, Heidelberg, Germany;

We consider elliptic optimal control problems with total variation penalty of the control. Using the predual of the BV space, we provide a new form of optimality conditions. We dis- cuss algorithmic opportunities to solve the control problems and present numerical results.

Control in the coefficients of the obstacle problem

Nicolai Simon (1), Winnifried Wollner (1),

(1) Universität Hamburg, FB Mathematik, Hamburg, Germany;

Within this talk, we will discuss the optimal control of an obstacle problem by the choice of the coefficients.

$$\min J(q,u) = j(u) + \frac{\alpha}{2}\|q\|^2$$
$$\text{s.t.} \begin{cases} (q\nabla u, \nabla(v-u)) \geq (f, v-u) & \forall v \in K, \\ u \in K, & q \in Q^{\mathrm{ad}}, \end{cases}$$

where $K \subset H_0^1(\Omega)$ and $Q^{\mathrm{ad}} \subset L^2(\Omega; \mathbb{R}^{d\times d}_{\mathrm{sym}})$ are suitable closed and convex subsets To cope with the product in the main part of the operator, tools from H-convergence, see, e.g., [1]. Further, we discuss first order optimality conditions obtained by extending a limiting approach for the control of the obstacle problem by the load f, see, e.g., [2,3]. The results are published in [4]. This work has been supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – Projektnummer 314067056

[1] F. Murat, L. Tartar. H-convergence, in *Topics in the mathematical modelling of composite materials*, Birkhäuser, **31**:21–43, 1995.

[2] K. Kunisch, D. Wachsmuth. Sufficient optimality conditions and semi-smooth Newton methods for optimal control of stationary variational inequalities, *ESAIM Control Optim. Calc. Var.*, **18**(2):520–547, (2012).

[3] C. Meyer, A. Rademacher, W. Wollner. Adaptive optimal control of the obstacle problem, *SIAM J. Sci. Comput.*, **37**(2), A918–A945, (2015).

[4] N. Simon, W. Wollner. Control in the Coefficients of an Obstacle Problem, *Math. Control Rel. Fields* (2025).

52. Operator and Algebraic Methods for Games 15:10 – 16:25

Chair: **S. Gaubert, G. Vigeral** **FH HS 3**

Stochastic games with vanishing stage duration

Sylvain Sorin (1),

(1) IMJ, Sorbonne University, Paris, France;

We study two person zero-sum stochastic games with vanishing stage duration. The analysis relies on associated differential games and deals with several public information structures.

[1] S. Sorin. Limit value of dynamic zero-sum games with vanishing stage duration,*Mathematics of Operations Research*, 43, 51-63, 2018.

Two models of stochastic games with stage duration

Ivan Novikov (1),

(1) CEREMADE, Université Paris Dauphine, Paris, France;

Zero-sum stochastic games with stage duration $h \in (0, 1]$ are used to approximate games played in continuous time. Namely, we consider a family of discrete-time stochastic games $\{G_h\}_{h\in(0,1]}$ with some conditions. There are two models:
Model from [1]: the game G_h proceeds as in continuous time, but players can choose actions only at times $0, h, 2h, \ldots$
Model from [2]: the stage payoff and the leaving probabilities in G_h are proportional to h. We are going to discuss the connection between these models. We consider the case of full observation of the state and the state-blind case. The talk is based on the article [3].

[1] Sorin S., Limit value of dynamic zero-sum games with vanishing stage duration, *Mathematics of Operations Research*, 43:51–63, 2018.

[2] Neyman A., Stochastic games with short-stage duration, *Dynamic Games and Applications*, 3:236–278, 2013.

[3] Novikov I., Zero-Sum State-Blind Stochastic Games with Vanishing Stage Duration. Dynamic Games and Applications (2024). https://doi.org/10.1007/s13235-024-00599-y.

Stochastic homogenization of HJ equations: a differential game approach

Andrea Davini (1), Raimundo Saona (2), Bruno Ziliotto (3),

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Two general approaches can be used to analyze the value of zero-sum dynamic games. The first is the operator approach, which characterizes the value through dynamic programming equations, such as the Bellman equation, the Shapley equation, or the Hamilton-Jacobi equation, depending on whether the model involves one or two players and whether it is in discrete or continuous time. The second is the strategic approach, which focuses on identifying "good" strategies for the players and deriving properties of the value from them. In this work, we adopt the strategic approach to study a classical problem in the Hamilton-Jacobi framework: stochastic homogenization. This problem investigates the long-term behavior of solutions to Hamilton-Jacobi equations when the Hamiltonian depends on a random parameter. We prove that, for a class of non-convex Hamiltonians, these solutions almost surely converge to a deterministic limit as the time and space scales become large. Our proof leverages the differential game interpretation of these solutions and the strategic approach.

53. Learning methods in optimal and predictive control 15:35 – 16:25

Chair: **L. Grüne, K. Worthmann** **FH HS 5**

Data-driven finite-extent LQ control of $2D$-discrete linear quarter-plane causal systems

Paolo Rapisarda (1), Yueqing Zhang (1),

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We consider quarter-plane causal dynamical systems described by partial difference equations over 2 independent variables. We are given: a "*sufficiently informative*" trajectory generated by the system; a "structured" finite-cardinality set ("*frame*"); boundary conditions; a quadratic cost functional of the system variables. The *data-driven finite-extent LQ-problem* consists in computing, *only using information from the data*, a trajectory w^* from the frame and its boundary such that: w^* satisfies the system laws on the frame and its boundary; w^* satisfies the boundary conditions; w^* has minimal cost among the trajectories satisfying (1)-(2). To solve this problem we use the concept of *informativity for identification* (see [1]) and the *data-driven parametrization* of all restrictions of trajectories of a system to a "structured" set (an "*unfolding*"), see [2].

[1] Chu, B., Rapisarda, P., Rocha, P., "Data-driven analysis and control of 2D Fornasini-Marchesini models", *IEEE Contr. Syst. Lett.*, vol. 8, pp. 664-669, 2024.

[2] P. Rapisarda and Y. Zhang, "An Input-Output Fundamental Lemma for Quarter-Plane Causal 2-D Models", *IEEE Contr. Syst. Lett.*, vol. 8, pp. 2475-2480, 2024.

Data-driven stochastic prediction and optimal control with residual disturbance

Guanru Pan (1), Ruchuan Ou (1), Dirk Peter Reinhardt (2), Sebastien Gros (2),
Timm Faulwasser (1),

(1) Institute of Control Systems, Hamburg University of Technology, Hamburg, Germany; (2) Department of Engineering Cybernetics, Norwegian University of Science and Technology, Gjøvik, Norway;

We develop a data-driven framework for the prediction and optimal control of stochastic systems. Leveraging the framework of Polynomial Chaos Expansion (PCE), this work extends Willems' fundamental lemma to stochastic systems, enabling representing stochastic system using recorded input/output/disturbance data. To consolidate all unmeasured or unmodeled disturbances, a residual disturbance—which can be statistically modeled and estimated from input/output data—is introduced. This leads to a data-driven stochastic prediction scheme that ensures effective predictions with PCE-based confidence intervals. A real-world case study using residential building data validates the effectiveness of the proposed prediction method. Building on this predictive capability, the talk further introduces a data-driven stochastic predictive control strategy with output feedback and closed-loop guarantees. Through online designs of initial conditions, the approach ensures recursive feasibility of the proposed output-feedback scheme. Moreover, via a data-driven design of the terminal ingredients, we establish an average asymptotic performance bound for the closed-loop system.

[1] G. Pan, D. Reinhardt, S. Gros, T. Faulwasser. Data-driven stochastic prediction with residual disturbances: application to residential building data. *In preparation.*

[2] G. Pan, R. Ou, T. Faulwasser. On data-driven stochastic output-feedback predictive control. *IEEE Transactions on Automatic Control* (Early Access), 2024.

[3] G. Pan, R. Ou, T. Faulwasser. On a stochastic fundamental lemma and its use for stochastic optimal control. *IEEE Transactions on Automatic Control* , **68**(10):5922-5937, 2023.

Data-driven optimal control through a behavioral lens

Jaap Eising (1), Florian Dörfler (1),

(1) Automatic Control Laboratory (IfA), ETH Zürich, Switzerland;

A plethora of recently developed methods in input-output data-driven control are based on Willems' fundamental lemma, that is, on the possibility of parametrizing all finite length windows of trajectories of a system in terms of measurements. These methods generally take one of two paths. First there are methods based on receding horizon control, which employ the trajectories as is. On the other hand are methods inherently based on state reconstruction or kernel representations. In this talk, we will take the viewpoint from behavioral control theory and consider systems *separate* from their representation. In fact, the same system can be represented in many manners, and specific representations might be preferable in certain situations. While kernel and state space realizations are undeniably useful tools, the contribution of this paper is to define an alternative representation of linear time-invariant dynamical systems using finite-time windows directly. We will show that such representations can be used for the same class of systems representable by state space and kernel representations; can be directly derived from 'rich enough' measurements, without any processing; and unlock controller design using simple tools from linear algebra as opposed to polynomial matrices.

54. Industry session

15:10 – 16:25

***Chair*: P. Filzmoser**

FH HS 7

Introduction

From PhD to Practice – Bridging the Gap Between Research and Industry

Christoph Mühlmann (1), Christopher Rieser (1),

(1) HDAnalytics GmbH, Vienna, Austria;

As two former researchers turned co-founders of a data science company, we want to share our perspective on how scientific methods can successfully transition into real-world applications. Starting from our academic background, through our Ph.D. and postdoc years, we will reflect on the journey of founding our company, HDAnalytics, and discuss how we connect with industry clients.

The talk will focus on questions such as: How do we find and approach clients? What are their needs, and what do we deliver? We will share practical insights from our daily work and touch upon the landscape of players in applied data science, such as research institutions or private consultancies.

A central part of this talk will explore how to align academic interests with industrial challenges and what companies truly benefit from when working with applied scientists. Finally, we will reflect on how this journey is also a continuous learning process for us, turning scientific curiosity into value-driven, practice-oriented solutions.

Nothing is as practical as a good theory ??

Christoph Krischanitz (1),

(1) Profi-Aktuar Christoph Krischanitz e.U., Gerasdorf, Austria, lecturer at TU Wien and former president of the Actuarial Association of Austria;

With 30 years of experience in the financial industry I will discuss the applicability and relevance of mathematical theory in actuarial practice. I will trace the metamorphosis from a young mathematician with high theoretical ambitions to a senior actuary who has to take personal responsibility for results based on a regulatory framework. I will show two examples where the academic focus and the actuarial reality drift apart.

55. Mean Field Control and Mean Field Games and Economic Applications

15:10 – 16:00

Chair: **D. Ghilli, M. Leocata** **FH HS 3**

Finite-Sample Convergence Bounds for Trust Region Policy Optimization in Mean-Field Games

Antonio Ocello (2), Daniil Tiapkin (1), Lorenzo Mancini (1), Mathieu Laurière (2), Eric Moulines (1),

(1) CMAP, École Polytechnique, Institut Polytechnique de Paris, Palaiseau, France; (2) NYU Shanghai, 567 West Yangsi Road, Pudong New District, Shanghai, China 200124;

We introduce Mean-Field Trust Region Policy Optimization (MF-TRPO), a novel algorithm designed to compute approximate Nash equilibria for ergodic Mean-Field Games (MFGs) in finite state-action spaces.

Building on the well-established performance of TRPO in the reinforcement learning (RL) setting, we extend its methodology to the MFG framework, leveraging its stability and robustness in policy optimization. Under standard assumptions in the MFG literature, we provide a rigorous analysis of MF-TRPO, establishing theoretical guarantees on its convergence. Our results cover both the exact formulation of the algorithm and its sample-based counterpart, where we derive high-probability guarantees and finite sample complexity. This work advances MFG optimization by bridging RL techniques with mean-field decision-making, offering a theoretically grounded approach to solving complex multi-agent problems.

Differential games with sparse interactions and infinite-dimensional systems of nonlinear PDEs

Davide Francesco Redaelli (1),

(1) Department of Mathematics, University of Rome Tor Vergata, Rome, Italy;

The theory of Mean Field Games (MFGs) represents the most influential contribution to the rapid development of the study of differential games with many players in the last two decades. It provides an effective paradigm to deal with players interacting in such a way that they are indistinguishable, and individually negligible in the large population limit. In the last years, we have witnessed an increasing interest in the understanding of large population limits of Nash equilibria – or particle systems, in general – under more general assumptions on the interactions. On the one hand, if those are governed by dense graphs, then Graphon MF(G) theory helps to describe effectively such limits; on the other, as of today, less is known about sparse interactions. I will discuss some recent results on this last topic, mainly obtained in collaboration with Marco Cirant (University of Padua), showing how Nash equilibria of games with sparse interactions can be described – in the large population limit – by infinite-dimensional systems of Hamilton–Jacobi–Bellman equations, and giving insights on how such systems can be dealt with.

56. Dynamic Macroeconomics

16:00 – 16:25

***Chair*: R. Neck**

FH HS 3

Ramsey's conjecture in dynamic contests

Kerim Keskin (1), Sıla Pehlivan (2), Çağrı Sağlam (2),

(1) School of Business, Ada University, Baku, Azerbaijan; (2) Department of Economics, Bilkent University, Ankara, Türkiye;

We analyze the extent of Ramsey's conjecture in an infinite-horizon model with two agents in a strategic interaction. Our model can also be interpreted as a dynamic contest model with resource constraints and an endogenous winning prize. We show that the most patient agent holds the entire productive resources in the long run (in line with Ramsey's conjecture) if his productivity is sufficiently high. However, different parameter values may also lead to a steady state where both agents have positive amounts of resources or even where the least patient agent holds the entire resources. Our results shed light on the comparative effects of ability vs. patience on the long-run equilibrium outcome in dynamic contests.

57. Variational Methods and Transportation Problems 16:40 – 17:30

Chair: **L. Mallozzi** **FH Nöbauer 8**

Unilateral parabolic obstacle problems for nonlinear operators

Gabriella Zecca (1),

(1) Università degli Studi di Napoli Federico II, Napoli, Italia; In this talk I will focus on some nonlinear and

non-coercive second-order differential operators, whose model appears in diffusion-convection problems. I will present existence results for solutions of parabolic obstacle problems for such operators under 'optimal' assumptions on the *convection* term and when the obstacle function is irregular with respect to the time variable. These results have been obtained in collaboration with F. Farroni, L. Greco and G. Moscariello.
Frencesca Angrisani (1),

(1) Laboratoire Jacques Louis Lions (LJLL), Sorbonne, Paris, France;

A function $u : [a, b] \to U \subseteq \mathbb{R}^n$ is called a *control* for a system of O.D.E. if there exists a process (x, u) which verifies equations of type $x'(t) = h(t, x(t), u(t))$.
In an ongoing work with Emmanuel Trelat,we generalize this idea of control and we allow controls to be Borel measures.
In particular we consider control systems of this type:

$$\begin{cases} \dfrac{dx}{dt} = (f_1 \circ x \star \mu_1)(t) + (f_2 \circ x \star \mu_2)(t), \quad \text{a.e. } t \in [0, T], \\ x(t) = \phi(t) \quad \text{if} \quad t \leq 0 \end{cases} \tag{1}$$

where $\phi \in C^2(\mathbb{R}, \mathbb{R}^n)$, the μ_i are bounded causal Borel measures and we denote by $(f_i \circ x \star \mu_i)(t)$ the quantity $(f_i \circ x \star \mu_i)(t) = \int_{\mathbb{R}} f_i(x(t-s))d\mu_i(s)$.
Although the idea of considering measure controls is already present in literature, issues related to controllability, optimality conditions, stabilization and other problems have never been studied. These topics are the subject of our study.
The presence of a non-local time delay provide a further aspect of analysis in our work.

58. Set-valued mappings in approximation, control and optimization 17:30 – 18:20

Chair: **R. Baier, E. Farkhi** **FH Nöbauer 8**

On set differences, regularity of set-valued maps and differential inclusions

Robert Baier (1), Elza Farkhi (2),

(1) Chair of Applied Mathematics, University of Bayreuth, Bayreuth, Germany; (2) School of Mathematical Sciences, Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv, Israel;

Following a general approach, various notions of continuity (or other regularity) of set-valued maps may be defined by using various differences of two sets. For instance, any notion of a difference of two

sets, $A \ominus B$, induces Lipschitz continuity of the set-valued map F as $\|F(x) \ominus F(y)\| \leq L\|x - y\|$. The differences of sets may be naturally applied to define also derivatives of set-valued functions. The main difficulty in this approach is that it is not easy to define a good difference of general sets that may induce a linear (Banach) space in which these sets can be embedded. This can be done successfully for convex compacts and partially for compacts (not necessarily convex) in R^n. We briefly survey the embedding of the cone of convex compacts in R^n in the Banach space of directed sets. In the second part we focus on some extensions of the Lipschitz continuity, namely the one-sided Lipschitz (OSL) and the strengthened OSL conditions, and Filippov-type approximation theorems for solutions of differential inclusions.

Linear estimates for trajectories of a state-constrained differential inclusion

Stoyan Apostolov (1), Mikhail Krastanov (2), Nadezhda Ribarska (1),

(1) FMI, Sofia University, Sofia, Bulgaria; (2) IMI, Bulgarian Academy of Sciences, Sofia, Bulgaria;

We consider differential inclusions $\dot{x} \in F(x)$ for Lipschitz set-valued map $F : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ with phase constraint closed set $K \subset \mathbb{R}^n$, i.e. we are interested in solutions of the inclusion, whose values belong to the set K. Let $\mathcal{C}$ denote the solution of the inclusion, i.e. the set of pairs $(x, y) \in L^\infty \times L^1$ such that $\dot{x} = y$ and $y \in F(x)$. Let $\mathcal{K}$ denote the set of pairs $(x, y) \in L^\infty \times L^1$ such that the values of x belong to K. Let $(\bar{x}, \dot{\bar{x}}) \in \mathcal{C} \cap \mathcal{K}$. Our main goal is to show that under some additional hypothesis on K, the sets $\mathcal{C}$ and $\mathcal{K}$ are subtransversal at $(\bar{x}, \dot{\bar{x}})$. Unraveling the definition of subtransversality, this essentialy means, that there exists a constant M such that given an element $(x, \dot{x}) \in \mathcal{C}$ which is close enough to $(\bar{x}, \dot{\bar{x}})$, there exists an element of $(y, \dot{y}) \in \mathcal{C} \cap \mathcal{K}$ for which $\operatorname{dist}((x, \dot{x}), (y, \dot{y})) \leq M \operatorname{dist}((x, \dot{x}), \mathcal{K})$. The distances here are taken with respect to the natural norm in $L^\infty \times L^1$. Similar linear estimates do exist in the literature, although they do not seem to put the problem in the context of subtransversality of sets. The known results give similar linear estimates when the set K has smooth enough boundary, or when it is convex. We aim to relax this assumptions.

59. Recent progress in PDE Constrained optimization 16:40 – 17:55

Chair: P. Rösch, J. Pfefferer **FH HS 6**

Optimality conditions for infinite horizon control problems under a stabilizability assumption

Eduardo Casas (1), Karl Kunisch (2),

(1) DAMCS, Universidad de Cantabria, Santander, Spain; (2) IMSC, University of Graz, Graz, Austria;

First and second order optimality conditions are derived for the following problem:

$$(\mathrm{P}) \quad \min_{(y,u)\in\mathcal{A}_{ad}} J(y,u) := \frac{1}{2}\int_0^\infty \|y(t) - y_d(t)\|^2_{L^2(\Omega)}\,\mathrm{d}t + \frac{\nu}{2}\int_0^\infty \|u(t)\|^2_{L^2(\omega)}\,\mathrm{d}t,$$

$$\begin{cases} \dfrac{\partial y}{\partial t} + Ay + f(x,t,y) = g + u\chi_\omega \ \text{ in } Q = \Omega \times (0,\infty), \\ \partial_{n_A} y = 0 \ \text{ on } \Sigma = \Gamma \times (0,\infty),\ y(0) = y_0 \ \text{ in } \Omega, \end{cases} \tag{1}$$

where $\omega \subset \Omega$, $\mathcal{A}_{ad} = \{(y,u) \in W(0,\infty) \cap L^\infty(Q) \times \mathcal{U}_{ad} : (y,u) \text{ satisfies (1)}\}$ with

$$\mathcal{U}_{ad} = \{u \in L^2(0,\infty; L^2(\omega)) : u(t) \in K_{ad} \text{ for a.a. } t \in (0,\infty)\}.$$

Above K_{ad} denotes a closed, convex, and bounded subset of $L^2(\omega)$ containing 0, $\nu > 0$, $y_0 \in L^\infty(\Omega)$, and $y_d, g \in L^p(0,\infty;L^2(\Omega)) \cap L^2(Q)$, $p > \frac{4}{4-n}$ if $n = 2, 3$ and $p \geq 2$ if $n = 1$. The function f is not monotone with respect to y. Furthermore, the convergence of families of finite horizon optimal control problems to the infinite horizon problem is investigated.

Output-based receding horizon stabilizing control for linear parabolic equations

Behzad Azmi (1), Sérgio Rodrigues (2),

(1) Department of Mathematics and Statistics, University of Constance, Universitätstr. 10, D-78457 Konstanz, Germany; (2) Johann Radon Institute for Computational and Applied Mathematics (RICAM), ÖAW, Altenbergerstr. 69, 4040 Linz, Austria;

A receding horizon control framework is coupled with a Luenberger observer to construct an output-based control input stabilizing parabolic equations. The actuators and sensors are indicator functions of small subdomains, representing localized actuation and localized measurements. It is shown that, for a class of explicitly given sets of actuators and sensors, we can guarantee the stabilizing property of the constructed input. Results of numerical simulations are presented validating the theoretical findings. The computation of a stabilizing receding horizon control input $u_i = u_i(y(t_i))$ is based on a sequence of optimization problems defined in finite time-horizon intervals $(t_i, t_i + T)$, for an appropriate divergent increasing sequence $0 \leq t_i < t_{i+1} \to \infty$, and require the state of the system $y(t_i)$ at the initial time t_i. In applications this state is often not fully available and an estimate $\widehat{y}(t_i)$ will be used instead to compute an optimal input in the form $\widehat{u}_i = \widehat{u}_i(\widehat{y}(t_i))$. In this talk we take this fact into consideration and discuss the asymptotic stability of the system coupling the receding horizon control strategy with a dynamic Luenberger observer providing us with the estimate $\widehat{y}(t_i)$ for the unknown $y(t_i)$.

A least-squares space-time approach to parabolic shape optimization

Michael Hinze (1), Michael Stahl (1),

(1) Mathematical Institute, University of Koblenz, Koblenz, Germany;

In this talk we extend the work in [1] to shape optimization with parabolic PDE constraints, where we consider time-dependent as well as time-independent domains. We use the method of mappings and consider steepest descent, BFGS- and Newton-type directions for the solution of the underlying optimization problems. The appearing PDEs for the state and the adjoint are formulated using a least-squares space-time approach assuming minimal regularity of the PDE solutions. The descent directions for the shape transformations are computed in the Lipschitz topology, where the respective linear and quadratic programmes are numerically solved with an interior point optimizer. We illustrate our approach at hand of a selection of numerical examples which demonstrate the performance of our method.

[1] K. Deckelnick, P. J. Herbert, and M. Hinze. 2024. PDE-Constrained Shape Optimization with First-Order and Newton-Type Methods in the $W^{1,\infty}$ Topology. *Optimization Methods and Software, Dec. 2024, pp. 1–27*, doi:10.1080/10556788.2024.2424525

Solving mixed-integer optimal control problems with TV-regularization

Jonas Marko (1), Gerd Wachsmuth (2),

(1,2) Chair of Optimal Control, BTU Cottbus-Senftenberg;

MIOCPs are often motivated by real-time control applications, which poses challenges in designing solution algorithms. A popular approach for handling MIOCPs involves the combinatorial integral approximation introduced in [1], which often produces rapidly switching solutions. Avoiding this using combinatorial constraints or the aforementioned TV-regularization invalidates the approximation guarantees of the method given in e.g. Corollary 12 of [1].

In the talk, we want to investigate a trust-region framework to solve an abstract class of TV-regularized MIOCPs, motivated by the ideas of [2]. We will take a look at optimality conditions and explain how the trust-region subproblems can be solved efficiently. Furthermore, convergence properties of sequences generated by this algorithm will be inspected. The numerical verification of these results is presented on two examples, constrained by an ODE and a PDE.

The framework is openly available in [3].

[1] S. Sager, M. Jung, C. Kirches. Combinatorial Integral Approximation. *Mathematical Methods of Operations Research* 73, pp. 363-380, 2011.

[2] L. Leyffer, P. Manns. Sequential linear integer programming for integer optimal control with total variation regularization. *ESAIM: Control, Optimisation and Calculus of Variations* 28, p. 66, 2022.

[2] J. Marko, G. Wachsmuth. Mixed-Integer Optimal-Control - Algorithm & Tools. 2025. Available at `github.com/Jonas477/mixed-integer-optimal-control--algorithm-tools`.

60. Operator and Algebraic Methods for Games

16:40 – 17:55

Chair: **S. Gaubert, G. Vigeral**

FH HS 4

Absorbing games with irrational values

Miquel Oliu-Barton (1), Guillaume Vigeral (2),

(1) EconomiX, U. Paris Nanterre, France; (2) CEREMADE, U. Paris-Dauphine, France;

Some classes of two-player zero-sum stochastic games satisfy the orderfield property: if all payoffs and transition probabilities belong to some field, so does the limit value. We show that it is not the case for the simple class of absorbing games: for any integer m we exhibit a $m \times m$ absorbing game with rational data whose limit value is algebraic of degree m. We also show that the orderfield property does hold if transitions are deterministic and one player has at most two actions, but does not for deterministic 3×3 absorbing games. Finally, we prove that any algebraic number of degree 2 is the limit value of a 2×2 absorbing game, which leads to the conjecture that any algebraic number of degree m is the limit value of an $m \times m$ absorbing game.

Marginal values of a stochastic game

Luc Attia (1), Miquel Oliu-Barton (2), Raimundo Saona (3),

(1) Université Paris-Dauphine, Paris, France; (2) Paris Nanterre University, Paris, France; (3) Institute of Science and Technology Austria, Klosterneuburg, Austria;

Zero-sum stochastic games are parameterized by payoffs, transitions, and possibly a discount rate. We study how the main solution concepts, the discounted and undiscounted values, vary when these parameters are perturbed. We focus on the marginal values, introduced by Mills in 1956 in the context of matrix games, that is, the directional derivatives of the value along any fixed perturbation. Our contributions are formulas for: (1) the marginal value of a discounted stochastic game; (2) (under mild assumptions) the limit of the marginal value as the discount rate vanishes; and (3) (under mild assumptions) the marginal values of an undiscounted stochastic game. Note that the last two differ in general.

Eikonal equations and representing functions via their metric slopes

Trí Minh Lê (1), Sebastián Tapia García (, 1Vador, TU Wien, Vienna, Austria

Eikonal equations in metric spaces have strong connections with the local slope operator (or the De Giorgi slope). In this talk, we consider an analogous model based on the global slope operator, expressed as $\lambda u + G[u] = \ell$, where $\lambda \geq 0$. The case $\lambda = 0$ is naturally related to the so-called discrete weak KAM theory. In strong contrast with the classical theory, the global slope operator relies neither on the local properties of the functions nor on the structure of the space. Therefore, new insights are required to analyze the above equation. Under mild assumptions on the metric space X and the given data ℓ, we primarily discuss the well–posedness of this equation, from which we derive a new integration formula based on the global slope of the given function.

61. Learning methods in optimal and predictive control 16:40 – 17:55

Chair: **L. Grüne, K. Worthmann** **FH HS 5**

Optimal value function approximation with neural networks: addressing the curse of dimensionality via separability

Lars Grüne (1), Dante Kalise (2), Luca Saluzzi (3), Mario Sperl (1),

(1) Mathematical Institute, University of Bayreuth, Bayreuth, Germany; (2) Department of Mathematics, Imperial College London, London, United Kingdom; (3) Sapienza, University of Rome, Rome, Italy;

In this talk, we discuss how separable approximations can be used to mitigate the curse of dimensionality in the approximation of high-dimensional value functions for optimal control. These approximations exploit intrinsic decaying sensitivity properties of the system, where the influence of one state variable on another diminishes with an increasing spatial distance. This structure enables the optimal value function to be efficiently represented as a sum of localized contributions. We also explore connections to neural networks, highlighting how separable structures facilitate scalable and computationally efficient representations of high-dimensional value functions. This talk is based on [1].

[1] M. Sperl, L. Saluzzi, D. Kalise and L. Grüne. Separable approximations of optimal value functions and their representation by neural networks. Preprint, arXiv: 2502.08559, 2025.

Neural networks for optimal control: A compositional framework and ensuring convexity assumptions via stage costs

Lars Grüne (1), Konrad Kleinberg (2), Thomas Kruse (2), Mario Sperl (1),

(1) Chair of Applied Mathematics, Mathematical Institute, University of Bayreuth, Bayreuth, Germany;
(2) Applied and Computational Mathematics, Department of Mathematics & Informatics, University of Wuppertal, Wuppertal, Germany;

Deep neural networks (DNNs) have emerged as powerful tools for solving high-dimensional optimal control problems. In particular, their compositional structure has been shown to enable efficient approximation of high-dimensional functions, making them powerful tools for addressing the curse of dimensionality in optimal control problems. In this talk, we build upon the theoretical framework developed by Kang and Gong [1], particularly their results on DNN approximations for compositional functions in optimal control. A key result, [1, Theorem 6.2], establishes that, under suitable assumptions on the compositional structure and its associated features, optimal control problems with strictly convex cost functionals admit a curse-of-dimensionality-free approximation of the optimal control by DNNs. We extend this result in two directions. First, we analyze the strict convexity requirement on the cost functional and demonstrate that reformulating a discrete-time optimal control problem with linear transitions and stage costs as a terminal cost problem ensures the necessary strict convexity. Second, we establish a variant of [1, Theorem 6.2] which provides weak error bounds for optimal control approximations by DNNs even when the cost functional is only convex rather than strictly convex.

[1] W. Kang and Q. Gong. Feedforward Neural Networks and Compositional Functions with Applications to Dynamical Systems. SIAM Journal on Control and Optimization, **60**(2):786-813, 2022.

Receding Horizon Games: Bridging competitive games and optimal control

Sophie Hall (1), Florian Dörfler (1),

(1) Automatic Control Laboratory, ETH Zürich, Switzerland;

Game-theoretic MPC (or Receding Horizon Games) is an emerging control methodology for multi-agent systems that generates control actions by solving a dynamic game with coupling constraints in a receding-horizon fashion. This control paradigm has recently received increasing attention in various application fields, including robotics, autonomous driving, traffic networks, and energy grids, due to its ability to model the competitive nature of self-interested agents with shared resources while incorporating future predictions, dynamic models, and constraints into the decision-making process. A theoretical formalization of Receding Horizon Games and the first stability analysis based on dissipativity and monotone operator theory has been presented in [1]. Building on this foundation, we present recent results connecting the optimality conditions of classical optimal control problems in both primal and dual spaces with those of generalized Nash equilibrium (GNE) problems, the underlying solution concept of Receding Horizon Games. By doing so, we will be able to point out in which sense exactly these competitive games differ from optimal control and what the consequences for a receding-horizon implementation are. In addition, we will discuss how in the context of learning, one can intuitively interpret GNE seeking as a market mechanism in which through bids and asks agents learn or converge to a single clearing price, akin to a Walrasian competitive market equilibrium. GNEs solved and applied in a receding-horizon fashion can be considered a dynamic pricing scheme that ensures economic efficiency as every agent trades off cost versus benefit from resources at every time step.

[1] S. Hall, G. Belgioioso, F. Dörfler and D. Liao-McPherson Stability Certificates for Receding Horizon Games. *Arxiv preprint*, https://arxiv.org/pdf/2404.12165v1, 2024

Control randomisation approach for policy gradient and application to reinforcement learning in optimal switching

Robert Denkert (1), Huyên Pham (2), Xavier Warin (3),

(1) Humboldt-Universität zu Berlin; (2) CMAP, Ecole Polytechnique; (3) EDF R&D and FiME, Laboratoire de Finance des Marchés de l'Energie;

We propose a comprehensive framework for policy gradient methods tailored to continuous time reinforcement learning. This is based on the connection between stochastic control problems and randomised problems, enabling applications across various classes of Markovian continuous time control problems, beyond diffusion models, including e.g. regular, impulse and optimal stopping/switching problems. By utilising change of measure in the control randomisation technique, we derive a new policy gradient representation for these randomised problems, featuring parametrised intensity policies. We further develop actor-critic algorithms specifically designed to address general Markovian stochastic control problems in continuous time. Our framework is demonstrated through its application to optimal switching problems, with two numerical case studies in the energy sector focusing on real options.

62. Industry session — P. Filzmoser

Chair: 16:40 – 17:55 — FH HS 7

Railway Optimization – Experiences and Results of Applied Research Projects

Jan Fabian Ehmke (1),
Gislind Baumgartner (1), Kanchan Joshi (1), Fatih Kocatürk (1),

(1) Department of Business Decisions and Analytics, University of Vienna, Austria;

This presentation provides experiences and results from two industry-related projects, *GreenTrAInPlan* and *VIPES*, that were carried out recently in collaboration with the Austrian Federal Railways (ÖBB). The projects were focusing on locomotive scheduling, a critical operational challenge.

GreenTrAInPlan targeted energy-efficient freight locomotive assignment. We introduce the Green Locomotive Assignment Problem (GLAP), minimizing net energy consumption based on empty runs, idling, and heating energy. The model is enhanced with deadheading and train-to-train connections. Three databases had to be merged to create an energy dataset used in both analytical and machine learning models, which are integrated into the scheduling and simulation frameworks to account for energy use and reliability under disruptions.

VIPES focused on the reliability of passenger and freight trains using a bi-objective locomotive scheduling model that minimizes fleet size and empty-run distance. The model employs a hierarchical solution approach and is paired with an agent-based simulation to assess schedule robustness. Realistic delay propagation and a chance-constrained optimization method help quantify and improve the reliability of circulation plans.

For both projects, results from mathematical optimization were tested on use cases based on ÖBB real data. The results show that these projects help advance the development of greener and more reliable railway operations through integrated optimization and simulation.

Uncertainty-aware prediction of battery knee points

Tobias Heinzle (1),

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Predicting the onset of accelerated battery aging—commonly referred to as the knee point—is valuable for scheduling battery replacements or evaluating second-life applications. However, due to inconsistent definitions and heuristic labeling strategies, accurate localization of this point is not straightforward. In collaboration with AVL (Graz), we propose a probabilistic labeling approach using Gaussian heatmaps to model the inherent uncertainty in knee annotations. Ensemble learning techniques are then employed to predict the resulting distributions, producing interpretable outputs that indicate both the likely knee location and the model's confidence. This approach contributes to robust decision-making under uncertainty, a theme closely aligned with optimal control in industrial systems.

Smart operation of wind turbines under icing conditions

Thomas Burchhart (1), Martin Gruber (1), Simon Kloiber (2), Georg Fritze (3), David Gruber (3), Paul Froidevaux (4), Franziska Gerber (4), Radu I. Boţ (5,6), Michael Drauch (5,6),

(1) VERBUND Green Power GmbH, Vienna, Austria; (2) VERBUND Energy4Business GmbH, Vienna, Austria; (3) AIT Austrian Institute of Technology GmbH, Vienna, Austria; (4) Meteotest AG, Bern, Switzerland; (5) Research Network Data Science @ Uni Vienna, Vienna, Austria; (6) Faculty of Mathematics, University of Vienna, Vienna, Austria;

In this applied research project funded by the Austrian Research Promotion Agency (FFG) we investigated optimised heating strategies for wind turbines operating in adverse weather conditions. The aim was to minimise production losses due to icing while ensuring safe operation of the turbines. The team by the University of Vienna was responsible for the development of machine learning models for optimised rotor blade heating control, based on heterogeneous industrial production data (Supervisory Control and Data Acquisition, weather forecasts, and ice sensors). In a final step, these models were deployed online in shadow operation to demonstrate the viability of the developed system.

63. Dynamic Macroeconomics

16:40 – 18:20

***Chair*: R. Neck**

FH HS 3

Limited factors, why Ramsey led to destruction

Benteng Zou (3), Carmen Camacho (1), Weihua Ruan (2),

(1) Paris School of Economics and CNRS, France; (2) Purdue University Northwest, USA; (3) Luxembourg University, Luxembourg;

We study the classic Ramsey model of 1928 with time discount, taking into account that production factors are always inevitably limited and must remain positive. Using Pontryagin (1962)'s technique, we turn these state constraints into control constraints, and solve completely the model, for all impatience rates under a linear production function. We find the same threshold value for impatience as in all previous literature. However, because limitedness of inputs was not included, the behaviors this threshold delimits

are opposite. This far, the literature had only dealt with mildly impatient agents. We cover here all ranges of impatience. When agents are very patient, the policy maker ensures that the economy reaches the maximum level of capital in finite time. It privileges first investment in new capital in detriment to consumption, which implies that consumption is an increasing function of time. Beyond the classical threshold, capital and consumption always decrease monotonically with time. Furthermore, there exists a second threshold for impatience, beyond which there is no investment at all at any time.

A substitution game in cobweb markets

Simone Baricca (1), Lorenzo Cerboni Baiardi (2),

(1) Department Economics, University of Bologna, Bologna, Italy; (2) Department Mathematics, University of Bologna, Bologna, Italy;

We consider two competitive markets, each one associated to a type of non storable homogeneous goods, and two suppliers, each one producing one type of good. The two products are partial substitutes. We assume that each producer can estimate the consumers' demand through market analysis and can regulate the degree of substitutability of its product with that of the opponent. Since individual choices on substitution levels reflect on producers' profits, a game theoretic framework is defined. We show the existence of a Nash equilibrium characterized by intermediate degrees of substitutability. Moreover, we study markets' stability under different treatments. By means of both analytical and numerical methods, we show the rise of complex dynamic behaviors.

[1] J.-M. Boussard. When risk generates chaos. *Journal of Economic Behavior & Organization*, 29(3):433–446, 1996

[2] C. Chiarella and X. He and H. Hung and P. Zhu. An analysis of the cobweb model with boundedly rational heterogeneous producers. *Journal of Economic Behavior & Organization*, 61(4):750–768, 2006.

[3] R. Dieci and F. Westerhoff. Interacting cobweb markets. *Journal of Economic Behavior & Organization*, 75(3):461–481, 2010.

A Neoclassical Insight into the Chinese and Indian Growth Miracles

Fernando Del Rio (1), Francis-Xavier Lores (1),

(1) ECOBAS, University of Santiago de Compostela, Santiago de Compostela, Spain;

Wedge-Growth Accounting reveals that the economic growth miracles in China and India were primarily driven by improvements in the efficiency wedge and secondarily to enhancements in the investment wedge. In China, the improvement in the investment wedge was consistent from 1982 to 2019, whereas in India, the improvement was concentrated between the late 1990s and 2005. In both countries, the labour wedge had a slightly negative impact on economic growth. In India, its poor performance led to a reduction in per capita working hours, while in China, per capita working hours increased slightly over the entire period, with the exception of 2005–2008. During this period, the labour wedge sharply worsened, reducing per capita working hours despite the positive contributions from the efficiency and investment wedges. Both economies experienced a slowdown in growth in the second decade of the 21st century, but the decline was more pronounced in China, driven to a larger extent by a slowdown in the growth of the efficiency wedge. In this paper we offer an overview of the economic policies that can explain the dynamics described.

Democracy between anarchy and dictatorship: A fragile balance

Reinhard Neck (1), Dieter Grass (2), Stefan Wrzaczek (3), Gustav Feichtinger (4),

(1) Institut für Volkswirschaftslehre, Universität Klagenfurt, Austria; (2) TU Wien, Vienna, Austria; (2) TU Wien, Vienna, Austria; (2) IIASA, Vienna, Austria;

This paper investigates the situation of liberal democracies using a dynamic model originating from biology, the Holling type III functional response, augmented by negative external effects, between two groups in a society, the elite and civil society. Although we start from observations and basic questions as posed by Acemoglu and Robinson in their work on the "narrow corridor", our approach delivers a greater variety of dynamic behavior and additional insights into challenges for liberal democracy. We analyze the dynamics of an uncontrolled system, a system with one-sided control by the elite, and a game model with strategic interactions between the two groups using dynamic systems theory, bifurcation theory, optimal control theory, and dynamic game theory. A low prevalence of negative effects between groups, a low inequality of initial endowments, and a low discount rate of future events are helpful for establishing a liberal democracy. Somewhat unexpectedly, the competitive dynamic game model seems to grant slightly better chances for democracy, thereby questioning the idea of Thomas Hobbes' "Leviathan". Keywords: Political economy; democracy; dictatorship; anarchy; dynamic system; bifurcations; optimal control and dynamic games.

Thursday, July 17^{th}

64. Control of systems in Wasserstein spaces

08:30 – 09:45

Chair: **H. Frankowska, M. Quincampoix**

FH Nöbauer 8

A global stochastic maximum principle for mean-field forward-backward stochastic control systems with quadratic generators

Rainer Buckdahn (1), Juan Li (2), Yanwei Li (2), Yi Wang (2),

(1) Laboratoire de Mathématiques de Bretagne Atlantique, Univ Brest, Brest, France; (2) School of Mathematics and Statistics, Shandong University, Weihai, P. R. China;

Our talk is devoted to the study of Peng's stochastic maximum principle (SMP) for a stochastic control problem composed of a controlled forward stochastic differential equation (SDE) as dynamics and a controlled backward SDE which defines the cost functional. Our studies combine the difficulties which come, on one hand, from the fact that the coefficients of both the SDE and the backward SDE are of mean-field type (i.e., they do not only depend on the control process and the solution processes but also on their law), and on the other hand, from the fact that the coefficient of the BSDE is of quadratic growth in Z. Our SMP is novel, it extends in a by far non trivial way existing results on SMP. The talk is based on a joint work with Rainer Buckdahn (UBO, France), Yanwei Li (SDU, China), Yi Wang (SDU, China).

Optimal control problems with generalized mean-field dynamics and viscosity solution to Master Bellman equation

Rainer Buckdahn (1), Juan Li (2), Zhanxin Li (2),

(1) Laboratoire de Mathématiques de Bretagne Atlantique, Univ Brest, Brest, France; (2) School of Mathematics and Statistics, Shandong University, Weihai, P. R. China;

In this talk we study an optimal control problem of generalized mean-field dynamics with open loop controls, where the coefficients depend not only on the state processes and controls, but also on the joint law of them. The value function V defined in a conventional way, but it does not satisfy the Dynamic Programming Principle (DPP for short). For this reason we introduce subtly a novel value function ϑ, which is closely related to the original value function V, such that, a description of ϑ, as a solution of a partial differential equation (PDE), also characterizes V. We establish the DPP for ϑ. By using an intrinsic notion of viscosity solutions, we show that the value function ϑ is a viscosity solution to a Master Bellman equation on a subset of Wasserstein space of probability measures. The uniqueness of viscosity solution is proved for coefficients which depend on the time and the joint law of the control process and the controlled process. The talk is based on joint work with Juan Li (SDU, China), Zhanxin Li (SDU, China).

Generalized differentiation in Wasserstein space and application to multiagent control problem

Rossana Capuani (1), Antonio Marigonda (2), Marc Quincampoix (3),

(1) Department of Mathematics, University of Arizona, USA; (2) Department of Computer Sciences, University of Verona, Italy; (3) LMBA, University of Brest, France;

Several concepts of generalized differentiation in Wasserstein space have been proposed in order to deal with the intrinsic nonsmoothness arising in the context of optimization problems in Wasserstein spaces. We introduce a concept of *admissible variation* encompassing some of the most popular definitions as special cases. This enables us to define a generalized differentiation in Wasserstein space that we show to coincide in the smooth case to several other differentiation concepts already existing in the literature. The relevance of such new concept of variation lies in the fact that it enables us to define sub/super differentials to study viscosity solutions to Hamilton Jacobi Bellman equations. We will investigate an optimal multiagent control problem where this concept is used and we will obtain a comparison theorem for the corresponding Hamilton Jacobi equation.

65. Recent progress in PDE Constrained optimization 08:30 – 09:45

Chair: **P. Rösch, J. Pfefferer** **FH HS 6**

An SQP method for abstract optimization and applications to control-constrained optimal control problems of PDEs.

Eduardo Casas (1), Mariano Mateos (1),

(1) Dpto. de Matemática Aplicada y Ciencias de la Computación, Universidad de Cantabria, Santander, Spain; (2) Dpto. de Matemáticas, Universidad de Oviedo, Gijón, Spain;

Among the different optimization algorithms than can be used to solve control-constrained optimal control problems governed by a partial differential equation, the sequence of quadratic programs obtained by the application of Newton's method to a generalized equation, Newton-SQP method, is usually seen as a very efficient one because of its quadratic order of convergence. In [1], Fredi Tröltzsch presents a proof of quadratic convergence of the Lagrange-Newton-SQP method for a problem governed by a parabolic PDE. Although the sketch of the proof has since been translated to control problems governed by other kind of PDEs, this effort has to be done on a case by case basis. In our work, we present an SQP method for constrained optimization problems in an abstract framework that is well suited for application to different control-constrained optimal control problems governed by PDEs. We consider a local solution $\bar{u}$ satisfying no-gap second order sufficient conditions and a strict complementarity condition and prove that an initial point in an L^2-neighborhood of $\bar{u}$ will lead to a sequence of admissible controls which is uniquely determined in that L^2-neighborhood and converges quadratically to $\bar{u}$ in L^∞. We will show several examples of control problems that fit in our framework and a numerical experiment to compare the performance of this version of the Newton-SQP algorithm with the Lagrange-Newton-SQP method studied in [1].

References

[1] Fredi Tröltzsch. On the Lagrange–Newton–SQP method for the optimal control of semilinear parabolic equations. *SIAM Journal on Control and Optimization*, 38(1):294–312, 1999.

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A globalized inexact semismooth Newton for strongly convex optimal control problems

Daniel Wachsmuth (1),

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We investigate a globalized inexact semismooth Newton method applied to strongly convex optimization problems in Hilbert spaces. Here, the semismooth Newton method is appplied to the dual problem, which has a continuously differentiable objective. We prove global strong convergence of iterates as well as transition to local superlinear convergence. The latter needs a second-order Taylor expansion involving semismooth derivative concepts. The convergence of the globalized method is demonstrated in numerical examples, for which the local unglobalized method diverges.

Accelerated generalized conditional gradient methods for total variation regularization

Giacomo Cristinelli (1), Daniel Walter (2), José A. Iglesias (1),

(1) University of Twente; (2) Humboldt-Universität zu Berlin;

Total variation regularization is a valuable tool for a wide array of applications ranging from inverse problems to optimal control and machine learning. This is particularly attributed to the observation that TV-penalties often favor piecewise constant minimizers with well-behaved jumpsets. On the downside, their intricate properties significantly complicate every aspect of their analysis, from the derivation of first-order optimality conditions to their discrete approximation and the choice of a suitable solution algorithm. In this

talk, we present two accelerated conditional gradient algorithms for this specific problem based on different, constrained surrogate formulations. The resulting methods rely on iterates given by conic combinations of characteristic functions of sets of finite perimeter. In every iteration, the algorithms alternate between proposing new sets to add to the iterate based on the resolution of prescribed curvature problems and finite-dimensional LASSO-like problems to update the associated weights. We discuss their convergence properties, providing an asymptotic linear rate of convergence if the level-sets of the minimizer are strictly stable. The talk is concluded by numerical experiments for PDE-constrained optimization problems, highlighting the practical relevance of the proposed method

66. Operator and Algebraic Methods for Games

08:30 – 09:45

Chair: **S. Gaubert, G. Vigeral**

FH HS 4

The competitive spectral radius of families of nonexpansive mappings

Marianne Akian (1), Stéphane Gaubert (1), Loïc Marchesini (1),

(1) Inria, CMAP, Ecole Polytechnique, Institut Polytechnique de Paris, CNRS;

We consider a new class of repeated zero-sum games in which the payoff is the escape rate of a switched dynamical system, where at every stage, the transition is given by a nonexpansive operator depending on the actions of both players. This generalizes to the two-player (and non-linear) case the notion of joint spectral radius of a family of matrices. We show that the value of this game does exist, and we characterize it in terms of an infinite dimensional non-linear eigenproblem. This provides a two-player analogue of Mañe's lemma from ergodic control and extends to the two-player case results of Kohlberg and Neyman (1981), Karlsson (2001), and Vigeral and the second author (2012), concerning the asymptotic behavior of nonexpansive mappings. We discuss two special cases of this game: order preserving and positively homogeneous self-maps of a cone equipped with Funk's and Thompson's metrics, and groups of translations. This allows us to prove the existence of the value of *matrix multiplication games*, introduced by Asarin et al. ([1]), for positive and invertible sets of matrices.

[1] E. Asarin, J. Cervelle, A. Degorre, C. Dima, F. Horn and V. Kozyakin. Entropy Games and Matrix Multiplication Games. *Proc. of STACS'2016* Springer series *DMEEF*, vol. 25, 2020.

[2] M. Akian, S. Gaubert, and L. Marchesini. The Competitive Spectral Radius of Families of Nonexpansive Mappings. *arXiv:2410.21097.* 2024.

Extending Markov decision processes: negative probabilities and priority dynamics

Xavier Allamigeon (1), Pascal Capetillo (1), Stéphane Gaubert (1),

(1) INRIA and CMAP, École polytechnique, IP Paris, CNRS, France;

Dynamical systems governed by priority rules arise in the modeling of emergency organizations and road traffic. These systems can be represented by piecewise linear time-delay models, similar to the dynamic programming equations that govern the value function of a finite-horizon Markov (or semi-Markov) decision process:

$$x_i(t) = \min_{a \in \mathcal{A}_i} \left(r_i^a + \sum_{\tau \in \mathcal{T}} \sum_{j \in [n]} (P_\tau^a)_{ij} x_j(t-\tau) \right),\ i \in [n],\ t \geq 0\,. \qquad (D)$$

Here, $\mathcal{T}$ is a finite subset of $\mathbb{N}$, $\mathcal{A}_i$ is a finite set for each $i \in [n]$, and r_i^a as well as $(P_\tau^a)_{ij}$ are real numbers, for all $a \in \mathcal{A}_i$ and $j \in [n]$. The coefficients $(P_\tau^a)_{ij}$ play the roles of transition probabilities but may take *negative* values. Every function $x_i(t)$ represents the number of events of a given type as a function of time. We show that a broad class of piecewise-linear dynamical systems arising in the modelling of priority rules admits a stationary solution—i.e., a solution of the form $x(t) = u + \rho t$ for all $t \geq 0$, for some $u, \rho \in \mathbb{R}^n$. This extends a theorem of Kohlberg [2] on the existence of invariant half-lines of piecewise-linear maps to a class of maps that are not necessarily nonexpansive . The proof relies on methods from topological degree theory, and on the concept of "Blackwell optimality". We illustrate this approach with real-world examples from road traffic management and emergency call center operations, where stationary solutions help determine performance indicators, such as throughput, based on available resources.

[1] X. Allamigeon, P. Capetillo, and S. Gaubert. Stationary regimes of piecewise linear dynamical systems with priorities. To appear in the *Proceedings of HSCC 2025*. arXiv:2411.12437

[2] E. Kohlberg. Invariant half-lines of nonexpansive piecewise-linear transformations. *Mathematics of Operations Research*, 5(3):366–372, 1980.

Computing Stackelberg Equilibrium with Memory in Sequential Games

Aditya Aradhye (1), Branislav Bošanský (2),

(1) Ashoka university, Sonepat, India; (2) Czech Technical University in Prague, Czech republic;

Stackelberg equilibrium is a solution concept that describes optimal strategies to commit: Player 1 (*the leader*) first commits to a strategy that is publicly announced. Then Player 2 (*the follower*) plays a best response to the leader's commitment. We study the problem of computing Stackelberg equilibria in sequential games, when players can play history-dependent strategies.

Using an alternate formulation called strategies with memory, we establish that strategy profiles with polynomial memory size can be described efficiently. We show a polynomial time algorithm which computes the Strong Stackelberg Equilibrium in sequential games defined on directed acyclic graphs, where the strategies depend on the linear memory size. We extend this result to games on directed graphs with cycles.

67. Learning methods in optimal and predictive control

08:30 – 09:45

Chair: **L. Grüne, K. Worthmann**

FH HS 5

Shaping the Koopman dictionary on the Grassmannian

Roland Schurig (1), Pieter van Goor (2), Karl Worthmann (3), Rolf Findeisen (1),

(1) Control and Cyber-Physical Systems Laboratory, Technische Universität Darmstadt, Germany; (2) Robotics and Mechatronics group, University of Twente, The Netherlands; (3) Optimization-based Control Group, Technische Universität Ilmenau, Germany;

The Koopman operator theory is a powerful framework for constructing linear predictors of nonlinear dynamical systems. To represent the predictor in numerical algorithms, extended dynamic mode decomposition computes a data-driven approximation of the Koopman operator in a finite-dimensional space.

However, the accuracy of the predictor depends heavily on the choice of observable functions, which remains a challenge. We propose a systematic framework for identifying observable dictionaries that reduce projection errors and span approximately invariant subspaces. Our approach uses Grassmann manifold optimisation, exploiting geometric properties for computational efficiency. Numerical results show improved invariance properties. This work advances the design of Koopman dictionaries and improves data-driven modelling and control of complex systems.

Guarantees in data-driven nonlinear predictive control: Proportional error bounds and kernel methods

Lea Bold (1), Lars Grüne (2), Friedrich M. Philipp (1), Manuel Schaller (3), Irene Schimperna (4), Karl Worthmann (1),

(1) Optimization-based Control Group, Institute of Mathematics, TU Ilmenau, Germany; (2) Mathematical Institute, University of Bayreuth, Germany; (3) Faculty of Mathematics, Chemnitz University of Technology, Germany; (4) Civil Engineering and Architecture Department, University of Pavia, Italy;

Extended Dynamic Mode Decomposition (EDMD) is a widely used data-driven technique for approximating the behavior of dynamical control systems through observable functions. In this talk, we explore how EDMD, along with associated finite-data error bounds, can be applied within data-driven Model Predictive Control to ensure asymptotic stability [1,2]. A central aspect is the use of proportional error bounds that vanish at the origin and which may be obtained very naturally when leveraging observables in a Reproducing Kernel Hilbert Space [3]. These proportional bounds make it possible that key system-theoretic properties—such as asymptotic stability of autonomous systems or cost controllability of control systems—are preserved in the data-driven model, provided a sufficient number of data samples is available.

[1] L. Bold, L. Grüne, M. Schaller, K. Worthmann. Data-driven MPC with stability guarantees using extended dynamic mode decomposition. *IEEE Transactions on Automatic Control*, **70**(1):534-541, 2025.

[2] L. Bold, M. Schaller, I. Schimperna, K. Worthmann. Kernel EDMD for data-driven nonlinear Koopman MPC with stability guarantees. Accepted for presentation at NOLCOS 2025. To appear in *IFAC PapersOnLine*.

[3] L. Bold, F.M. Philipp, M. Schaller, K. Worthmann. Kernel-based Koopman approximants for control: Flexible sampling, error analysis, and stability. Preprint arXiv:2412.02811, 2024.

Koopman-based control of nonlinear systems with closed-loop guarantees

Robin Strässer (1), Julian Berberich (1), Manuel Schaller (2), Karl Worthmann (3), Frank Allgöwer (1),

(1) Institute for Systems Theory and Automatic Control, University of Stuttgart, Stuttgart, Germany; (2) Faculty of Mathematics, Chemnitz University of Technology, Chemnitz, Germany; (3) Optimization-based Control Group, Institute of Mathematics, Technische Universität Ilmenau, Ilmenau, Germany;

This talk presents recent achievements in data-driven controller design for nonlinear dynamical systems, focusing on ensuring closed-loop stability through Koopman operator theory. The Koopman operator allows us to rewrite a nonlinear control-affine system as an infinite-dimensional bilinear system, which may be approximated by Extended Dynamic Mode Decomposition (EDMD) using a finite dictionary

and finite data. By incorporating robust control strategies, we develop purely data-driven state-feedback controllers that explicitly account for approximation errors. In particular, we discuss the Stability and-certificate-oriented EDMD (SafEDMD) framework [1], [2], which provides rigorous stability guarantees through proportional error bounds, enabling a reliable controller design. This method, applicable to both continuous-time and discrete-time systems, results in a nonlinear controller parameterization that can be solved efficiently using semidefinite programming [3]. Numerical examples are presented to demonstrate the successful application of the SafEDMD controller and its advantages over existing Koopman-based control methods.

[1] R. Strässer, M. Schaller, K. Worthmann, J. Berberich, and F. Allgöwer. Koopman-based feedback design with stability guarantees. *IEEE Transactions on Automatic Control*, vol. 70, no. 1, pp. 355–370, 2025.

[2] R. Strässer, M. Schaller, K. Worthmann, J. Berberich, and F. Allgöwer. SafEDMD: A certified learning architecture tailored to data-driven control of nonlinear dynamical systems. *arXiv:2402.03145*, 2024.

[3] R. Strässer, J. Berberich, M. Schaller, K. Worthmann, and F. Allgöwer. Koopman-based control using sum-of-squares optimization: Improved stability guarantees and data efficiency. *arXiv:2411.03875*, 2024.

68. Mathematical Models for Human Impact on the environment 08:30 – 09:45

Chair: **G. Fabbri, S. Faggian, F. Gozzi** **FH HS 7**

Evolutionary dynamics in fishery exploitation

Gian Italo Bischi (1), Lucia Sbragia (2),

(1) University of Urbino, Italy; (2) Durham University Business School, UK;

We propose an overview of some dynamic models to describe the exploitation of a common pool renewable natural resource (e.g. fisheries) when agents can switch between different harvesting strategies. The share of the exploiters' population adopting the more profitable strategy will increase in the next time period according to a switching mechanism, denoted as profit-driven exponential replicator. This may represent a case of authorities adopting a "libertarian paternalism" approach, as they give the possibility of freely choosing among different harvesting strategies , according to profit maximization considerations. Due to the particular form of cost functions that characterize fisheries exploitation, harvesting is generally less profitable when the fish stock is more depleted. Moreover, overexploitation implies lower market prices, another self-regulation mechanism. The examples proposed include models that describe the creation of marine protected areas, models where cooperative behaviors are suggested instead of individualistic and aggressive ones, models describing the possibility to harvest just one fish species according to profit maximization arguments. As usual in this context, an interdisciplinary approach is proposed, involving competencies from economics, ecology, social sciences,as well as a mathematical approach involving a trade-off between analytical and numerical methods.

[1] G.I. Bischi and F. Lamantia. Harvesting dynamics with protected and unprotected areas. *Journal of Economic Behavior & Organization*, 62: 348-370, 2007.

[2] G.I. Bischi, F. Lamantia, and L. Sbragia. Strategic interaction and imitation dynamics in patch differentiated exploitation of fisheries. *Ecological Complexity*, 6:353-362, 2009.

[3] G.I. Bischi, F. Lamantia, and D. Radi. Multi-species exploitation with evolutionary switching of harvesting strategies. *Natural Resource Modeling*, 26(4): 546-571, 2013.

[4] D. Radi, F. Lamantia, and G.I. Bischi. Benefits and Perils of Integrated Data Systems in Managing Sustainable Fishing Quotas. *Environmental and Resource Economics*, 2025.

Political cycles around the roundabout

Stefano Bosi (1), David Desmarchelier (2),

(1) EPEE, Université Paris-Saclay, France; (2) BETA, Université de Lorraine, France;

We develop a unified framework at the crossroads of economics, political and environmental science, and, to some extent, epidemiology. Populism is equated with climate skepticism and seen as an opinion that spreads through the population. Drawing on compartmental models in epidemiology, the population is divided into two groups that interact with each other: climate skeptics, almost always populists, and environmentalists. The political building block is integrated into a Ramsey model with a pollution externality originated from production. We introduce a Pigouvian tax to finance depollution according to a balanced-budget rule. To take account of populist pressure against environmental policies, we assume also that the tax rate decreases in the share of skeptics in population. Our unified approach reveals an interesting result: populism generates stable limit cycles through a Hopf bifurcation around the steady state, whatever the pollution effect on the consumption demand. Importantly, without populism, it was not the case under a negative distaste effect. Thus, populism exacerbates pollution-induced volatility: populist parties focusing on economic issues should manage excess volatility without rejecting environmental policies out of hand.

Optimal harvesting and the value of information in a resource model with stochastic collapse

Thorsten Upmann (1), Stefan Wrzaczek (2), Michael Kuhn (2),

(1) Fraunhofer Institute for Ceramic Technologies and Systems IKTS, Halle (Saale), Germany. Helmholtz-Institute for Functional Marine Biodiversity at the University of Oldenburg HIFMB, Oldenburg, Germany. Bielefeld University, Faculty of Business Administration and Economics, Bielefeld, Germany.; (2) International Institute for Applied Systems Analysis (IIASA), Laxenburg, Austria. Wittgenstein Centre for Demography and Global Human Capital (IIASA, VID/OeAW, University of Vienna), Austria.;

We apply and extend the concept of the value of information to a classical optimal harvesting problem when the renewable resource is threatened by a sudden collapse. The decision maker faces twofold uncertainty: the time of a collapse is stochastic, and the hazard rate of a shock, which determines the survival time of the resource, is unknown. While the former cannot be learnt, the latter can be if sufficient investments are made. If the hazard is unknown, the intertemporal decision problem becomes substantially more involved, as the decision maker has to decide on information acquisition to get more information on the hazard rate before deciding on optimal harvesting. We show that irrespective of the information acquired, the optimal harvesting path is given by the most rapid approach path (MRAP) followed by a singular solution. To assess the impact of information on the harvesting problem, we apply the concept of the value of information, which gives the expected increase in the maximised objective function if, once the information is acquired, the optimal policy path is adjusted accordingly. Since the problem is not autonomous, the steady state path and the singular solution may shift over time. We show that information acquisition

may substantially increase the optimal objective value, and to what extent this increase depends on the model parameters. Only in the case of perfect information, i.e., when information acquisition unveils the actual hazard rate, an analytic solution can be found; in all other cases, numerical methods have to be applied.

27. Dynamic games in economics

08:30 – 08:55

Chair: **S. Wrzaczek, R. Neck**

FH HS 2

Hospital competition with age-structured patients and congestion effects: a differential game approach

Michael Kuhn (1), Dominika Machowska (2), Andrzej Nowakowski (3),
Agnieszka Wiszniewska-Matyszkiel (2), Stefan Wrzaczek (1),

(1) International Institute for Applied Systems Analysis, Austria; Wittgenstein Center, University of Vienna, Austria.; (2) University of Warsaw, Faculty of Mathematics, Informatics and Mechanics, Institute of Applied Mathematics and Mechanics, Warsaw, Poland; (3) Faculty of Mathematics and Computer Science, University of Łódź, Poland.;

We explore a noncooperative game framework involving two hospitals, where treatment quality suffers under congestion. Recognizing that healthcare demand is significantly influenced by patient age, we incorporate a continuous age distribution into our model. Each hospital aims to determine the optimal treatment (age-structured) intensity that maximizes its objective: for a public hospital, this involves enhancing the cross life-expectancy as measure for the number and quality of treated patients (public hospital); for a private hospital, the goal is to maximize profits based on public payments for treatment. The resulting problem leads to the introduction of differential games with the closed-loop information structure. The paper formulates conditions for verifying whether a given strategy profile constitutes an ε-Nash equilibrium with the dual closed-loop information structure. The verification theorem is then used to develop a numerical algorithm for determining ε-Nash equilibria in a finite number of steps. The numerical simulations illustrate how changes in hospital objectives can lead to shifts in the Nash equilibrium.

28. Supply chain and environmental

08:30 – 08:55

Chair: **S. Wrzaczek, R. Neck**

FH HS 2

The Relative Impact of Underestimating the Cost of Pollution and Omitting the Switching Effect from Pollution Sinks to Pollution Sources

Fouad El Ouardighi (), Dieter Grass (), Sara Mesrar (1),
(1) ESSEC Business School, Paris, France;

This study examines the relative impacts of underestimating pollution costs and omitting nonlinear feedback dynamics responsible for capturing the switching effect of pollution sinks to sources. In this paper, we analyze how inaccuracies in pollution cost estimation (parameter c) and neglecting nonlinear absorption effects or release effect (parameter e) influence environmental and economic outcomes. We find that ignoring nonlinear feedback e has a more substantial negative effect, leading to multiple equilibria, increased instability, and higher risks of reaching environmental tipping points. Conversely, underestimating

the pollution cost c results primarily in higher steady-state pollution levels and greater production effort, with comparatively less severe systemic instability. The analysis highlights that accurately modeling nonlinear environmental dynamics is critical for robust policy design, particularly in the context of managing irreversible ecological risks and optimizing long-term welfare.

Innovative strategies for sustainable and efficient last-mile delivery

Gabriella Colajanni (1), Patrizia Daniele (1), Daniele Sciacca (1),

(1) Department of Mathematics and Computer Science, University of Catania, Catania, Italy;

The exponential growth of e-commerce and the complexities of urban logistics have spurred the development of innovative last-mile delivery solutions. Among these, parcel lockers have emerged as a key strategy to enhance delivery efficiency, reduce costs, and improve customer satisfaction. By offering secure, 24/7 accessible pickup points, lockers eliminate missed deliveries and minimize failed delivery attempts, benefiting both consumers and logistics providers. Their strategic placement in urban environments contributes to reduced carbon emissions and congestion, supporting sustainability goals. In parallel, hybrid delivery models, such as truck-drone systems, and crowdsourced logistics are gaining traction. Truck-drone systems combine the range and capacity of trucks with the speed and flexibility of drones, reducing delivery times and energy consumption. Optimization of these systems requires addressing challenges like drone battery limitations, regulatory constraints, and route synchronization. Meanwhile, the integration of green vehicles and occasional drivers (crowdsourcing) offers scalable solutions for dynamic delivery networks. Mathematical models, including variational inequality frameworks, have been employed to optimize routing, costs, and environmental impact. We explore how these complementary innovations—parcel lockers, truck-drone collaboration, and crowdsourced green delivery—can be jointly leveraged to create more efficient, sustainable, and consumer-centered urban logistics ecosystems.

[1] Archetti, C., Savelsbergh, M., Speranza, M.G. The Vehicle Routing Problem with Occasional Drivers. In *European Journal of Operational Research*, **254** (2016), 472-480.

[2] Boccia, M., Mancuso, A., Masone, A., Sterle C. Exact and heuristic approaches for the32 Truck–Drone Team Logistics Problem. In *Transportation Research Part C: Emerging Technologies*, **165** (2024), 104691.

Inventory holding and pollution accumulation in a vertically and horizontally competitive supply chain

Alain Bensoussan (1), Fouad El Ouardighi (2), Federico Pasin (3), Moein Qaisari Hasan Abadi (2),

(1) NJSM, University of Texas at Dallas, Dallas, USA; (2) DAO, ESSEC Business School, Cergy-Pontoise, France; (3) OLM, HEC-Montreal, Montréal (Québec), Canada;

This study analyzes the interplay between inventory holding, price competition, and environmental externalities in a supply chain where a manufacturer supplies products to two competing retailers. We formulate a dynamic game model incorporating inventory and pollution accumulation as state variables. By considering four inventory-holding structures—bilateral, downward, upward, and no-inventory (just-in-time)—we derive non-cooperative equilibrium strategies for the players. Our results highlight how inventory decentralization shifts environmental costs, pricing policies, and profits. We provide managerial insights into optimal pollution abatement strategies and supply chain coordination mechanisms. This

work offers novel perspectives on balancing economic performance and environmental sustainability in competitive supply chains.

69. Control of systems in Wasserstein spaces

10:05 – 11:20

Chair: **H. Frankowska, M. Quincampoix**

FH Nöbauer 8

Overview of Constrained Mean Field Games and New Perspectives

Rossana Capuani (1),

(1) Department of Mathematics, The University of Arizona, USA;

In this talk, I will present an overview of mean field games with state constraints, a topic of growing interest due to its relevance in applications such as pedestrian dynamics and macroeconomic models. State constraints are naturally encountered in these settings, representing boundaries within which agents operate. However, incorporating constraints significantly complicates the analysis, as traditional equilibrium concepts and solutions based on classical PDE systems are no longer applicable. In addition to providing a comprehensive overview of the field, I will discuss new perspectives and emerging directions, including potential extensions.

Optimal transport map approximation via linear-control systems

Alessandro Scagliotti (1,2), Sara Farinelli (3),

(1) CIT School, Technical University of Munich, Munich, Germany; (2) Munich Center for Machine Learning (MCML), Munich, Germany; (3) DIMA, University of Genoa, Genoa, Italy;

In this presentation, we tackle the problem of reconstructing the optimal transport map T between two absolutely continuous measures $\mu, \nu \in \mathcal{P}(\mathbb{R}^n)$, and for this approximation we employ flows generated by linear-control systems in $\mathbb{R}^n$ of the form $\dot{x} = \sum_{j=1}^{k} F_i(x)u_i$.
After recalling the connection between deep Residual Neural Networks (ResNets) and control systems, we first show that, under suitable assumptions on the controlled vector fields, the flows generated by the system can approximate in the C_c^0-topology any diffeomorphism isotopic to the identity [1]. When the measures μ, ν are sufficiently regular, it turns out that the optimal transport map T is a diffeomorphism isotopic to the identity, and we apply the machinery developed before for its approximation. Pursuing a data-drive approach, we suppose that discrete approximations μ_N, ν_N of the measures μ, ν are available, and we use a discrete optimal transport plan to set up an optimal control problem. With a Γ-convergence argument, we prove that its solutions corresponds to flows that provide approximations of the optimal transport map T (see [2]). Finally, we propose an iterative scheme for the numerical resolution, resulting in an algorithm for the practical computation of approximations of the optimal transport map.

[1] A. Agrachev and A. Sarychev. Control on the manifolds of mappings with a view to the Deep Learning. *Journal of Dynamical and Control Systems*, **28**:989-1008, 2022.

[2] A. Scagliotti and S. Farinelli. Normalizing flows as approximations of optimal transport maps via linear-control neural ODEs. *Nonlinear Analysis*, to appear, 2025.

Stochastic Euler schemes and dissipative evolutions in the space of probability measures

Giulia Cavagnari (1), Giuseppe Savaré (2), Giacomo Enrico Sodini (3),

(1) Dipartimento di Matematica, Politecnico di Milano, Milan, Italy; (2) Department of Decision Sciences and BIDSA, Bocconi University, Milan, Italy; (3) Institut für Mathematik - Fakultät für Mathematik, Universität Wien, Vienna, Austria;

We study dissipative evolutions of probability measures driven by Multivalued Probability Vector Fields $\mathbf{F} \subset \mathcal{P}_2(\mathrm{T}\mathbb{R}^d)$. Under suitable dissipativity and boundedness conditions on $\mathbf{F}$, we prove that an appropriate measure on the interpolating paths of the Explicit Euler scheme converges to the unique probabilistic representation of the Implicit Euler scheme governed by a maximal dissipative extension of the barycenter of $\mathbf{F}$. As a possible application, this theory gives a general and unifying measure-theoretic approach to study the convergence of stochastic schemes and interacting particle systems in continuous time.

70. Feedback control and stabilization of nonlinear PDEs 10:05 – 11:20

Chair: **S. S. Rodrigues** **FH HS 6**

Semiglobal exponential stabilization of nonisothermal Cahn-Hilliard systems via finite-dimensional feedback control

Behzad Azmi (1), Marvin Fritz (2), Sérgio S. Rodrigues (2),

(1) Department of Mathematics and Statistics, University of Konstanz, Konstanz, Germany; (2) Johann Radon Institute for Computational and Applied Mathematics, ÖAW, Linz, Austria;

This talk addresses the semiglobal exponential stabilization of the nonisothermal Cahn-Hilliard equations toward reference time-dependent trajectories. The stabilizing control input is constructed via explicit feedback operators based on suitable oblique projections acting through a finite set of actuators. These actuators are modeled by indicator functions whose supports are appropriately distributed across the domain. Notably, the total volume occupied by the actuators can be made arbitrarily small and predetermined, with their locations explicitly identified for convex polygonal or polyhedral domains. The results are also valid for the isothermal Cahn-Hilliard system. Numerical experiments illustrate the stabilizing performance of the proposed feedback operators.

Feedback stabilization of a surface tension system modeling the motion of a two-dimensional soap bubble

Sébastien Court (1),

(1) Department of Mathematics, University of Innsbruck, Austria; (2) Digital Science Center, University of Innsbruck, Austria;

Our aim is to design a feedback operator for stabilizing in infinite time horizon a system modeling the interactions between a viscous incompressible fluid and the deformation of a soap bubble. The latter is represented by an interface separating a 2-dimensional bounded domain into two connected parts filled with viscous incompressible fluids. The interface is a smooth perturbation of the 1-sphere, and the surrounding fluids satisfy the incompressible Stokes equations in time-dependent domains. The mean curvature of the

surface defines a surface tension force which induces a jump of the normal trace of the Cauchy stress tensor. The response of the fluids is a velocity trace on the interface, governing the time evolution of the latter, via the equality of velocities. The data are assumed to be sufficiently small, in particular the initial perturbation, that is the initial shape of the soap bubble is close enough to a circle. Our approach relies on the approximate controllability of the linearized system. The latter is rewritten as an evolution equation dealing with the displacement of the soap bubble interface, involving in particular a Neumann-to-Dirichlet type operator. The control function is a surface tension type force on the interface. We design it as the sum of two feedback operators: The first one is explicit, and the resulting operator defines an analytic semigroup of contraction, which implies that the unstable modes are now of finite number. The second operator is finite-dimensional, and is obtained as the solution of a Riccati equation. Both feedback operators are functions of the tangential derivatives of the displacement field. Their sum enables us to define a control operator that stabilizes locally the soap bubble to a circle with an arbitrary exponential decay rate, up to translations, and up to non-contact with the outer boundary.

[1] Court, S. Feedback stabilization of a two-fluid surface tension system modeling the motion of a soap bubble at low Reynolds number: The two-dimensional case.
J. Math. Fluid Mech. 26, 7 (2024). https://doi.org/10.1007/s00021-023-00841-4.

[2] Court, S. (2023). Approximate controllability of a 2D linear system related to the motion of two fluids with surface tension. *Proceedings in Applied Mathematics and Mechanics*, 23, e202300083. https://doi.org/10.1002/pamm.202300083

Feedback stabilization for the McKean–Vlasov equation

Dante Kalise (1), Lucas Moschen (1), Greg Pavliotis (1),

(1) Department of Mathematics, Imperial College London, London, UK;

The McKean–Vlasov equation is a nonlinear PDE modeling large-scale interacting particle systems [1]. It arises in models of synchronization, phase transitions, and collective behavior across various fields. Depending on the interaction and confining potentials, the equation can exhibit multiple equilibria, slow convergence rates to steady states, and phase transitions. We propose a feedback control framework, inspired by [2], that modifies the confining potential using time-dependent controls derived from a linear-quadratic optimal control problem formulated for the linearization around a steady-state solution. This approach leads to an optimal feedback law obtained via an operator Riccati equation, motivating ongoing analysis of stabilization properties within this framework. We illustrate our approach through preliminary numerical experiments using spectral methods, demonstrating both acceleration toward equilibria and stabilization of unstable states in examples such as the Kuramoto model and the O(2) model with a magnetic field.

[1] A. S. Sznitman. Topics in propagation of chaos. In *Hennequin, P.-L. (ed.) École d'Été de Probabilités de Saint-Flour XIX - 1989. Lecture Notes in Mathematics*, **vol. 1464**, Springer, Berlin, Heidelberg, 1991.

[2] T. Breiten, K. Kunisch, and L. Pfeiffer. Control strategies for the Fokker–Planck equation. *ESAIM: Control, Optimisation and Calculus of Variations*, **24**(2):741-763, 2017.

71. Topology optimization: theory, numerics and applications 10:05 – 11:20

Chair: K. Sturm, P. Gangl FH HS 4

Topological derivative for Kirchhoff-Love shells

Samuel Amstutz (1), Michael Gfrerer (2),

(1) LMA, Avignon University, France; (2) TU Graz, Austria;

The concept of topological derivative measures the sensitivity of a shape functional with respect to a small topology perturbation. In this vein I will present a recent work dedicated to the linear Kirchhoff-Love shell model, considering an elastic inclusion. It extends former works on Kirchhoff-Love plates, but significant additional difficulties arise due to curvature, in a more or less direct way. Notably, the coupling between the tangential and bending deformations mixes first and second order derivatives of displacements at every step of the analysis. A practical consequence is that the computation of the polarization tensor is highly involved, and so far we are only able to obtain a closed formula at an umbilical point, i.e. when the two principal curvatures are equal. I will present a few elements of derivation, the main theoretical results and some numerical validations.

Topology optimization for additive manufacturing

Luise Blank (1), Maximilian Urmann (1),

(1) Faculty of Mathematics, University of Regensburg, Regensburg, Germany;

Additive manufacturing of rigid structures leads to a topology optimization problem where on one hand the mean compliance of the final structure under external forces shall be minimized and on the other hand large overhangs during the production shall be avoided to guarantee constructability. In the talk we introduce the arising control problem using a diffuse interface approach and present analytical results concerning well-posedness and differentiability of the reduced cost functional. The problem is numerically solved using a projected gradient type method in function space. Second order information can be included by changing the underlying inner product in every iteration. We also mention some details on the implementation for additional speed up. Convergence in function space indicates that the iteration numbers are bounded independently of the discretization level and of the number of construction layers. Numerical evidence of this and the efficiency of the method are illustrated. The choice of weights for the penalization of overhangs during the building phase is discussed. Furthermore, we present results in 2D and 3D as well as with three materials and for various manufacturing problem settings.

Robust topology optimization of electric machines using topological derivatives

Peter Gangl (1), Theodor Komann (2), Nepomuk Krenn (1), Stefan Ulbrich (2),

(1) RICAM, ÖAW, Linz, Austria; (2) TU Darmstadt, Darmstadt, Germany;

Designing high-performance electric machines that maintain their efficiency and reliability under uncertain material and operating conditions is crucial for industrial applications. We present a novel framework for robust topology optimization with partial differential equation constraints to address this challenge.

The robust optimization problem is formulated as a min-max optimization problem, where the inner maximization detects the worst case with respect to predefined uncertainties, while the outer minimization aims to find an optimal topology that is robust to these uncertainties using the topological derivative. The shape of the domain is represented by a level set function, which allows for arbitrary perturbations of the domain. The robust optimization problem is solved using a modification of Danskin's Theorem to compute the topological derivative of the worst case function. This allows the min-max problem to be solved efficiently and ensures that we find a design that performs well even in the presence of uncertainty. Finally, numerical results for a two-material optimization of a permanent magnet synchronous machine demonstrate both the effectiveness of the method and the improved performance of robust designs under uncertain conditions.

72. Learning methods in optimal and predictive control 10:05 – 11:20

Chair: **L. Grüne, K. Worthmann** **FH HS 5**

Koopman-based Control for Stochastic Systems: Application to Enhanced Sampling

Lei Guo (1,2), Jan Heiland (3), Feliks Nüske (1),

(1) Max-Planck-Institute for Dynamics of Complex Technical Systems, Magdeburg, Germany; (2) Otto von Guericke Universität Magdeburg, Germany; (3) Technische Universität Ilmenau, Germany;

Stochastic dynamics with metastability are a recurring theme in many scientific disciplines, for instance, in simulations of macro-molecules, in climate systems, and in applications of uncertainty quantification. Metastability describes the existence of long-lived macro-states in a dynamical system's state space, such that transitions between these macro-states are rare events. It is thus also closely related to control systems. There is a wide range of biased sampling algorithms, which seek to overcome the rare event nature of the dynamics using a time-dependent input. In this study, we join the ideas of Koopman-based modeling and biased sampling [3]. The key ingredient is the generator extended dynamic mode decomposition algorithm (gEDMD)[1], a variant of EDMD to approximate the Koopman generator. [2] shows that for control-affine stochastic differential equations (SDEs), application of gEDMD reduces the Kolmogorov backward equation into an ODE that is bi-linear in expectation and input. This simplified structure can be utilized for designing controllers which are geared towards accelerated sampling of rare events. In this talk, I will report on recent progress concerning the data-driven analysis of metastable systems using Koopman generators. First, I will introduce Koopman operators for (controlled) stochastic systems, the gEDMD method, and its application to optimal control problem. Second, I will present the numerical results showing that the gEDMD method for control-affine SDEs can be used to a) accurately predict the expectation of observable functions of interest for fixed control input; b) solve optimal control problems (OCPs) with integrated running cost and terminal cost; c) design OCPs which enforce accelerated transitions between metastable states.

[1] Klus, S., Nüske, F., Peitz, S., Niemann, J.H., Clementi, C., and Schütte, C. Data-driven approximation of the Koopman generator: Model reduction, system identification, and control. *Physica D: Nonlinear Phe- nomena*, **406**, 2020.

[2] Peitz, S., Otto, S.E., and Rowley, C.W. Data- Driven Model Predictive Control using Interpolated Koopman Generators. *SIAM Journal on Applied Dy- namical Systems*, **19**(3), 2162–2193, 2020.

[3] Guo, L., Heiland, J., and Nüske, F. Koopman-based Control for Stochastic Systems: Application to Enhanced Sampling. *arXiv preprint arXiv:2410.09452*, 2024

Variational approximation of reversible small noise diffusions

Carsten Hartmann (1), Lara Neureither (1), Markus Strehlau (1),

(1) Institute of Mathematics, BTU Cottbus–Senftenberg, Cottbus, Germany;

We study the variational approximation of small noise diffusions of the form

$$dX_t = -\nabla V(X_t)\,dt + \sqrt{2\epsilon}\,dW_t\,, \quad X_0 = x \tag{1}$$

on an unbounded time interval $[0,\infty)$ for a smooth potential V that has several local minima. In doing so, we follow the approach in [1] where a local Gaussian approximation of the path measure on a bounded time interval $[0,T]$ in Kullback-Leibler divergence has been derived. The extension to an infinite time horizon is based on the following two ingredients: firstly, a Freidlin-Wentzell type approximation of the transition rates between metastable sets, secondly, a variational approximation of the Bayes posterior akin to [2], given observations of (1) and using the local Gaussian approximation as prior measure. We explain the central steps necessary to extend the finite time result to infinite time and show that the formulation leads to a linear quadratic tracking problem, for which we discuss various possible approximations that are meaningful in the small noise regime. Theoretical results will be illustrated with suitable numerical examples.

[1] D. Sanz-Alonso and A.M. Stuart. Gaussian approximations of small noise diffusions in Kullback-Leibler divergence. *Commun. Math. Sci.*, **15**(7):2087–2097, 2017.

[2] M. Opper and C. Archambeau. The variational Gaussian approximation revisited. *Neural Comput.*, **21**(3):786-792, 2009.

On Heat-Resilient Health Care:
Towards a Differential Equation based Mathematical Prediction Model

Aditi Jain (1), Michael Heinrich Baumann (1), Lars Grüne (1),

(1) University of Bayreuth, Germany;

Climate change is causing a global rise in temperatures leading to an increase in frequency, duration, and intensity of extreme heat events. In the era of climate change, heat event effects on human health are a serious concern as they lead to increased mortality rates, particularly among vulnerable populations. Heat modeling in epidemiology focuses on understanding how heat events affect human health and disease patterns. We present a explorative model for heat-related affected people using ordinary differential equations (ODEs), incorporating temperature variations in a heat accumulating stress function. This model can in a further step be used for prediction, too. The model defines the dynamics of affected people due to heat events, utilizing the heat stress function that becomes active when ambient temperatures exceed a critical threshold. This critical threshold temperature is based on the relationship between heat events and its impact on the health of a given population in a particular region. The system is solved numerically, e.g., via Runge-Kutta schemes, enabling time-dependent predictions of the share of affected people. Model parameters, such as heat stress accumulation rate, are estimated through a loss resp. error function that measures the distance between model predictions and real-world data. This loss function is optimized using numerical techniques such as Interior-Point or Sequential Quadratic Programming (SQP) methods. The results demonstrate the model's potential for forecasting extreme heat impact on human health. Future research may focus on machine learning-assisted parameter estimation. Another important

point is to include control functions to (optimally) influence the impact of heat events to human health. By combining data-driven ODE modeling with real-world policy frameworks, this approach can provide useful suggestions to reduce heat-related mortality against during (extreme) heat events. Further, multi-scale modeling, i.e. to model in different (connected) clusters, shall be implemented to enhance the model's accuracy and usability for policymakers.

73. Mathematical Models for Human Impact on the environment 10:05 – 11:20

Chair: **G. Fabbri, S. Faggian, F. Gozzi** **FH HS 7**

How to avoid the tragedy of the commons in the imperfect world

Agnieszka Wiszniewska-Matyszkiel (1), Rajani Singh (2),

(1) Institute of Applied Mathematics and Mechanics, University of Warsaw, Poland; (2) Department of Digitalization, Copenhagen Business School, Denmark;

How to counteract "*the tragedy of the commons*" despite limitations of real world economies? Consider a marine fishery divided into exclusive economic zones of countries. Each of the countries fishes to maximize their profit, not taking into account that its fishing effort generates "*the tragedy of the commons*". The Nash equilibrium results in overexploitation, leading to depletion of the fishery, while it is optimal to preserve it. We present a differential game modelling this process and various ways to solve "*the tragedy*". To answer the opening question, we derive regulatory tax-subsidy systems and self-enforcing environmental agreements for a common fishery divided into exclusive economics zones. The first limitation is that the feasible class of tax-subsidy systems may have a complicated form, e.g. there are transition periods for smooth reduction of fishing. Alternative limitation is that there is no institution that can impose taxes or subsidize, so sustainability can be achieved only by self-enforcing international agreements. Next, we assume that it takes time to detect a defection. We study these enforcement tools in a continuous-time version of a Fish Wars game with n countries. The Nash equilibrium leads to depletion of fish, while the social optimum results in sustainability. For partial cooperation, only 2-country coalitions are stable. We propose an algorithm that looks for a system enforcing social optimum in an arbitrary class of regulatory tax-subsidy systems. For the international agreement with imperfect monitoring, we are interested in the maximal detection delay for which the agreement remains self-enforcing. Counter-intuitively, the more players, the more stable the agreement. The talk is mainly based on [1].

[1] A. Wiszniewska-Matyszkiel, R. Singh, How to avoid the tragedy of the commons in an imprefect world, *Journal of Public Economic Theory* **26**(5):e12713, 2024

Managing invasive alien species with differential games and cost optimization

Andrea Caravaggio (1), Andrea Di Liddo (2), Angela Martiradonna (2),

(1) University of Siena, Siena, Italy; (2) University of Foggia, Foggia, Italy;

Invasive alien species pose significant threats to ecosystems, biodiversity, and global economies. Their rapid spread, driven by globalization and international trade, results in substantial ecological damage and economic costs, particularly in sectors like agriculture, forestry, fisheries, and public health. This study employs dynamic game theory and optimal control to investigate the management of invasive species

through strategic interactions among multiple agents [1,2,3]. Agents determine treatment strategies based on population densities and ecological factors, optimizing resource allocation and intervention timing. The goal is to minimize costs, including damages and treatment contributions. Cooperation or financial incentives may enhance the success of the removal strategy. The results should offer policymakers both qualitative and quantitative insights to balance rapid responses and long-term control, ultimately improving ecological and economic outcomes in the context of human-driven environmental challenges.

[1] C.M. Baker, F. Diele, D. Lacitignola, C. Marangi, A. Martiradonna. Optimal control of invasive species through a dynamical systems approach. *Nonlinear Analysis: Real World Applications*, **49**: 45-70, 2019.

[2] Di Liddo, A., Martiradonna, A. Effective management of invasive alien species in an optimal control framework. *Decisions in Economics and Finance.* Under revision.

[3] A. Lampert, Multiple agents managing a harmful species population should either work together to control it or split their duties to eradicate it. *Proceedings of the National Academy of Sciences*, **117**(19): 10210-10217, 2020.

Symbiosis emergence and abandonment in nature: a coordination game approach

Simon Levin (1), Ted Loch-Temzelides (2),

(1) Princeton University, Princeton, USA; (2) Rice University, Houston, USA;

We employ an n-player coordination game to model certain mutualisms in nature. The game payoff structure captures the insight that such "biological markets" cannot operate effectively, unless they reach a certain scale. Only when a threshold is reached, mutualistic strategies lead to robust advantages. The game gives rise to three types of Nash equilibria, which correspond to steady states of a dynamical system describing the underlying evolutionary process. We then study stochastic evolutionary dynamics using large deviation theory for discrete-time Markov processes. We provide a sharp analytical characterization of the stochastic steady states and of the transition dynamics across Nash equilibria and employ simulations to illustrate these results in special example cases. We find that the mutualism is abandoned and re-established several times through evolutionary time, but the mutualism persists the majority of time. Environmental and other changes that reduce the benefit/cost ratio associated with the symbiosis increase the likelihood of its abandonment. While the symbiosis establishment and abandonment could result from direct transitions across the mutualistic and non-mutualistic states, it is far more likely for such transitions to occur indirectly through intermediate partially mutualistic states. Thus, a mutualism might be (partially or fully) abandoned even if it is associated with overall superior fitness.

104. Environmental — 10:30 – 11:20

Chair: **S. Wrzaczek, R. Neck** — **FH HS 2**

Optimal green shift, R&D-generated growth, and the risk of environmental Disaster

Tapio Palokangas (1),

(1) HeGSE, University of Helsinki, Helsinki, Finland; (2) IIASA, Laxenburg, Austria;

Heterogeneous monopolists produce goods using either brown technology, which relies on labor and carbon energy, or green technology, which relies solely on labor. R&D firms enhance productivity using labor to outcompete existing monopolists, thereby driving economic growth. The extraction of carbon energy releases pollutants that harm production and increase the risk of environmental disasters. R&D should neither be taxed nor subsidized. A benevolent government can optimally mitigate the distortions caused by pollution by a two-part Pigouvian tax on carbon energy, with one part being precautionary, applied only before any disaster occurs. The optimal tax rate is determined.

Carbon intensity, endogenous emission, and technology transition

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We propose a tractable dynamic model of optimal technology transition that considers technological progress and endogenous emissions when facing a fixed emission target settled by the policy maker. This model allows us to study the extent to which these factors contribute to transition opportunities and abatement activities. We find that lower-cost abatement technologies and a high-growth market environment are favorable but not necessary conditions for enterprises to engage in transition. The primary driving forces are moderately active markets and increasing abatement burdens. Additionally, we find that the discount level and technological progress together determine the actual rate of return on emission reduction. When this rate is higher (or lower) than the market growth level, the decision to reduce emissions results in a corresponding increase (or decrease) in activity. Moreover, economic activities during the carbon peak period significantly impact the deployment of abatement capital, leading to abrupt behavior at certain key points in time.

74. Control of systems in Wasserstein spaces

11:30 – 12:45

Chair: **H. Frankowska, M. Quincampoix**

FH Nöbauer 8

Mean-field optimal control of multi-agent systems

Giulia Cavagnari (1), Mauro Bonafini (2), Antonio Marigonda (2),

(2) Department of Computer Science, University of Verona, Italy; (1) Department of Mathematics, Polytechnical of Milan, Italy;

Multi-agents systems are systems where the number of possibly interacting agents is so large that only a statistical description is viable. In this presentation, we introduce an optimal control problem for multi-agent systems with non-local cost fostering simultaneous aggregation of particles. This is done introducing a time-dependent notion of multiplicity whose intrinsic dynamical nature differs from more established geometric-like definitions used in branched transport theory. Applications to optimal design of chain supplies or to optimal logistic network for good's supplies are discussed.

Time-delayed opinion dynamics with leader-follower interactions

Cristina Pignotti (1),

(1) DISIM, University of L'Aquila, L'Aquila, Italy;

We study time-delayed variants of the Hegselmann–Krause opinion formation model. In particular, we focus on a model featuring a small group of leaders and a large group of non-leaders. In our model, leaders influence all agents but only interact among themselves, while non-leaders update their opinions via interactions with both their peers and the leaders, with time delays accounting for communication and decision-making lags. We prove that the system achieves consensus with an exponential decay rate and establish uniform l_∞-stability with exponentially decaying transients. Furthermore, we analyze the mean-field limit in two regimes: (i) with a fixed number of leaders and an infinite number of non-leaders, and (ii) with both populations tending to infinity, obtaining existence, uniqueness, and exponential decay estimates for the corresponding macroscopic models.
Based on joint papers with Young-Pil Choi, Chiara Cicolani, and Alessandro Paolucci.

Atomic gradient flow

Christian Amend (1), Marcello Carioni (1), Leonardo Del Grande (1), Konstantinos Zemas (2),

(1) University of Twente; (2) University of Bonn;

One of the popular approaches for optimization problems in the space of measures such as

$$\inf_{\mu \in M_+(\Omega)} F\left(\int_\Omega \phi d\mu\right) + \int_\Omega V d\mu,$$

is the so-called **Particle Gradient Flow (PGF)**. The PGF **restricts to linear combinations of Dirac deltas** $\sum_{i=1}^n c_i \delta_{x_i}$ and then takes an euclidean gradient flow in the weights and positions, significantly simplifying computations. Results of Chizat and Bach [1, 2] have shown that in this setting, PGFs perform exceptionally well, both theoretically and numerically.
In this talk, I will show how to generalize PGFs to arbitrary Banach spaces with separable predual and variational problems of the form

$$\inf_{u \in \mathcal{X}} \mathcal{F}(Ku) + \mathcal{R}(u)\,.$$

We call this approach the **Atomic Gradient Flow**. The crucial idea is to replace the Dirac deltas with **extremal points of the regularizer** $\mathcal{R}$, so work on $\mathcal{B} := \overline{\mathrm{Ext}(\{\mathcal{R} \leq 1\}}^*$ and assume for this metric space to have **non-positive curvature**.
Using as in [3] Choquet's theorem, I will present how to lift the problem into the Wasserstein space $\mathcal{P}_2(\mathbb{R} \times \mathcal{B})$, and study **convexity, existence and uniqueness properties** of metric gradient flows in both the lifted and non-lifted setting. Finally, I will discuss the relations between **the lifting of the atomic gradient flow** and metric gradient flows in $\mathcal{P}_2(\mathbb{R} \times \mathcal{B})$. Lastly, I will showcase applications such as $\mathcal{R}$ to be the Benamou-Brenier functional, or K to be a linear differential operator with $\mathcal{R}$ the TV-norm.

[1] L. Chizat and F. Bach. On the global convergence of gradient descent for over-parameterized models using optimal transport. In *Advances in neural information processing systems*, vol. 31, 2018.

[2] L. Chizat. Sparse optimization on measures with over-parameterized gradient descent. In *Math. Program.*, vol. 194, 2022.

[3] K. Bredies, M. Carioni, S. Fanzon, D. Walter. Asymptotic linear convergence of fully-corrective generalized conditional gradient methods. In *Math. Program.*, vol. 205, 2023.

75. Feedback control and stabilization of nonlinear PDEs 11:30 – 12:45

Chair: S. S. Rodrigues FH HS 6

The polynomial-polynomial regulator for high dimensional nonlinear control

Nick Corbin (1), Boris Kramer (1),

(1) Department of Mechanical and Aerospace Engineering, University of California San Diego, USA;

We consider the optimal regulation problem for nonlinear control-affine dynamical systems. Whereas the linear-quadratic regulator (LQR) considers optimal control of a linear system with quadratic cost function, we study polynomial systems with polynomial cost functions; we call this problem the polynomial-polynomial regulator (PPR). The resulting polynomial feedback laws provide two potential improvements over linear feedback laws: 1) they more accurately approximate the optimal control law, resulting in lower control costs, and 2) for some problems they can provide a larger region of stabilization. We derive explicit formulas—and a scalable, general purpose software implementation—for computing the polynomial approximation to the value function that solves the optimal control problem. The method is illustrated first on a low-dimensional aircraft stall stabilization example, for which PPR control recovers the aircraft from more severe stall conditions than LQR control. Then we turn to semi-discretized partial differential equations (PDEs), which lead to high-dimensional ODEs. We show further improvements to the above method based on subspace projections, where higher-order polynomial terms can be computed in low-dimensional subspaces, enabling scalability of the approach to thousands of degrees of freedom. Several PDE examples will be provided to demonstrate the scalability and efficiency of the controls.

[1] N. Corbin and B. Kramer. Computing solutions to the polynomial-polynomial regulator problem. *63rd IEEE Conference on Decision and Control (CDC)*, 2689 - 2696, 2024. DOI: 10.1109/CDC56724.2024.10885897

[2] N. Corbin and B. Kramer. Scalable Computation of $\mathcal{H}_\infty$ Energy Functions for Polynomial Control-Affine Systems *IEEE Transactions on Automatic Control)*, (early access), 2025. DOI: 10.1109/TAC.2024.3494472

Control of falling liquid films using a hierarchical model approach

Susana Gomes (1), Oscar Holroyd (1,2), Radu Cimpeanu (1),

(1) Mathematics Institute, University of Warwick, Coventry, UK; (2) HetSys CDT, University of Warwick, Coventry, UK;

The flow of a thin film down an inclined plane is a canonical setup in fluid mechanics and associated technologies, with applications such as coating, where the liquid-gas interface should ideally be flat, and heat or mass transfer, where an increase of interfacial area is desirable. In each of these applications, we would like to robustly and efficiently manipulate the flow in order to drive the dynamics to a desired interfacial shape. Because of the nonlinearities of the problem, this has not been tackled from a control theoretical point of view until recently. In this talk, I will present several results obtained when controlling falling liquid films using same fluid blowing and suction through the wall. The controls are developed on a simplified model, allowing for theoretical results on controllability, and used throughout a hierarchy of nonlinear models, where we investigate their ability to translate across the hierarchy into real-life situations by using direct numerical simulations of the Navier-Stokes equations. I will discuss distributed controls as well as (more realistic) point-actuated controls, their robustness to parameter uncertainties and validity across the hierarchy of models.

On the role of unique continuation in uniform stabilization of MHD systems

Irena Laseicka (1,2), Buddhika Priyasad (3), Roberto Triggiani (1),

(1) Department of Mathematical Sciences, University of Memphis, Memphis, TN 38152 USA; (2) IBS, Polish Academy of Sciences, Warsaw, Poland; (3) Department of Mathematics and Statistics, Universität Konstanz, Konstanz, Germany;

This work addresses the uniform stabilization of the d-dimensional MagnetoHydroDynamics (MHD) system ($d = 2, 3$) posed on a smooth bounded domain, with low-regularity initial data and homogeneous boundary conditions. We consider destabilizing external sources and construct static, finite-dimensional, interior-localized feedback controls that stabilize the system near an unstable equilibrium. These controls act on arbitrarily small subdomains and are minimal in number. The analysis is carried out in tight Besov spaces $\widetilde{\mathbf{B}}^{2-\frac{2}{p}}_{q,p}(\Omega) \times \widetilde{\mathbf{B}}^{2-\frac{2}{p}}_{q,p}(\Omega)$, which allow handling nonlinearities while avoiding boundary compatibility constraints. A key component of our approach is the establishment of a Unique Continuation Property (UCP) for a corresponding overdetermined MHD eigenvalue problem, which is equivalent to the Kalman-type controllability condition for the unstable finite-dimensional component. This UCP result, proved via a Carleman-type estimate adapted from techniques used in Navier-Stokes and Boussinesq systems, plays a central role in enabling the feedback design. In this paper, we solve the linear uniform stabilization problem and establish maximal L^p-regularity up to infinite time for the stabilized linearized system, paving the way for nonlinear stabilization results presented in a forthcoming work.

76. Topology optimization: theory, numerics and applications 11:30 – 12:45

Chair: K. Sturm, P. Gangl **FH HS 4**

Mesh Quality Constraints for Shape Optimization

Sebastian Blauth (1), Christian Leithäuser (1),

(1) Transport Processes, Fraunhofer ITWM, Kaiserslautern, Germany;

In shape optimization, the quality of the underlying mesh is of crucial importance. The mesh quality is particularly important when shape optimization problems constrained by partial differential equations (PDEs) are considered, as the mesh quality is essential for the numerical approximation of the PDE's solutions. Even when starting with a very good mesh, the mesh quality usually tends to deteriorate over the course of a shape optimization. This either results in a premature stop of the optimization algorithm or a costly remeshing operation must be performed to continue with the optimization. In this talk, we present a novel approach for enforcing constraints on the mesh quality in shape optimization which guarantee a minimum mesh quality over the entire optimization. To do so, the angle of triangular and the solid angle of tetrahedral mesh cells is bounded from below. These constraints are treated with with Rosen's gradient projection method which ensures that in each iteration the mesh is feasible w.r.t. the constraints. We present some numerical results of the proposed approach which highlights the applicability and performance of approach. In particular, with our approach, new shape optimization problems, particularly those relevant for industrial applications, which could not been solved with alternative approaches, can now be treated.

Topology optimization of two bipolar plate models of hydrogen electrolysis cells

Leon Niklas Baeck (1,3), Sebastian Blauth (1), Kevin Sturm (2), Christian Leithäuser (1), René Pinnau (3),

(1) Transport Processes, Fraunhofer ITWM, Kaiserslautern, Germany; (2) Institute of Analysis and Scientific Computing, TU Vienna, Vienna, Austria; (3) Technomathematics Group, RPTU Kaiserslautern-Landau, Kaiserslautern, Germany;

Hydrogen technologies are essential for achieving climate-neutral mobility and fostering a clean energy sector, with electrolysis cells playing a critical role in hydrogen production. This talk explores the topology optimization of two distinct bipolar plate models in proton exchange membrane (PEM) electrolysis cells. We first present the two models: the first describes fluid flow within the plate using Stokes equations, while the second introduces a porous medium to relax the original formulation. We derive the topological derivative for both models and employ the approach from [1] to numerically compute multiple local minimizers, leading to innovative designs for these plates. Additionally, we demonstrate that the relaxed model serves as a robust approximation of the original model. Through both numerical simulations and theoretical analysis, we show that the relaxed states, adjoints, and a version of the normalized generalized topological derivative converge strongly to their counterparts in the original model.

[1] L. Baeck, S. Blauth, C. Leithäuser, R. Pinnau, and K. Sturm. A novel deflation approach for topology optimization and application for optimization of bipolar plates of electrolysis cells. *arXiv https://arxiv.org/abs/2406.17491*, 2025.

Continuation methods for higher-order topology optimization

Michael Winkler (1), Peter Gangl (2),

(1) RICAM, Linz, Austria; (2) RICAM, Linz, Austria;

We aim to solve a topology optimization problem where the distribution of material in the design domain is represented by a density function. To obtain candidates for local minima, we want to solve the first order optimality system via Newton's method. This requires the initial guess to be sufficiently close to the a priori unknown solution. Introducing a stepsize rule often allows for less restrictions on the initial guess while still preserving convergence. In topology optimization one typically encounters nonconvex problems where this approach might fail. We therefore opt for a homotopy (continuation) approach which is based on solving a sequence of parameterized problems to approach the solution of the original problem. The arising Newton-type method also allows for employing deflation techniques for finding multiple distinct solutions as well as for efficiently tracing Pareto optimal points in multi-objective optimization problems. First numerical results for PDE-constrained design optimization problems are presented.

77. Using Model Predictive Control, Reinforcement Learning, and Related Methods for Risk Control

11:30 – 12:45

***Chair*: W. Semmler**

Saal

Multi-objective optimal control with carbon emission and temperature constraints: for achieving a low-fossil-fuel economy

Helmut Maurer (1), Willi Semmler (2),

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(2) The New School, Department of Economics, 6E 16th St., New York, 10003, USA
University of Bielefeld, Bielefeld, Germany; IIASA, Laxenburg, Austria;

In this paper we propose multi-objective control to deal with climate change and climate risks and the transition to a low carbon economy. Extending our previous collaborative work as in [1] we again build on the Nordhaus type DICE model to include various optimal macroeconomic policies such as mitigation, adaptation and climate-related infrastructure investment studying the dynamics of the decarbonizing of the economy. Based on a finite horizon model that includes the threats of climate disasters arising from emissions and temperature rise, we deal with preventive measures such as adaptation reducing disaster effects. Our optimal control problem of finite horizon is consisting of a dynamical system with five-dimensional state vector representing stocks of private capital, green capital, public capital, stock of brown energy in the ground,carbon emissions, and temperature AQ2. The objective function captures preferences over consumption but is also impacted by atmospheric, climate risks events and by mitigation and adaptation policies. Given the numerous challenges to climate change policies with multiple objectives the control vector is eight-dimensional including mitigation, adaptation and infrastructure investment. The optimal control problem is studied under various state constraints. In two scenarios we compute the Pareto front for a bi-objective control problem. Optimization over the Pareto front [2] provides us with suitable weights for the two objectives. In particular, we explore the role of constraints, as the Kyoto Protocol has suggested, and temperature constraints, as the Copenhagen–Paris agreements have proposed.

[1] M. Atolia, P. Loungani, H. Maurer, W. Semmler. Optimal control of a global model of climate change with adaptation and mitigation. *Mathematical Control and Related Fields*, **13** (2): 583–604, 2023.

[2] Y.C. Kaya, H. Maurer. Optimization over the Pareto front of nonconvex multi-objective optimal control problems. *Computational Optimization and Applications*, **86**: 1247–1274, 2023.

Model predictive control for risk measures: challenges and first solutions

Jonas Schießl (1), Rouchuan Ou (2), Michael H. Baumann (1), Timm Faulwasser (2), Lars Grüne (1),

(1) Chair of Applied Mathematics, University of Bayreuth, Bayreuth, Germany; (2) Institute of Control Systems, Hamburg University of Technology, Hamburg, Germany;

Model Predictive Control (MPC) is a control scheme that can be used to approximate infinite horizon optimal control problems by a series of finite-horizon optimal control problems. Recently, such an approximation property has been made mathematically rigorous for MPC applied to a class of stochastic optimal control problems [1]. However, for structural reasons, which will be explained in this talk, this result only applies to stage costs that are expectations over deterministic functions, which excludes many interesting risk measures. As a remedy, through a turnpike-based analysis we demonstrate that for a particular class of risk measures, which includes, e.g., the averaged value-at-risk, this result can still be leveraged to formulate an implementable risk-averse MPC scheme, resulting in near-optimal averaged performance. We explain the resulting MPC algorithm and illustrate its performance by means of numerical experiments.

[1] J. Schießl, R. Ou, T. Faulwasser, M.H. Baumann, and L. Grüne, Near-optimal performance of stochastic economic MPC. In: Proceedings of the 63rd IEEE Conference on Decision and Control (CDC 2024), Milan, Italy, pp. 2565–2571, 2024.

European Sovereign Debt Control Through Deep Reinforcement Learning

Tato Khundadze (2), Willi Semmler (1),

(1) The New School, New York, USA; (2) The New School, New York, USA;

The resilience of economic systems largely depends on coordination among key stakeholders during macroeconomic or external shocks. A lack of such coordination can lead to financial and economic crises. This paper draws on experiences from global and regional shocks, notably the Eurozone crisis (2009–2012) and the COVID-19 pandemic (from 2020 onward), to demonstrate the importance of monetary and fiscal policy cooperation in responding to macroeconomic non-resilience, with a focus on public debt management. The Euro area serves as the empirical setting, as its resilience critically relies on coordination among diverse national actors. Given the asymmetric nature of shocks within the EU and the heterogeneity in economic structures and policies, coordinated responses are essential. Building on Semmler and Haider (2018), who employed Nonlinear Model Predictive Control (NMPC) to trace macroeconomic dynamics under cooperative and non-cooperative fiscal scenarios, this paper extends their approach by applying Deep Reinforcement Learning. Specifically, it tests the performance of the Soft Actor-Critic (SAC) algorithm in modeling the dynamics of inflation, interest rates, the output gap, government debt, and net lending in a macroeconomic environment.

78. Mathematical Models for Human Impact on the environment 11:30 – 12:45

Chair: **G. Fabbri, S. Faggian, F. Gozzi** **FH HS 7**

Stability of pollution reduction agreements in presence of non-signatories

Agnieszka Wiszniewska-Matyszkiel (1), Jakub Mierzwa (2),

(1) Institute of Applied Mathematics and Mechanics, University of Warsaw, Warsaw, Poland; (2) Faculty of Mathematics, Informatics and Mechanics, University of Warsaw, Warsaw, Poland;

We study a differential game that models the international pollution reduction policy for a global stock pollutant. This means that the polluting substance influences the welfare of countries by its aggregate stock. The control is by an international agreement whose signatories can reduce their own emissions and impose costly sanctions to overpolluting non-signatories. Imposing sanctions is also costly to the signatories and the signatories share those costs. We examine how equilibrium welfare of signatories and non-signatories depends size of the agreement, decide what size is stable, then examine the influence on the global emissions, including their dynamic behavior and the resulting welfare. We compare the results with those for problems without sanctions and without any agreement, showing that presence of sanctions is vital to a large-scale agreement and significant reduction of pollution. Finally, we make the choice of severity of the sanction scheme an additional stage of the pollution agreement supergame. Although it could have been expected that introducing and increasing sanctions results in increasing the size of stable agreement, this is true only to some extent: for sufficiently high severity and specific behavior of both costs of sanctions. Introducing sanctions with low severity may decrease the size of a stable agreement compared to no sanctions, or even make cooperation impossible.

Network-based optimal control of pollution growth

Fausto Gozzi (1), Marta Leocata (1), Giulia Pucci (2),

(1) Department of Economics and Finance, Luiss University, Rome, Italy.; (2) Department of Mathematics, KTH Royal Institute of Technology, Stockholm, Sweden;

In this talk, we present a model for the optimal control of pollution diffusion over time and space by a centralized economic agent (the planner). The space is modeled by a network of interconnected locations. The planner's controls include investments in production—of both green and non-green types—as well as depollution efforts, with the objective of maximizing an intertemporal utility function that increases with consumption while incorporating the costs of pollution. We provide explicit solutions in cases where pollution costs are either strictly convex or linear. We examine the trade-off between green and non-green technologies within this dynamic, network-based framework. Additionally, we propose a first simplified model for output movement between nodes and present preliminary results on how such transfers influence the optimal strategy. Finally, we conduct numerical investigations using a quadratic renewable cost function to further analyze the role of renewable investment and the impact of geographic and economic heterogeneities on optimal investment strategies.

Understanding the quality of the environment: act local, think global?

Marta Leocata (1), Emmanuelle Augeraud Véron (1), Daria Ghilli (2), Fausto Gozzi (3), Marta Leocata (4),

(1) Bordeaux School Economics; (2) Università degli studi di Pavia; (3) Luiss Guido Carli; (4) Luiss Guido Carli;

In the first part of the talk, we consider a N-players game where agents maximize a given utility function depending on local environmental quality (specific to each location) and on a global enviromental quality, which is the interaction term. We are interested in understanding whether taking into account global environmental quality (thinking globally) may better local environmental quality. We show that local environmental quality may not necessarily be better by this consideration. We validate our choices in the model by an empirical analysis. In the second part of the talk, we analyse the case in which there is an infinite number of locations (and then agents) and agents are still competitive, that is, the *Mean Field Game* associated. Finally, we study the case in which agents are cooperative and there is a social planner who optimizes, that is the *Mean Field Control* associated.

79. Semi-plenary

14:00 – 14:50

Chair: **M. Quincampoix**

FH Nöbauer 8

Second-order analysis for bang-bang control of PDEs

Gerd Wachsmuth (1),

(1) BTU Cottbus-Senftenberg, Cottbus, Germany;

In this talk, we consider optimal control problems of the form

$$\text{Minimize} \qquad \frac{1}{2}\|S(u) - y_d\|^2_{L^2(\Omega)} \qquad \text{w.r.t. } u \in L^2(\Omega), u_a \le u \le u_b,$$

where S is the solution operator of a semilinear elliptic partial differential equation (PDE) equipped with homogeneous Dirichlet or Neumann boundary conditions.

The main feature of this problem is the missing L^2-Tikhonov term. We are interested in the situation that the optimal control is bang-bang, i.e., it attains only the values u_a and u_b. Under a regularity assumption on the adjoint state, we perform a second-order analysis of the problem. In particular, we address no-gap second-order conditions, stability and differential sensitivity of solutions. We also address the application of these results to numerical methods. We show that the problem can be solved by a proximal-gradient method and by Newton's method.

80. Semi-plenary — 14:00 – 14:50

Chair: **L. Grüne** — **FH HS 5**

Dynamic Stochastic Variational Inequalities and Their Applications in AI

(1) Department of Applied Mathematics, The Hong Kong Polytechnic University;

The dynamic stochastic variational inequality (DSVI) is an ordinary differential equation whose right-hand side is defined by the two-stage stochastic variational inequality (SVI). The DSVI provides a unified modelling framework for various applications in which dynamics, uncertainties and equilibrium are present. We show the existence and uniqueness of a solution for two classes of the DSVI in the space of continuously differentiable functions with the space of measurable functions. The first class is defined by a strongly monotone SVI in the second stage, and the second class pertains to a box-constrained P-matrix linear SVI in the second stage. We develop sample average approximation and time-stepping schemes to compute a solution of the DSVI. The uniform convergence and exponential convergence are established for the discrete approximation. We show applications of the DSVI for optimizing multimodal lager model-based embodied intelligence system for the elderly, where the two-stage SVI arises from stochastic bilevel optimization.

81. Control of systems in Wasserstein spaces — 15:10 – 15:35

Chair: **H. Frankowska, M. Quincampoix** — **FH Nöbauer 8**

Optimal control of particle systems with absorption or stopping time

Pierre Cardaliaguet (1), Joe Jackson (2), Panagiotis Souganidis (2),

(1) Ceremade, Université Paris-Dauphine, Paris, France; (2) The University of Chicago Department of Mathematics, Chicago, USA;

In this joint work with J. Jackson and P. Souganidis (U. Chicago), we discuss the optimal control of systems of particle which can be either absorbed by a boundary or removed from the system. We show that, as the number of particle tends to infinity, the limit is an optimal control problem of a flow of subdensities and characterize the associated value function in terms of Hamilton-Jacobi equations on the space of subdensities.

82. Continuous Optimization: Theory and Applications — 15:35 – 16:25

Chair: **R. Bot, Y. Malitsky, A. Daniilidis** — **FH Nöbauer 8**

Maximal Monotone Operators with unique Representation

Sotiris Armeniakos (1), Aris Daniilidis (1),

(1) VADOR, TU Wien, Vienna, Austria;

For a monotone operator $A : X \rightrightarrows X^*$, we define $\mathcal{F}_A$ as the family of all representations of A through convex lower semicontinuous functions on $X \times X^*$. These representations facilitate the application of convex analysis techniques in the study of monotone operators.

Bartz et al. have shown that when the operator is the subdifferential of a sublinear function, $\mathcal{F}_A$ is a singleton. In this work, we investigate the inverse problem: what conditions must an operator satisfy to have a unique representation? We establish specific properties and, under certain assumptions, provide a complete characterization of such operators. This research is conducted in collaboration with A. Daniilidis.

Horoballs and a splitting subgradient algorithm

Ariel Goodwin (1), Adrian Lewis (2), Genaro López (3), Adriana Nicolae (4),

(1) CAM, Cornell University, Ithaca, USA; (2) ORIE, Cornell University, Ithaca, USA; (3) IMUS, University of Seville, Seville, Spain; (4) Department of Mathematics, Babeş-Bolyai University, Cluj-Napoca, Romania;

Many of the primal ingredients of convex optimization extend naturally from Euclidean to Hadamard spaces — nonpositively curved metric spaces like Euclidean, Hilbert, and hyperbolic spaces, metric trees, and more general CAT(0) cubical complexes. Linear structure, however, and the duality theory it supports are absent. Nonetheless, we introduce a new type of subgradient for convex functions on Hadamard spaces, based on Busemann functions. This notion supports a splitting subgradient method with guaranteed complexity bounds. In particular, the algorithm solves p-mean problems in general Hadamard spaces: we illustrate by computing medians in BHV tree space.

[**1**] M. Bačák. Convex Analysis and Optimization in Hadamard Spaces, De Gruyter, Berlin, 2014.

[**2**] M.R. Bridson and A. Haefliger. Metric Spaces of Non-Positive Curvature, Springer-Verlag, Berlin, 1999.

[**3**] A. Goodwin, A. Lewis, G. López, A. Nicolae. A subgradient splitting algorithm for optimization on nonpositively curved metric spaces, arXiv:2412.06730, 2024.

83. Set-valued mappings in approximation, control and optimization 15:35 – 16:25

Chair: **R. Baier, E. Farkhi** **FH HS 6**

Hermite interpolation and divided differences of convex set-valued maps via directed sets

Robert Baier (1),

(1) Department of Mathematics, University of Bayreuth, Bayreuth, Germany;

Problems of interpolation or finding zeros and fixed sets of set-valued maps in $\mathbb{R}^n$ often require either negative coefficients in the linear combination of sets, suitable set-valued divided differences or set-valued derivatives. Since several options for the subtraction of convex compact sets in $\mathbb{R}^n$ suffer from disadvantages – compared to the addition with the well-known Minkowski sum –, we use the framework of the Banach space $\mathcal{D}(\mathbb{R}^n)$ of directed sets [1]. This space provides an embedding of convex compacts and set-valued arithmetic operations with a subtraction that is inverse to the addition. The two components of a directed set in $\mathbb{R}^n$ are both parameterized by unit vectors. The first one – present only for n greater than 1 – is an $(n-1)$-dimensional directed set based on supporting faces for a given unit direction, while the second one is a scalar continuous function. The arithmetic operations for directed sets are performed recursively on each of the two components and are applied only to common directions. Convex and non-convex visualization parts of a directed set help to identify the main parts of the directed set. We review applications of directed sets to set-valued interpolation and zero-finding problems. By embedding the images of the set-valued map we obtain a function with values in the Banach space $\mathcal{D}(\mathbb{R}^n)$. For univariate functions the Lagrange/Hermite interpolation polynomial, its Newton form with divided differences or derivatives are computed [2]. Furthermore, a problem of the form $F(x) = G(x)$ with two set-valued maps F, G is studied and a secant method with multivariate divided differences is applied [3].

[1] R. Baier and E. Farkhi. Differences of convex compact sets in the space of directed sets. Parts I & II: The space and visualization of directed sets. *Set-Valued Analysis*, **9**(3):217–245, 247–272, 2001.

[2] R. Baier and G. Perria. Set-valued Hermite interpolation. *Journal of Approximation Theory*, **163**(10):1349–1372, 2011.

[3] R. Baier and M. Hessel-von Molo. Newton's method and secant method for set-valued mappings. In *Proc. 8th International Conference on "Large-Scale Scientific Computations" (LSSC 2011).* I. Lirkov, et al., Eds., Lecture Notes in Computer Science, vol. 7116, pp. 91–98, 2012.

On optimality conditions and an exact penalty approach based on the directed subdifferential in nonsmooth optimization

Wolfgang Achtziger (1), Robert Baier (2), Elza Farkhi (3), Vera Roshchina (4),

(1) Dept. of Math., FAU Erlangen-Nürnberg, Erlangen, Germany; (2) Dept. of Math., University of Bayreuth, Bayreuth, Germany; (3) School of Math. Science, Tel-Aviv University, Tel-Aviv, Israel; (4) School of Math. and Stat., UNSW Sydney, Sydney, Australia;

Based on the Banach space of directed sets [1], we consider the directed subdifferential. The definition of the directed subdifferential is straightforward for DC functions in n real variables and can be extended to so-called directed subdifferentiable functions, a large class containing, e.g., quasidifferentiable functions [2].
As a benefit of directed sets, the definition of the directed subdifferential leads to advantageous calculus rules as, e.g., the sum rule as an equation. Furthermore, the application of the visualization apparatus for directed sets [1] leads to the so-called visualization of the directed subdifferential and its visualization parts being subsets of $\mathbb{R}^n$. Relations of the visualization to known subdifferentials in $\mathbb{R}^n$ show that the directed subdifferential collects the essential information on local function behaviour. In view of unconstrained optimization problems, this gives rise to the formulation of local optimality conditions based on the visualization parts.
The talk presents the application of these optimality conditions to the standard technique of exact l_1-penalty approaches in constrained Nonlinear Programming and on closely related classical theory like

Lagrange duality and saddle point optimality conditions. These approaches are illustrated by examples for which the directed subdifferential can be analytically calculated. It turns out that the directed subdifferential can give additional information on the nature of critical points. The talk supplements first results once presented in [3].

[1] R. Baier, and E. Farkhi. Differences of convex compact sets in the space of directed sets. Parts I & II. *Set-Valued Analysis*, **9**(3), 217-245, 247-272, 2001.

[2] R. Baier, E. Farkhi, and V. Roshchina. From quasidifferentiable to directed subdifferentiable functions: Exact calculus rules. *Journal of Optimization Theory and Applications*, **171**(2), 384-401, 2016.

[3] W. Achtziger, R. Baier, E. Farkhi, and V. Roshchina. Exact penalty and Lagrange duality via the directed subdifferential. *Pure and Applied Functional Analysis*, **2**(2):183-220, 2017.

On polar convexity in finite-dimensional Euclidean spaces

Shubhankar Bhatt (1), Hristo Sendov (2),

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(2) Department of Statistical and Actuarial Sciences and Department of Mathematics, The University of Western Ontario, 1151 Richmond Str., London, ON, N6A 5B7 Canada;

Let $\hat{\mathbb{R}}^n$ be the one point compactification of $\mathbb{R}^n$ obtained by adding a point at infinity. We say that a subset $A \subseteq \hat{\mathbb{R}}^n$ is $\mathbf{u}$-*convex* if for every pair of points $\mathbf{z}_1, \mathbf{z}_2 \in A$, the arc of the unique circle through $\mathbf{u}, \mathbf{z}_1$ and $\mathbf{z}_2$, from $\mathbf{z}_1$ to $\mathbf{z}_2$ and not containing $\mathbf{u}$, is contained in A. In this case, we call $\mathbf{u}$ a pole of A. When the pole $\mathbf{u}$ approaches infinity, $\mathbf{u}$-convex sets become convex in the classical sense. The notion of polar convexity in the complex plane has been used to analyze the behavior of critical points of polynomials. In this talk, we extend the notion to finite-dimensional Euclidean spaces. The goal of this work is to start building the theory of polar convexity and to show that the introduction of a pole creates a richer theory. For example, polar convexity enjoys a beautiful duality that does not exist in classical convexity. We formulate polar analogues of several classical results of the alternatives, such as Gordon and Farkas lemmas. Finally, we give a full description of the convex hull of finitely many points with respect to finitely many poles.

84. Health

15:10 – 16:00

Chair: **M. Freiberger**

FH HS 5

Analysis of parameters and approximate optimal control of drags in the mathematical model of Osteoporosis

Anita Krawczyk (1), Andrzej Nowakowski (2),

(1) Faculty of Mathematics & Computer Sciences, University of Lodz, Banacha 22, Lodz, Poland;
(2) Faculty of Mathematics & Computer Sciences, University of Lodz, Banacha 22, Lodz, Poland;

Osteoporosis is an aging disease characterized by loss of bone mass, where bones become fragile and more likely to fracture. Bone density begins to decrease after about 50 years, and a state of osteoporosis is

defined by loss of more than 25%. Cellular senescence is a permanent arrest of the normal cell cycle while maintaining cell viability. The number of senescent cells increases with age. It is natural to consider the question to what extent senescent cells induce loss of bone density and osteoporosis. In [**?**] the authors developed the mathematical model of osteoporosis depending on several parameters and controls (senolytic drugs such as fisetin and quercetin). Parameters are adjusted by making simulations or experiments. Similarly, drug treatment is done by making simulations and comparing with empirical data. The model consists of fourteen ordinary differential equations. The essential point is that these equations depend on many parameters of which part is chosen flexible. One of the objectives of this article is to optimize the choice of parameters according to some rules. The rules means a kind of functional that should measure the behavior of interesting result with respect to these parameters. By an interesting result, we mean bone density and the number of cellular senescence. That functional also contains a term related to the treatment of drugs, as well as of a time when we must start treatment. Next, we formulate an optimal control problem. The second aim of the paper is to formulate new sufficient optimal conditions in a form of verification theorem. As a final result, based on the verification theorem, we state an algorithm that allows efficient calculation to find optimal parameters and optimal treatment of drugs in the best time.

Food waste: you can't always want what you get

Carmen Camacho (1), Agustín Pérez-Barahona (2), Çağrı Sağlam (3),

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Food waste constitutes a significant economic inefficiency and should therefore be a central policy issue. While in low-income countries food waste is often associated with poor harvesting, storage and transportation conditions, in middle- and high-income countries consumers' behavior is considered to be the main driver of this problem. The general aim of our paper is to contribute to the understanding of food waste. We focus on household food waste and the economic mechanisms behind it. We propose a stylized model in which food waste appears as an economic decision of households. Our framework of "rational food waste" relies primarily on consumer behavioural biases, which could be further encouraged by aggressive pricing strategies such as quantity discounts.

The model describes how a representative household prepares a meal using different types of ingredients. Although households can grow part of their food, they can also purchase and store some of the ingredients. Meals are then made by combining three categories of ingredients: home-grown ingredients and purchased fresh and processed foods. Considering the production of home-grown ingredients, as well as storage and meal preparation technologies, households get utility from the corresponding composite meal. However, we also assume that a household's preferences may depend on the processed foods consumed. In our stylized framework, this specific taste for processed foods encapsulates the sales and marketing practices of the processed and ultra-processed food industry to attract consumers. These strategies range from including additives in (ultra-)processed foods in order to increase their palatability and appeal to aggressive marketing techniques (see, for instance, Moodie et al., 2013; Hall et al., 2019; and Perona, 2022). We show that this taste can create a mismatch between households' preferences and the nutritional contribution of processed foods, leading to food waste. Moreover, if this bias is systematically nurtured, food waste would be persistent over time. We also study the role played by non-linear food prices in this regard. We find that quantity discounts can sustain even higher levels of food waste.

85. Learning methods in optimal and predictive control 15:35 – 16:25

Chair: L. Grüne, K. Worthmann FH HS 5

Gaussian-process-based learning for model predictive control

Maik Pfefferkorn (1), Sebastian Hirt (1), Philipp Holzmann (1), Yongpeng Zhao (1,2), Rolf Findeisen (1),

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(2) Group Innovation, Volkswagen AG, Wolfsburg, Germany;

Safety-critical constraints and performance requirements must be reliably met in many control systems. Model predictive control (MPC) is widely used to address these challenges, but its performance relies on careful tuning of cost functions, prediction models, and constraints. In practice, incomplete or uncertain information about the system or its environment can degrade closed-loop performance and reliability. To address this, machine learning methods, such as Gaussian process regression, can be integrated into MPC to infer missing information and improve adaptability. Recent interest has focused on learning these components directly from closed-loop performance data. However, ensuring robust safety and stability during both learning and operation remains a significant challenge, limiting most approaches to non-safety-critical applications. In this talk, we explore Gaussian process regression and Bayesian optimization for closed-loop learning and propose a novel, stability-informed approach to address these limitations.

[1] P. Holzmann, M. Pfefferkorn, J. Peters, and R. Findeisen. Learning Energy-Efficint Trajectory Planning for Robotic Manipulators using Bayesian Optimization. *European Control Conference*, 1374-1379, 2024.

[2] S. Hirt, M. Pfefferkorn, and R. Findeisen. Safe and Stable Closed-Loop Learning for Neural-Network-Supported Model Predictive Control. *Conference on Decision and Control*, 2024.

[3] S. Hirt, A. Höhl, J. Pohlodek, J. Schaeffer, M. Pfefferkorn, R. D. Braatz, and R. Findeisen. Safe Learning-Based Optimization of Model Predictive Control: Application to Battery Fast-Charging. *American Control Conference*, 2025.

[4] Y. Zhao, M. Pfefferkorn, M. Templer, and R. Findeisen. Efficient Learning of Vehicle Controller Parameters via Multi-Fidelity Bayesian Optimization: From Simulation to Experiment. *Intelligent Vehicles Symposium*, 2025.

Data-driven models in predictive control: Uncertainty quantification and robust designs

Haldun Balim (1), Andrea Carron (1), Melanie N. Zeilinger (1), Johannes Köhler (1),

(1) ETH Zürich, Institute for Dynamic Systems and Control, Zürich, Switzerland;

In this talk, we present a principled design for predictive controllers based on data. This approach does not assume existence of a physical model, which significantly enhances the practical applicability. Instead, the proposed framework extracts model information and uncertainty description from experimental data and then leverages this information in a robust design [1]. The approach is applicable to linear systems with output measurements and stochastic process and measurement noise. Parameter identification uses

Expectation-Maximization, while incorporating structural constraints. Uncertainty quantification uses the asymptotic Gaussian distribution of the estimation error. We reformulate the parametric uncertainty to synthesize robust dynamic output-feedback controllers. Lastly, we present a predictive controller that accounts for the stochastic noise and the parametric uncertainty, ensuring recursive feasibility and satisfaction of chance constraints. This methodology is complimented by a code framework that implements all the described steps for a general class of linear systems, thus automating the design of robust and predictive controllers from data. Efficacy is demonstrated with a numerical example involving a 10-dimensional spring-mass-damper system.

[1] H. Balim, A. Carron, M.N. Zeilinger, J. Köhler. From Data to Predictive Control: A Framework for Stochastic Linear Systems with Output Measurements. Submitted to Automatica, preprint available online at arXiv:2407.17277, 2024.

Neural network based inverse optimal control

Kathrin Flaßkamp (1), Sofya Maslovskaya (2), Sina Ober-Blöbaum (2), Christian Offen (2), Amine Othmane (1),

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(2) Department of Mathematics, Paderborn University, Paderborn, Germany;

The problem of recovering the optimality principles from observations in nature for modeling and design of digital twins leads to the inverse optimal control problem. The observed trajectories, solutions of an optimal control problem (OCP), are used to find an unknown cost, which can be further used for generating new solutions. One of the central questions in inverse optimal control is the identification of a suitable parameterization for a given class of problems. In this work, we consider a class of nonlinear OCPs defined on an infinite time horizon. In our case, the Hamilton-Jacobi-Bellman equation admits a simplified form and provides a perfect framework for the inverse problem. We use a parameterization through the value function, which allows us to both reconstruct a cost and generate feedback controls for new simulations in a straightforward manner. We formulate a learning algorithm based on a suitable choice of neural network parameterization and form of the loss, which ensures the optimality and stability of the generated feedback. The obtained results are validated on numerical examples.

86. Dynamic management of natural resources — 15:35 – 16:25

Chair: **T. Upmann, D. Gromov** — **FH HS 7**

Destabilization, stabilization, and multiple attractors in saturated mixotrophic environments

Torsten Lindström (1), Yuanji Cheng (2), Subhendu Chakraborthy (3),

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The ability of mixotrophs to combine phototrophy and phagotrophy is now well recognized and found to have important implications for ecosystem dynamics. In this paper, we examine the dynamical consequences of the invasion of mixotrophs in a system that is a limiting case of the chemostat. The model is a hybrid of a competition model describing the competition between autotroph and mixotroph populations for a limiting resource, and a predator–prey-type model describing the interaction between autotroph and

herbivore populations. Our results show that mixotrophs are able to invade in both autotrophic environments and environments described by interactions between autotrophs and herbivores. The interaction between autotrophs and herbivores might be in equilibrium or cycle. We find that invading mixotrophs have the ability to both stabilize and destabilize autotroph-herbivore dynamics depending on the competitive ability of mixotrophs. The invasion of mixotrophs can also result in multiple attractors.

[1] T. Lindström, Y. Cheng, S. Chakraborthy Destabilization, stabilization, and multiple attractors in saturated mixotrophic environments *SIAM Journal of Applied Mathematics*, **476**(2):665-695, 2019.

On some Mathematical Models for Biodiversity and Agroecology

Emmanuelle Augeraud (1), Raouf Boucekkine (2), Alessandro Calvia (3), Daria Ghilli (4), Fausto Gozzi (5), Marta Leocata (6), Federica Masiero (7),

(1) Department of Economics, Université Bordeaux, France; (2) Department of Economics, Aix-Marseille Université, France; (3) Department of Mathematics, Politecnico di Milano, Italy; (4) Department of Economics and Management, Pavia, Italy; (5) Department of Economics and Finance, Luiss University, Roma, Italy; (6) Department of Economics and Finance, Luiss University, Roma, Italy; (1) Department of Mathematics, Universita di Milano Bicocca, Italy;

In this talk we present some ideas on how to model the evolution of biodiversity in the Antropocene (the era where the presence of homo sapiens is modifying deeply the biosphere) and its relations with the economic variables. In particular we focus on two aspects:

- on one side the relations of biodiversity loss, deforestation and agriculture in an optimal control model;
- on the other side the space-time evolution of biodiversity in a spatial mean field game.

Partial cooperation in resource extraction on networks

Silvia Faggian (1), Dominika Machowska (2), Agnieszka Wiszniewska-Matyszkiel (2),

(1) University of Venice Ca'Foscari; (2) Institute of Applied Mathematics and Mechanics, University of Warsaw, Warsaw, Poland;

We study a differential game of resource extraction in which players' resource deposits are interrelated. An example is extraction of a marine fishery divided into Exclusive Economic Zones of countries. The interactions can be asymmetric and various players areas may have different reproduction rates. The spatial relations between resources of players are modelled by a network. For this game, we study the potential of cooperative behaviour. We compare the Nash equilibrium payoff to payoffs if at least a fraction of players cooperate and check stability of various sizes of coalitions. We also study the problem of sustainability.

[1] A. Wiszniewska-Matyszkiel and R. Singh. Counteracting "the tragedy of the commons" in an imperfect world. In *Journal of Public Economic Theory*, Vol. 26 (4), 2024.

[2] G.Fabbri, S. Faggian, and G. Freni. On competition for spatially distributed resources in networks. *Theoretical Economics*, 19(2):743–781, 2024.

87. Continuous Optimization: Theory and Applications

16:40 – 17:55

Chair: **R. Bot, Y. Malitsky, A. Daniilidis**

FH Nöbauer 8

A quasi-optimization problem for a family of functions.

Mario Diego Emanuele Giordano (1), Monica Milasi (2),

(1) MIFT, University of Messina, Messina, Italy; (2) Department of Economics, University of Messina, Messina, Italy;

We deal with a quasi-optimization problem: finding a common maximum point for a family of objective functions on a set-valued map. We reformulate the problem in terms of a suitable variational problem to give the existence of the solution. In the variational approach, the notion of quasiconcave family, introduced in [1] represents a key role. Finally, we apply our study to the Kantian equilibrium introduced in [2].

[1] F. Flores-Bazán, Y. García, and N. Hadjisavvas. Characterizing quasiconvexity of the pointwise infimum of a family of arbitrary translations of quasiconvex functions, with applications to sums and quasiconvex optimization. *Mathematical Programming*, **189**(1):315-337, 2021.

[2] J.E. Roemer. Kantian Equilibrium. *Scandinavian Journal of Economics*, **112**(1): 1-24, 2010.

An augmented lagrangian algorithm with inexact proximal method and its application to a mean-field control problem

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In this presentation, we will describe our recent results on the convergence of the Augmented Lagrangian Algorithm with inexact proximal method, as well as its application to the numerical approximation of a solution to a mean-field control problem. The Augmented Lagrangian is an algorithm for the minimization of a convex cost function which can be written as the sum of two convex functions f and g. Motivated by problems in which the direct minimization of $f+g$ is difficult but that of the sum of f with a regularization of g is simpler, the Augmented Lagrangian algorithm proceeds by iteratively minimizing the latter sum, modifying the regularization of g at each iteration. This regularization is obtained through a dual principle, so that our regularized problem is the dual problem associated to an augmented dual cost which penalizes the distance to a changing dual parameter $\bar{\mu}$. A more complete definition can be found for instance in [1]. Our contribution consists here in studying the behaviour of this algorithm when we cannot find an exact solution of the regularized problems. More precisely, we use the Frank–Wolfe Algorithm for the resolution of these problems. We show a sublinear convergence speed for this method, in terms of the number of calls to the oracle for the Frank–Wolfe Algorithm. As an application, we show that this algorithm can be used to find a numerical solution of a mean-field control problem. Our main idea consists in putting the contribution of the running cost and the Fokker–Planck equation in the function f, and the other interaction terms, which are possibly nonsmooth, in the function g. The oracle then consists in a numerical scheme for the coupled system of PDEs Hamilton–Jacobi–Bellman-Fokker–Plank. This talk is based on an ongoing work by the authors.

[1] Jorge Nocedal and Stephen J. Wright, *Numerical optimization*, Springer, 1999.

Optimal control under uncertaintis

Teresa Scarinci (1),

(1) University of Cassino and Southern Lazio, Cassino, Italy;

The study of models with uncertainty plays an important role in scientific numerical simulations. This class of problems is strongly utilized in engineering, biology, and finance. In this talk, we discuss the importance of including uncertainty in optimal control. Randomness can be utilised to model applications where the data of the problem – such as the dynamic, the coefficients, or the time delay – are not known a priori and one knows only statistical information.

88. Set-valued mappings in approximation, control and optimization 16:40 – 17:55

Chair: **R. Baier, E. Farkhi** **FH HS 6**

On ranges of set-valued mappings

Radek Cibulka (1), Tomáš Roubal (2),

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(2) Department of Decision-Making Theory, Institute of Information Theory and Automation CAS, Prague, Czechia;

In this lecture, we derive conditions that ensure the range of a set-valued mapping with a compact convex domain covers a prescribed set. We begin our discussion in Fréchet spaces, where we approximate the set-valued mapping by a single-valued mapping whose inverse features convex fibers. We then extend these ideas to Banach and finite-dimensional spaces by focusing on approximations determined by a convex set of bounded linear mappings, including tools such as the Páles–Zeidan Jacobian, Clarke's generalized Jacobian, shields by T.H. Sweetser, and Neumaier's interval extensions of the derivative of a smooth mapping. Moreover, we illustrate how these techniques yield, as immediate corollaries in Euclidean spaces, the perturbation stability of metric semiregularity under additive perturbations by a single-valued mapping with a sufficiently small calmness modulus, as well as extensions of the non-smooth Lyusternik–Graves and Robinson's theorems.

Metric differentiability and metric local linear approximants of set-valued functions

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The differential calculus of set-valued functions (SVFs, multifunctions) plays a significant role in various areas of applied mathematics. Over the past decades, different notions of set-valued derivatives have been introduced to address different theoretical and practical needs. Much of the existing research focuses on multifunctions with convex images. Several concepts of differentiability — such as contingent derivatives, graphical derivatives, co-derivatives — have been developed and studied for SVFs with general images.

These notions are used mainly for deriving optimality conditions in variational analysis and optimization rather than for approximation purposes. In our work we introduce a new notion of metric differentiability at a point for a univariate set-valued function F with general compact (not necessarily convex) sets in $\mathbb{R}^n$ as values. Extending the classical approach, we define the notion of metric divided differences of first order anchored at a point $y \in F(x_0)$ and introduce one-sided (left and right) metric derivatives anchored at y as the one-sided limits (left and right) of the corresponding metric divided differences. Based on this framework, we construct and study a local metric linear approximant for metrically differentiable SVFs and establish error estimates.

89. Control of dynamic systems and games with applications 16:40 – 17:55

Chair: **N. Hayek, S. Pickenhain** **FH HS 4**

On the small-time local controllability of a class of polynomial systems

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(2) Faculty of Mathematics and Informatics, Sofia University "St. KLiment Ohridski", Sofia, Bulgaria ;

Small-time local controllability (STLC) is one of the basic properties of a nonlinear control system. It is crucial for solving different problems of the mathematical control theory. We follow a general geometrical approach proposed by Hermann, Hermes, Kawski, Krener, Sussmann, and etc. It is based on using different differential-geometric tools as the classical formula of Campbell-Baker-Hausdorff formula (taken from the Lie group theory), different symmetries (related to the structure of the control values), and etc.
Our approach is based on a suitable definition of a set $E^+(x_0)$ of tangent vector fields to the reachable set of a control system at the starting point x_0. The basic idea is that if the origin belongs to the interior of the convex hull of $E^+(x_0)$ then the corresponding control system is STLC at the point x_0. We study carefully the Lie algebra of the vector fields generated by the drift term f of the control system Σ and the constant vector fields generated by the set $U \cap \bar{\mathbf{B}}$. We prove that some "bad Lie brackets" (in the sense of Sussman) belong to the set $E^+(0)$, and hence they are not obstructions for small-time local controllability. As a corollary we obtain a new sufficient condition for the small-time local controllability of the control system Σ at the origin.

[1] Krastanov, M. I., Nikolova, M. N., A sufficient condition for small-time local controllability of a polynomial control system, *C. R. Acad. Bulg. Sci.* , **73**: 1638–1649, 2020.

[2] Krastanov, M. I., Nikolova, M. N.. On the small-time local controllability *Systems & Control Letters.* **177**, 2023.

[3] H.Sussmann, A general theorem on local controllability, *SIAM J. Control Optim.*, **25**: 158–194, 1987.

Lagrange multipliers in dynamic optimization

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This talk is based on the following condition as well as on the Lagrange multiplier rule previously obtained by the authors: Let X and Y be Banach spaces, $\varphi : X \times Y \longrightarrow \cup\{+\infty\}$ be a lower semi-continuous function, $L : Y \longrightarrow X$ be a bounded linear operator and

$$S := \{(Ly, y) \in X \times Y \ : \ y \in Y\}$$

be its graph. **Definition** It is said that φ satisfies the **variational condition** at $(\bar{x}, \bar{y}, \varphi(\bar{x}, \bar{y}))$ with correcting set $U \subset X \times Y \times$ iff there exists a positive real $\bar{\delta} > 0$ such that for every $\lambda \in [0, \bar{\delta}]$ the following inclusion holds true:

$$\operatorname{epi} \varphi \cap \left((\bar{x}, \bar{y}, f(\bar{x}, \bar{y})) + \bar{\delta} \cdot \bar{B}_{X \times Y \times R}\right) + \lambda \left(\bar{B}_X, 0, 0\right) \subset \operatorname{epi} \varphi + \lambda U.$$

Consider the optimization problem

$$(OP) \qquad \varphi(x, y) \to \min \quad \text{subject to} \quad (x, y) \in \widetilde{S},$$

where $(S + (\bar{x}, \bar{y})) \times (-\infty, \varphi(\bar{x}, \bar{y})]$. **Theorem (Lagrange multiplier rule)** Let in the above setting $(\bar{x}, \bar{y})$ be a solution of the problem (OP), the functional φ have finite value at $(\bar{x}, \bar{y})$ and satisfy the variational condition with a correcting set U (having the appearance $U = (U_X, U_Y, U)$). Let U be bounded and $\mu\left(U_X - T(U_Y)\right) < 1$ where μ denotes the ball measure of noncompactness (in X). Then there exists a triple $(\xi, \mu, \zeta) \in X^* \times Y^* \times \mathbb{R}$ such that

(i) $(\xi, \mu, \zeta) \neq (\mathbf{0}, \mathbf{0}, 0)$;

(ii) $\zeta \in \{0, 1\}$;

(iii) $\langle \xi, u \rangle \ + \langle \mu, v \rangle \ = 0$ for every $(u, v) \in S$;

(iv) $\langle \xi, u \rangle \ + \langle \mu, v \rangle \ + \zeta w \geq 0$ for every $(u, v, w) \in \widehat{T}_{epi\ \varphi}(\bar{x}, \bar{y}, \varphi((\bar{x}, \bar{y}))$.

In this talk some applications of the above result are going to be presented. They include: • necessary optimality conditions for optimal control problems whose dynamics is governed by a differential inclusion with Lipschitz continuous right-hand side in the presence of pure state constraints of "inequality type"; • necessary optimality conditions for optimal control problems whose dynamics is governed by a differential inclusion with Lipschitz continuous right-hand side in the presence of "soft" state constraints.

On the Euler equation

Mikhail Krastanov (1,2), Nadezhda Ribarska (2,1),

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We consider the following basic problem of calculus of variations

$$\int_a^b L(x(t), \dot{x}(t))dt \to \min$$

under the pure state constraints

$$x(t) \in \Phi \text{ for all } t \in [a, b],$$

where Φ is a closed subset of R^n. Moreover, the following terminal constraints are imposed: $x(a) = x_a$ and $x(b) = x_b$, where x_a and x_b are two fixed points of Φ. It is well known that the necessary optimality

condition for this problem without state constraints and with smooth integrand is given by the classical Euler–Lagrange equation. To find a non trivial necessary optimality condition in the presence of pure state constraints is important and there is a huge amount of papers devoted to this problem. Most of them are dealing with "inequality type constraints" (usually this means that the Clarke tangent cones to Φ at the points of the optimal trajectory have non empty interior). Here, we are interested in the problem when the Clarke tangent cones to Φ at the points of the optimal trajectory may have empty interior. The typical case is when the set Φ is determined by "equality type constraints". To our knowledge, the first necessary condition of Pontryagin type for such kind of problems is obtained by Gamkrelidze (1960). The aim of this talk is to present a version of the Euler–Lagrange equation in the presence of pure state constraints of equality type in a non smooth setting under suitable assumptions. These assumptions allow treating not only the problems, where Φ is a smooth manifold and the integrand is Lipschitz continuous, but also some problems, where Φ has singular points. Moreover, we obtain a generalization of the classical DuBois-Reymond lemma in the presence of pure state constraints. It sheds some light on the considered problem even in the case when the constraint is a smooth manifold.

90. Learning methods in optimal and predictive control

16:40 – 17:55

Chair: **L. Grüne, K. Worthmann**

FH HS 5

Tensor-Train-Based Semi-Lagrangian Schemes for High-Dimensional Mean Field Games

Elisabetta Carlini (1), Luca Saluzzi (2),

(1,2) Sapienza, University of Rome, Rome, Italy;

Mean field games (MFGs) provide a powerful framework for modeling decision-making in large populations of interacting agents. However, solving high-dimensional MFG problems remains a significant computational challenge due to the curse of dimensionality. In this talk, we present a novel approach based on combining Semi-Lagrangian (SL) schemes with tensor-train (TT) decompositions to efficiently approximate solutions of high-dimensional MFGs. Semi-Lagrangian (SL) schemes allow us to derive a semi discrete in time approximations of the Hamilton-Jacobi-Bellman (HJB) and Fokker-Planck-Kolmogorov (FPK) equations, inherent in MFG models. By combining TT representations with SL discretizations, we achieve a scalable and memory-efficient numerical scheme that mitigates the exponential growth in computational complexity. We discuss theoretical properties of the method, numerical accuracy, and demonstrate its effectiveness through computational experiments on high-dimensional test cases.

[1] S. Dolgov, D. Kalise, and L. Saluzzi. Data-driven Tensor Train Gradient Cross Approximation for Hamilton-Jacobi-Bellman Equations. *SIAM Journal on Scientific Computing*, **45**(5):A2153-A2184, 2023.

Online convex optimization for constrained control of nonlinear systems

Marko Nonhoff (1), Johannes Köhler (2), Matthias A. Müller (1),

(1) Leibniz University Hannover, Institute of Automatic Control, Hannover, Germany; (2) ETH Zürich, Institute for Dynamic Systems and Control, Zürich, Switzerland;

In this talk, we study the problem of controlling nonlinear systems subject to state and input constraints while minimizing time-varying and a priori unknown cost functions. We propose a modular approach that

combines the online convex optimization (OCO) framework and reference governors to solve this problem [1]. The presented method is general in the sense that we do not limit our analysis to a specific choice of OCO algorithm or reference governor. Instead, we provide design requirements for each of the components of the proposed OCO-RG framework, all of which are satisfied by various popular methods in the literature and, hence, are not restrictive. We derive performance guarantees for the closed loop by showing that the dynamic regret of the proposed framework (i.e., the difference between the closed-loop cost and the optimal steady-state cost) is bounded linearly in both the dynamic regret and the path length of the chosen OCO algorithm, even though the OCO algorithm by itself does not account for the underlying dynamical system. We prove that a linear bound with respect to the OCO algorithm's dynamic regret is optimal, i.e., cannot be improved upon. Finally, we demonstrate the modularity and flexibility of the proposed framework on a numerical simulation example.

[1] M. Nonhoff, J. Köhler, M. A. Müller. Online convex optimization for constrained control of nonlinear systems. *Submitted to Automatica*, preprint available online at arXiv:2412.00922, 2024.

Supervised Learning for Hamilton-Jacobi-Bellman PDEs using High-Order Information

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We propose a data-driven framework to approximate solutions of Hamilton-Jacobi-Bellman (HJB) equations in optimal control problems using second-order information. By exploiting the link between the Pontryagin Maximum Principle (PMP) and the Dynamic Programming Principle (DPP), we generate augmented datasets that include function values, gradients, and Hessians of the value function. Sparse polynomial regression with hyperbolic cross bases and regularization techniques enables accurate approximation of value functions and optimal feedback laws. We validate the approach through numerical experiments on problems of different nature and dimensionality, highlighting its performance and scalability.

[1] B. Adcock and Y. Sui. Compressive Hermite interpolation: Sparse, high-dimensional approximation from gradient-augmented measurements. *Constructive Approximation*, **50**(1):167–207, Aug 2019.

[2] B. Azmi, D. Kalise, and K. Kunisch. Optimal feedback law recovery by gradient-augmented sparse polynomial regression. *Journal of Machine Learning Research*, **22**(48):1–32, 2021.

[3] W. M. Czarnecki, S. Osindero, M. Jaderberg, G. Swirszcz, and R. Pascanu. Sobolev training for neural networks. In *Advances in Neural Information Processing Systems*, vol. 30, 2017.

[4] T. Nakamura-Zimmerer, Q. Gong, and W. Kang. Adaptive deep learning for high-dimensional Hamilton-Jacobi-Bellman equations. *SIAM Journal on Scientific Computing*, **43**(2):A1221–A1247, 2021.

Koopman-based Parameter Estimation for an Inverter-controlled Induction Machine

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The presented work focuses on an application of the usage of advanced control-techniques such as Model Predictive Control, Moving Horizon Estimation and Koopman-Estimation in the context of electrified

aviation. For this, we propose the usage of the Koopman-framework as a parameter estimation technique in a three phase, variable-speed induction motor drive connected to a three level inverter controlled by Model Predictive Control. The Koopman estimation technique proposed is constructed over a large set of data obtained through simulations using moving horizon estimation. An extended library is used to upgrade the dimensionality. Moreover, a dynamic mode decomposition is used to solve the optimisation problem by means of a singular value decomposition.

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[2] Yan, Mingxue and Han, Minghao and Law, Adrian Wing-Keung and Yin, Xunyuan. Self-tuning moving horizon estimation of nonlinear systems via physics-informed machine learning Koopman modeling. *AIChE Journal*, **Wiley Online Library**, vol. 71, no. 2 p. e18649, 2025.

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91. Dynamic management of natural resources

16:40 – 17:55

Chair: **T. Upmann, D. Gromov**

FH HS 7

Approximate optimal control of arid grazing systems

Faan Langelaan (1), Florian Wagener (1),

(1) CeNDEF, Amsterdam School of Economics, University of Amsterdam;

We apply optimal control to the management of arid grazing ecosystems, which are modeled by the dynamics of grassy vegetation and grazing herbivores. These systems generally exhibit bi-stability between a healthy vegetated state and a desert state. We show that, under suitable economic parameters, optimal control of herd size through extraction allows managers to pursue their economic incentives without causing desertification. The resulting two-state optimal control problem is solved numerically using Pontryagin's Maximum Principle, though interpreting the solution can be challenging. As an alternative, we propose linearizing the entire four-dimensional state-costate system around the steady state. This approach yields an easily interpretable linear feedback rule for the control, based on the two state variables. We show that this rule closely approximates the actual optimal control and converges to it asymptotically near the steady state. Additionally, this technique makes it easier to analyze the direct effects of various parameter changes on the control, making it a useful tool for policymakers. We also demonstrate that this approximation outperforms the approach of first linearizing the nonlinear two-dimensional system around the steady state and then applying control theory.

Pollution control under uncertainty: Integrating optimal control theory and value of information

Dmitry Gromov (1,4), Prateek Verma (1,2,3), Amelie Luhede (1,2,3), Thorsten Upmann (5,2,3),

(1) Carl von Ossietzky Univ. Oldenburg, Inst. for Chemistry and Biology of Marine Environments, Germany; (2) Helmholtz-Institute for Functional Marine Biodiversity, Germany; (3) Bielefeld University, Faculty of Business Administration and Economics, Germany; (4) Dept. Mathematics, University of Latvia, Latvia; (5) Fraunhofer Institute for Ceramic Technologies and Systems, Germany;

This contribution is devoted to evaluating the Value of Information (VoI) in a dynamic context using optimal control theory. Our analysis is based upon the classical pollution control model with a linear cost function. We consider uncertainty in both the initial state and the model parameter—the absorption coefficient. The results indicate that the VoI is strongly depends on the structure of uncertainty. In particular, when the absorption rate is uncertain, the VoI is found to be proportional to the variance of the parameter. Additionally, we compare two probabilistic approaches to handling uncertainty: a proper solution, based upon the method of optimal ensemble control, and a naive one. While the naive approach may appear a convenient shortcut for dealing with parameter uncertainty, we demonstrate that it is not only generally suboptimal but also leads to increased pollution emission.

Fuzzy set-based evaluation of the value of information in ecological applications

Martiņš Zemlītis (1), Svetlana Asmuss (1), Dmitry Gromov (1,2), Olga Grigorenko (1),

(1) Department of Mathematics, University of Latvia, Rīga, Latvia; (2) Carl von Ossietzky Universität Oldenburg, Germany;

In many real-world decision-making scenarios, achieving the optimal outcome is conditioned upon determining the values of key parameters. However, as these parameters are typically uncertain, the decision-makers have to undertake additional investigations, which incur associated costs. This assessment of the potential benefits of acquiring more information is formalized through the Value of Information (VoI), which quantifies the advantage of obtaining further insights before making a decision. Although VoI has gained increasing popularity (see, e.g., [1,2]), its applicability can be severely restricted in situations where modeling the problem probabilistically is difficult or infeasible, such as when the probability distribution of the variable of interest is unknown or cannot be reliably estimated. To overcome this difficulty, we propose an alternative approach that involves fuzzy sets to handle uncertainty. The suggested approach is applied to the problem of water quality monitoring. This research is funded by the Latvian Council of Science, project No. lzp-2024/1-0188 and the Deutsche Forschungsgemeinschaft (DFG), project No. 448981368.

[1] J. Eidsvik, T. Mukerji, D. Bhattacharjya, *Value of Information in the Earth Sciences: Integrating Spatial Modeling and Decision Analysis*, Cambridge Univ. Press, 2015.

[2] A. Luhede, H. Yaqine, R. Bahmanbijari, M. Römer, T. Upmann, The value of information in water quality monitoring and management, *Ecological Economics* **219**:108128, 2024.

92. Continuous Optimization: Theory and Applications 08:30 – 09:45

Chair: R. Bot, Y. Malitsky, A. Daniilidis FH Nöbauer 8

Asymptotic behavior of penalty dynamics for constrained variational inequalities

Juan Peypouquet (1), Mathias Staudigl (2), Siqi Qu (2),

(1) Rijksuniversiteit Groningen, Faculty of Science and Engineering, Systems, Control and Optimization — Bernoulli Institute, Groningen, The Netherlands; (2) Mannheim University, Department of Mathematics, B6 26, 68159 Mannheim, Germany;

This paper is concerned with the monotone inclusion problem

$$0 \in \Phi(x) = (x) + (x) + (x), \tag{P}$$

where :→ 2 is a maximally monotone operator on a real Hilbert space , :→ is monotone and $\frac{1}{\eta}$-Lipschitz, and = () ≠ ∅ is the set of zeroes of a μ-cocoercive operator :→. This is a three-operator formulation of a general class of variational problems, where a constrained equilibrium of the sum of two maximally monotone operators + is requested over a domain , which admits a representation of the set of zeroes of another single-valued monotone operator . This abstract formulation has many applications in optimal control and optimization, in particular those of a hierarchical nature. Associated with problem (P), we study dynamical systems for solving auxiliary problems governed by the inclusion

$$0 \in \Phi_{\epsilon,\beta}(x) = (+ + \epsilon\mathrm{Id} + \beta)(x).$$

Depending on whether is cocoercive or not, we establish strong convergence results of the trajectory to the least norm solution of the original problem (P) by leveraging a technical tracking argument using Lyapunov ideas. We then move by discussing extension of our basic method to the setting with multiple penalty terms and inertial effects. This paper is based on

[1] Staudigl, M., & Qu, S. (2024). Tikhonov regularized exterior penalty dynamics for constrained variational inequalities. IEEE Control Systems Letters.

[2] Qu, S., Staudigl, M., & Peypouquet, J. (2025). Asymptotic behavior of penalty dynamics for constrained variational inequalities. arXiv preprint arXiv:2503.03902 (submitted for publication)

On a stochastic differential equation with correction term governed by a monotone and Lipschitz continuous operator

Radu Ioan Bot (1), Chiara Schindler (1),

(1) Faculty of Mathematics, University of Vienna, Vienna, Austria;

In the pursuit of finding a zero for a monotone and Lipschitz continuous operator $M : \mathbb{R}^n \to \mathbb{R}^n$ amidst noisy evaluations, we explore an associated differential equation within a stochastic framework, incorporating a correction term. We present a result establishing the existence and uniqueness of solutions for the stochastic differential equations under examination. Additionally, assuming that the diffusion term is square-integrable, we demonstrate the almost sure convergence of the trajectory process $X(t)$ to a zero of M and of $\|M(X(t))\|$ to 0 as $t \to +\infty$. Furthermore, we provide ergodic convergence rates in expectation for $\|M(X(t))\|^2$ and $\langle M(X(t)), X(t) - x^* \rangle$, where x^* is an arbitrary zero of the monotone

operator. Subsequently, we apply these findings to a minimax problem. Finally, we analyze two temporal discretizations of the continuous-time models, resulting in stochastic variants of the Optimistic Gradient Descent Ascent and Extragradient methods, respectively, and assess their convergence properties.

A smoothing implicit gradient algorithm for optimization with parametric variational inequality constraints on a moving polyhedron

Yixuan Zhang (1), Xiaojun Chen (1), Jin Zhang (2),

(1) Department of Applied Mathematics, The Hong Kong Polytechnic University, Kowloon, Hong Kong, China; (2) Department of Mathematics, Southern University of Science and Technology, Shenzhen, Guangdong, China;

This work introduces a Smoothing Implicit Gradient Algorithm with Inexactness (SIGAI) to address optimization problems constrained by Parametric Variational Inequalities (PVI) defined on a moving polyhedron. Unlike prior work limited to fixed feasible sets, SIGAI handles moving constraints where the feasible region evolves with parameters. Key innovations of the proposed SIGAI framework include: (i) a smoothing approximation of the projection operator with a vanishing smoothing parameter; (ii) inexact sub-problem solutions requiring only finite iterations per sub-problem; and (iii) convergence guarantees to stationary points and convergence rate characterization. Numerical experiments validate the algorithm's convergence and efficiency, with applications to real-world portfolio management problems.

93. Stochastic Control and Applications

08:30 – 09:45

Chair: **A. Calvia, E. Bandini**

FH HS 5

Dam management in the era of climate change

Cristina Di Girolami (1), M'hamed Gaïgi (2), Vathana Ly Vath (3), Simone Scotti (4),

(1) Dipartimento di Matematica, Università Alma Mater Studiorum Bologna, Italy; (2) Université de Tunis El Manar, Ecole Nationale d'Ingénieurs de Tunis, Tunisia; (3) ENSIIE, Laboratoire de Mathématiques et Modélisation d'Évry, Université Paris-Saclay, France; (4) Dipartimento di Economia e Management, Università di Pisa, Italy;

Climate change has a dramatic impact, particularly by concentrating rainfall into a few short periods, interspersed by long dry spells. In this context, the role of dams is crucial. We consider the optimal control of a dam, where the water level must not exceed a designated safety threshold, nor fall below a minimum level to ensure functionality and sustainability for for the outgoing river. To model dry spells and intense rainfall events, commonly referred to as water bombs, we introduce a Hawkes process, a well-known example of a self-exciting process characterised by time-correlated intensity, which endogenously reproduces the concentration of events. The problem is formulated as an optimal switching problem with constraints. We establish existence results and propose numerical methods for approximating the solution. Finally, we illustrate the main achievements of this approach through numerical examples. The main and counterintuitive result of our numerical analysis is that the optimal water level inside the dam increases with the self-exciting parameter. This result shows that, when facing the dilemma of managing the opposing risks of dam overtopping and dry spells, the former ultimately dominates the latter. In conclusion, dams will increasingly lose their role as water reserves and take on a greater role in flood protection.

Mean viability theorems and second-order Hamilton–Jacobi equations

Christian Keller (1),

(1) Department of Mathematics, University of Central Florida, Orlando, Florida, United States;

We introduce the notion of mean viability for controlled stochastic differential equations and establish counterparts of Nagumo's classical viability theorems (necessary and sufficient conditions for mean viability). As an application, we provide a purely probabilistic proof of a comparison principle and of existence for contingent and viscosity solutions of second-order fully nonlinear path-dependent Hamilton–Jacobi–Bellman equations. We do not use compactness and optimal stopping arguments, which are usually employed in the literature on viscosity solutions for second-order path-dependent PDEs.

Optimal Stopping of Branching Diffusion Processes

Idris Kharroubi (1), Antonio Ocello (2),

(1) LPSM, Sorbonne Université, Paris, France; (2) CMAP, Ecole Polytechnique, Palaiseau, France;

This talk presents the analysis of an optimal stopping problem for branching diffusion processes. It consists in looking for optimal stopping lines, a type of stopping time that maintains the branching structure of the processes under analysis. By using a dynamic programming approach, we characterize the value function for a multiplicative cost, which may depend on the particle's label. We reduce the problem's dimensionality by setting a branching property and defining the problem in a finite-dimensional context. Within this framework, we focus on the value function, establishing uniform continuity and boundedness properties, together with an innovative dynamic programming principle. This outcome leads to an analytical characterization with the help of a nonlinear elliptic PDE. In the case of a regular value function, we provide an optimal stopping rule involving the elliptic PDE. We conclude by showing that the value function serves as the unique viscosity solution for this PDE, generalizing the comparison principle to this setting.

94. Industry Dynamics

08:30 – 09:45

Chair: **H. Dawid**

FH HS 7

The Signorini problem - Comparison of methods for solving real problems from industry

Lukas Kapera (1), Petr Beremlijski (1),

(1) VSB - Technical University of Ostrava, FEECS, Department of Applied Mathematics;

The Signorini problem is an elastostatics problem in linear elasticity. It consists of finding the elastic equilibrium configuration of an elastic body, which is by part of its boundary in possible contact with a rigid, frictionless obstacle. At all points along this part of the boundary, it is not a priori known which boundary condition applies - Dirichlet or homogeneous Neumann. However, at each point, exactly one of them applies, which is crucial for the subsequent solution of the task. The statement of the problem involves not only equalities but also inequalities; therefore, inequality-constrained optimization methods need to be used to solve the problem.Many numerical methods proposed to solve this problem are, unfortunately,

somewhat academic, and their application to real industrial problems is quite problematic. For example, since the condition number of the resulting system of linear equations is roughly an order of magnitude worse than Young's modulus, the Young's modulus used in academic examples is often extremely low to make the system of linear equations easier to solve. Furthermore, symmetric, orthogonal, and mainly simple geometries are frequently used, making the solution almost easy to guess in advance. As a result, such methods are often unsuitable for real-world industrial problems.In our contribution, we model Signorini problem boundaries using Bézier curves. We focus on convex quadratic programming with linear and box constraints, namely on the Active-set method, interior-point methods, and the Gradient projection method, which are directly intended for solving inequality-constrained optimization problems. Then, we compare the solution of the Signorini problem using these inequality-constrained optimization methods together with the classical BFGS method for the objective function with quadratic penalty.

Numerical solution of optimal control problems using quadratic transport regularization

Nicolas Borchard (1), Gerd Wachsmuth (1),

(1) Optimal Control Group, BTU Cottbus-Senftenberg, Cottbus, Germany;

We address optimal control problems on the space of measures for an objective containing a smooth functional and an optimal transport regularization. That is, the quadratic Monge-Kantorovich distance between a given prior measure and the control is penalized in the objective. We consider optimality conditions and reparametrize the problem using the celebrated structure theorem by Brenier. The optimality conditions can be formulated as a piecewise differentiable equation. This is utilized to formulate solution algorithms and to analyze their local convergence properties. We present a numerical example to illustrate the theoretical findings. This talk is based on the following preprint:

[1] N.Borchard and G. Wachsmuth. Numerical solution of optimal control problems using quadratic transport regularization, https://arxiv.org/abs/2503.07105, 2025.

Trend and buybacks. Model of behavioral price expectations with endogenous firm productivity.

Herbert Dawid (1), Tomasz Makarewicz (1), Philipp Harting (2),

(1) Bielefeld Universität, Bielefeld, Germany; (2) Université Cote d'Azur, Nice, France;

The literature on behavioral expectations demonstrates that learning often leads to violation of the Efficient Market Hypothesis, trend-following behavior and non-equilibrium asset prices [3]. However, these studies typically focus on forecasting and thus assume exogenous market fundamentals. An alternative approach was proposed by [2], who study a Bertrand market in which managers need to balance the trade-off between stock buyback, dividend stream and firm's productivity. The aim of this research is to combine these two threads of literature and study the interaction between the real and financial sectors in the presence of buybacks and chartist financial investors. We take the framework of Dawid et al. (2019) for the real side of the model. Following [1], boundedly rational traders learn how to forecast the prices of the firms' stocks through the social learning variant of the Genetic Algorithm, independently for each firm. Depending on the version of the model, they can learn how to respond to the announced buybacks, the past observed price trend, or both of these variables.

[1] M. Anufriev, C. Hommes, and T. Makarewicz. Simple forecasting heuristics that make us smart: Evidence from different market experiments. *Journal of the European Economic Association*, **17**(5):1538-1584, 2019.

[2] H. Dawid, P. Harting, and S. van der Hoog. Manager remuneration, share buybacks, and firm performance. *Industrial and Corporate Change*, **28**(3):681-706, 2019.

[3] C. Hommes. The heterogeneous expectations hypothesis: Some evidence from the lab. *Journal of Economic Dynamics and Control*, **35**(1):1-24, 2011.

95. Continuous Optimization: Theory and Applications

10:05 – 11:20

Chair: **R. Bot, Y. Malitsky, A. Daniilidis**

FH Nöbauer 8

Variational principles for monotone variational inequalities: the multivalued case

Pando Georgiev (1),

(1) Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Sofia, Bulgaria;

We consider a parameterized variational inequality (A, Y) in a Banach space E defined on a closed, convex and bounded subset Y of E by a multivalued maximal monotone operator A depending on a parameter. We prove that under suitable conditions, there exists an arbitrarily small maximal monotone perturbation of A such that the perturbed variational inequality has a solution which is a continuous function of the parameter, and is near to a given approximate solution. In the nonparametric case this can be considered as a variational principle for variational inequalities, an analogue of the Borwein-Preiss smooth variational principle [1]. Some applications are given: an analogue of the Nash equilibrium problem, defined by a partially monotone operator, and a variant of the parametric Borwein-Preiss variational principle [3] for parametrized convex functions under relaxed assumtions, which extends the parametric Ekeland's variational principle [2]. The tool for proving the main result is a useful lemma about existence of continuous ε-solutions of a multivalued variational inequality depending on a parameter. It has an independent interest and allows a direct proof of an analogue of Ky Fan's inequality for multivalued maximal monotone operators, introduced here, which leads to a new proof of the Kakutani fixed point theorem in arbitrary Banach spaces. This work extends the results from [4].

[1] J.M. Borwein, D. Preiss: A smooth variational principle with applications to subdifferentiability and to differentiability of convex functions, *Trans. Amer. Math. Soc.* **303**, 517-527, 1987.

[2] P.G. Georgiev, Parametric Ekeland's Variational Principle, *Appl. Math. Lett.* **14**(6),691–696, 2001.

[3] P.G. Georgiev, Parametric Borwein–Preiss variational principle and applications, *Proc. Amer. Math. Soc.* **133**(11), 3211–3225, 2005.

[4] P.G. Georgiev, Variational Principles for Monotone Variational Inequalities: the Single-Valued Case, *Serdica Math. J.* **49**, 77–96, 2023.

From Frank-Wolfe to DCA and beyond

Alp Yurtsever (1), Suvrit Sra (2),

(1) Umeå University, Sweden; (2) Technical University of Munich, Germany;

This work uncovers a simple but rather surprising connection: the classical Difference-of-Convex Algorithm (DCA) can be viewed as a special case of the Frank-Wolfe (FW) method. This connection not only provides insight of pedagogical value, but also enables the direct transfer of non-asymptotic convergence guarantees from FW to DCA. Building on this connection, we show that a broad class of optimization algorithms, such as the Proximal Point Method, Mirror Prox, and Expectation-Maximization, can also be interpreted within the FW framework. This viewpoint enables a unified analysis of seemingly distinct iterative optimization methods.

First and second order algorithms for bilevel optimization

Radu Ioan Bot (1), Enis Chenchene (1), Ërno Robert Csetnek (1), David Alexander Hulett (1),

(1) Faculty of Mathematics, University of Vienna, Vienna, Austria;

In a real Hilbert space, we address a convex bilevel optimization problem, where both the inner and outer levels have a composite smooth+nonsmooth structure. We study two algorithms, which at each step execute a proximal gradient step on a dynamically regularized objective function: one without momentum, in the spirit of the Bi-SG algorithm introduced by Merchav and Sabach in 2023, and one with momentum, in the spirit of the FBi-PG algorithm introduced by Merchav, Sabach and Teboulle in 2024. Attached to these algorithms we formulate a family of Lyapunov energy functionals which satisfy a certain descent property, and which allow us, in a streamlined way, to show convergence rates statements which match or surpass those already present in the literature. When the inner function satisfies a Hölderian error bound, we achieve small o rates for the inner and outer functional values along the last iterate, which is a first for this class of problems; additionally, the iterates converge weakly to a solution of the bilevel problem. These algorithms can be seen as forward discretizations of their continuous-time counterparts, which are respectively a first- and a second-order differential equation. The simpler, continuous-time Lyapunov analysis provides a guide and suggests the inequalities which should be reached in the discrete-time case.

96. Feedback control and stabilization of nonlinear PDEs 10:30 – 11:20

Chair: **S. S. Rodrigues** **FH HS 6**

Newton-type methods for detecting robotic arm position using camera

Michaela Bailová (1), Petr Beremlijski (1),

(1) Department of Applied Mathematics, FEECS, VSB-Technical University of Ostrava, Czechia;

The contribution presents a digital twin of an industrial camera for calibrating robotic manipulators in the manufacturing process. A full camera model (for more details, see [1]) was used to approximate the camera's characteristics, which allows for high accuracy. The aim is to estimate a robotic arm pose based on the captured image of a calibration pattern, framing it as an unconstrained quadratic programming problem.

The mapping $F : \mathbb{R}^6 \to \mathbb{R}^{10}$ describes the function assigning the camera's position and orientation to the image on the display. The vector $\overline{F}$ contains the prescribed coordinates of the five significant points of the specific geometric pattern.

The function being optimized is highly nonlinear and has various local minima. To create a method suitable for industrial applications, finding a global minimum in real time is also necessary. Therefore, a hybrid algorithm is introduced, combining the fast local convergence of Newton's method with the robustness of metaheuristic methods like differential evolution. Alternatively, a small neural network can approximate the inversion of F, providing an initial guess for Newton's method. These approaches offer significant reliability and speed. The presented approach is then compared with standard derivative-based and metaheuristic methods and a neural network.

[1] J. Konecny, P. Beremlijski, M. Bailova, Z. Machacek, and J. Koziorek. Industrial camera model positioned on an effector for automated tool center point calibration. *Scientific Reports*, **14**(1), 2024.

Optimal control of epidemic models with age and space structure

Behzad Azmi (1), Nicolas Schlosser (1),

(1) Department of Mathematics and Statistics, Universität Konstanz, Konstanz, Germany;

Epidemic modeling has long been an important field of research, and the recent Covid-19 pandemic has shown its importance to everyday life. Often, in order to increase accuracy and applicability of the models, a PDE approach is used, thus allowing for modeling spread of infections over countries or accounting for age-depend model parameters such as mortality, infectivity, etc. In this talk, we study a class of nonlinear epidemic models with age and space structure and nonlocal infection terms and introduce an additional control parameter interpreted as a vaccination rate. The main obstacle of our model is that due to its structure, it prevents the use of classical compactness theorems. We show under which conditions the existence of an optimal control can be guaranteed nontheless, and derive first-order optimality conditions for the problem. To conclude, we present numerical simulations of the model and discuss their behaviour.

Certifed reduced-order methods for model predictive control of evolution processes

Behzad Azmi (1), Michael Kartmann (1), Mattia Manucci (2), Jan Rohleff (1), Benjamin Unger (2), Stefan Volkwein (1

(1) University of Konstanz, Konstanz, Germany; (2) University of Stuttgart, Stuttgart, Germany;

In this talk model predictive control (MPC) is utilized to stabilize a class of evolution problems. In our examples, finite-dimensional control inputs and nonsmooth cost functionals are studied. Among stabilizability, the application of reduced-order models (ROM) to derive algorithms with closed-loop guarantees is discussed. Utilizing a-posteriori error estimates adaptive MPC-ROM algorithms are designed and numerically tested.

97. Topology optimization: theory, numerics and applications 10:05 – 11:20

Chair: K. Sturm, P. Gangl **FH HS 4**

Stabilized schemes for phase field modelling in topology optimization

Jiajie Li (1), Hui Yang (2), Shengfeng Zhu (2),

(1) School of Mathematical Sciences, Shanghai Jiao Tong University, Shanghai, China; (2) School of Mathematical Sciences, East China Normal University, Shanghai, China;

We consider topology optimization in nonlinear structural design and fluid flows using a phase field model. Semi-implicit schemes with generalized stabilization for the Allen-Cahn and Cahn-Hilliard types of gradient flows are proposed for solving the topology optimization model problems in geometrically nonlinear elasticity, Navier-Stokes flows, and thermal-fluid multi-physical coupling. Unconditional energy stabilities are shown for the stabilized schemes of the gradient flows in both continuous and discrete spaces. Numerical examples are presented to show the effectiveness and robustness of the optimization algorithms proposed.

[1]J. Li, H. Yang, and S. Zhu. Topology optimization of thermal-fluid coupling by a phase field method, *revision submitted,* 2025.

[2]J. Li, H. Yang, and S. Zhu. An unconditionally energy stable gradient flow for phase field modelling of structural topology optimization in geometrically nonlinear elasticity, *submitted,* 2025.

[3]J. Li and S. Zhu. Energy stable gradient flow schemes for shape and topology optimization in Navier-Stokes flows, *preprint arXiv:2405.05098, submitted,* 2024.

Approximation of shape tensors and its relation to shape gradients

Laura Hetzel (1), Gerhard Starke (1),

(1) Fakultät für Mathematik, Universität Duisburg-Essen, Germany;

The approximation of the shape tensor, introduced by Laurain and Sturm [3], in the L^p sense and is related to the computation of shape gradients in L^{p^*} $(1/p+1/p^* = 1)$. Subject to the constaint of symmetry this is shown in [2]. The limit $p \to 1$ provides an approach, different from the one by Deckelnick, Herbert and Hinze in [1], towards shape gradients in $W^{1,\infty}$. From the numerical realization using appropriate finite element spaces (of Raviart-Thomas type), shape gradient approximations can be obtained. For a number of shape optimization examples, the viability of our approach is shown. Finally, we discuss the implications on the topological derivative.

[1] K. Deckelnick, P. J. Herbert, and M. Hinze. A novel $W^{1,\infty}$ approach to shape optimisation with Lipschitz domains. *ESAIM Control Optim. Calc. Var.*, **28**:2, 2022.

[2] L. Hetzel, and G. Starke. Constrained L^p approximation of shape tensors and its role for the determination of shape gradients. *Mathematical Control and Related Fields*, `doi:10.3934/mcrf.2025019`, 2025.

[3] A. Laurain, and K. Sturm. Distributed shape derivative via averaged adjoint method and applications. *ESAIM Math. Model. Numer. Anal.*, **50**:1241-1267, 2016.

Inverse problems for Voronoi tessellations

Ernesto G. Birgin (1), Antoine Laurain (2), Danilo R. Souza (1),

(1) Institute of Mathematics and Statistics, University of São Paulo, Brazil; (2) Faculty of Mathematics, University of Duisburg-Essen, Germany;

We propose and analyze a numerical method for the inverse problem of recovering a piecewise constant coefficient with multiple phases. The unknown coefficient is either the potential or the scalar conductivity coefficient of a second-order elliptic equation. The phases are modeled using a Voronoi tessellation (Voronoi diagram) generated by a set of sites, which serve as control parameters. We first reformulate the inverse problem as an optimization problem with respect to the positions of the sites. By combining techniques from non-smooth shape calculus and the sensitivity analysis of Voronoi diagrams, we derive the gradient of the cost functional under standard non-degeneracy assumptions on the diagram. Singularities in the PDE solution arise near the diagram's vertices, which need to be carefully analyzed to prove the shape differentiability. We present two different formulas for the gradient, a distributed and an interface expressions, which are compared in numerical experiments. We provide several numerical experiments to investigate the dependence of the reconstruction on the problem parameters, such as noise, number of sites and initialization.

98. Stochastic Control and Applications

10:05 – 11:20

Chair: **A. Calvia, E. Bandini**

FH HS 5

An optimal advertising model with carryover effect and mean field terms

Fausto Gozzi (1), Federica Masiero (2), Rosestolato Rosestolato (3),

(1) Dipartimento di , Università LUISS - Guido Carli, Roma, Italy; (2) Dipartimento di Matematica e Applicazioni, Università di Milano Bicocca, Milano, Italy; (3) Dipartimento di Economia, Università di Genova, Genova, Italy;

We consider a class of optimal advertising problems under uncertainty for the introduction of a new product into the market, on the line of the seminal papers of Vidale and Wolfe, 1957 and Nerlove and Arrow, 1962 The main features of our model are that, on one side, we assume a carryover effect (i.e. the advertisement spending affects the goodwill with some delay); on the other side we introduce, in the state equation and in the objective, some mean field terms that take into account the presence of other agents. We take the point of view of a planner who optimizes the average profit of all agents, hence we fall into the family of the so-called "Mean Field Control" problems. The simultaneous presence of the carryover effect makes the problem infinite dimensional. Here we consider, as a first step, a simple version of the problem providing the solutions in a simple case through a suitable auxiliary problem.

Regularization in mean field control via infinite-dimensional common noise

François Delarue (1), Mattia Martini (1), Giacomo E. Sodini (2),

(1) Université Côte d'Azur, Nice, France; (2) University of Vienna, Vienna, Austria;

The aim of this talk is to present some regularizing effects of an infinite-dimensional common noise on mean-field control models. It is known that the usual finite-dimensional common noise is often insufficient to enhance well-posedness in mean-field systems with non-regular data. In contrast, an infinite-dimensional common noise is expected to enforce the uniqueness of solutions. However, constructing such a forcing requires the introduction of a diffusion process that takes values in the space of probability measures. In this work, we examine the impact of a Dirichlet-Ferguson type noise (see [1]). We first discuss a system of non-controlled interacting particles and the associated backward Kolmogorov equation on the space of probability measures. We then move on to a mean-field control problem and analyze the corresponding second-order Hamilton-Jacobi-Bellman equation. This presentation is based on joint work with François Delarue and Giacomo Sodini.

[1] L. Dello Schiavo. The Dirichlet–Ferguson diffusion on the space of probability measures over a closed Riemannian manifold, *The Annals of Probability*, 50(2), 591-648, 2022.

Mean-field control of non exchangeable systems

Anna De Crescenzo (1), Filippo de Feo (2), Marco Fuhrman (3), Idris Kharroubi (4), Huyên Pham (5),

(1) LPSM, Université Paris Cité; (2) Technische Universität Berlin; (3) Università degli Studi di Milano; (4) LPSM, Sorbonne Université; (5) CMAP, Ecole Polytechnique;

We study the optimal control of mean-field systems with heterogeneous and asymmetric interactions. We consider a family of controlled Brownian diffusion processes with dynamics depending on the whole collection of marginal probability laws. We prove the well-posedness of such systems and define the control problem together with its related value function. Leveraging tools tailored for this framework, such as derivatives along flows of measures and associated Itô calculus, we establish that the value function for this control problem satisfies a Bellman dynamic programming equation in a L^2-set of Wasserstein space-valued functions. To illustrate the applicability of our framework, we present a linear-quadratic graphon model with analytical solutions, and apply it to a systemic risk example involving heterogeneous banks.

[1] A. De Crescenzo, M. Fuhrman, I. Kharroubi, H. Pham. Mean field control of non exchangeable systems, *arXiv:2407.18635*

[2] A. De Crescenzo, F. de Feo, H. Pham: Linear-quadratic optimal control problems for non-exchangeable mean-field SDEs, *arXiv:2503.03318*

99. Infinite dimensional optimal control and differential games in Economics

10:05 – 10:55

***Chair*: A. Zanco** **FH HS 7**

Stochastic internal habit formation and optimality

M. Aleandri (1), F. Gozzi (1), A. Bondi (1),

(1) LUISS University, DEF, Roma, Italy;

Growth models with internal habit formation have been studied in various settings under the assumption of deterministic dynamics. The purpose of this paper is to explore a stochastic version of the model in [2,3], one the most influential on the subject. The goal is twofold: on one hand, to determine how far we can advance in the technical study of the model; on the other, to assess whether at least some of the deterministic outcomes remain valid in the stochastic setting. The resulting optimal control problem proves to be challenging, primarily due to the lack of concavity in the objective function. This feature is present in the model even in the deterministic case (see, e.g., [1]). We develop an approach based on Dynamic Programming to establish several useful results, including the regularity of the solution to the corresponding HJB equation and a verification theorem. There results lay the groundwork for studying the model optimal paths and comparing them with the deterministic case.

[1] Bambi, M. and Gozzi, F., Internal habits formation and optimality. In *Journal of Mathematical Economics,* **91**: 165–172, 2020

[2] Carroll, C.D., Overland, J. and Weil, D.N., Comparison Utility in a Growth Model. In *Journal of Economic Growth,* **2**: 339 – 367, 1997

[3] Carroll, C.D., Overland, J. and Weil, D.N., Saving and Growth with Habit Formation. In *American Economic Review,* **90**: 341–355, 2000

A class of deterministic and stochastic differential games with myopic Nash equilibria

Onésimo Hernández-Lerma (1), José E. Márquez-Prado (1), Héctor Jasso-Fuentes (1),

(1) Department of Mathematics, Cinvestav, Mexico City, Mexico;

We give conditions for a class of deterministic and stochastic differential games to have myopic Nash equilibria, that is, Nash equilibria obtained by solving static games independent of the state trajectory. We show how these conditions are the same for the deterministic and stochastic cases, which means that the certainty equivalence principle is satisfied. Moreover, for the infinite-horizon time-homogeneous case, we will see that Nash equilibria are obtained by solving a single static game; hence, Nash equilibria are constant functions.

[1] J. E. Márquez–Prado, O. Hernández–Lerma. Linear–State Control Problems and Differential Games: Deterministic and Stochastic Systems. *Journal of Optimization Theory and Applications.* 205:41 (2025).

[2] J. E. Márquez–Prado, O. Hernández–Lerma, H. Jasso-Fuentes. Myopic optimal strategies for a class of continuous-time deterministic and stochastic control problems. *Systems & Control Letters.* 196, 106016 (2025).

100. Continuous Optimization: Theory and Applications — 11:30 – 12:45

Chair: **R. Bot, Y. Malitsky, A. Daniilidis** — **FH Nöbauer 8**

Fast Krasnoselskii–Mann method with twice larger step sizes

Enis Chenchene (1), Radu I. Boţ (1), Jalal M. Fadili (2),

(1) Faculty of Mathematics, University of Vienna, Vienna, Austria; (2) Normandie Université, ENSICAEN, CNRS, GREYC, Caen, France;

We present a novel Fast Krasnoselskii–Mann-type method that maintains the fast $o(k^{-2})$ convergence rate on the fixed-point residual while allowing step sizes up to twice as large as those allowed in the original method introduced in [1]. This method reveals an intriguing connection between Nesterov and Halpern acceleration mechanisms, and offers a unified algorithmic framework to accelerate a wide range of splitting algorithms. We show that the enhanced parameter flexibility provides an efficient solution to solve Monge–Kantorovich optimal transport problems.

[1] R. I. Boţ and D.-K. Nguyen. Fast Krasnoselskii–Mann algorithm with a convergence rate of the fixed point iteration of $o(\frac{1}{k})$. *SIAM Journal on Numerical Analysis*, **61**(6):2813-2843, 2023.

A structured tour of optimization with finite differences

Marco Rando (1), Cesare Molinari (2), Lorenzo Rosasco (1), Silvia Villa (2),

(1) Machine Learning Genoa Center (MaLGa), Dipartimento di Informatica, Bioingegneria, Robotica e Ingegneria dei Sistemi (DIBRIS), Università degli Studi di Genova, Genova, Italia; (2) Machine Learning Genoa Center (MaLGa), Dipartimento di Matematica (DIMA), Università degli Studi di Genova, Genova, Italia;

Finite-difference methods are widely used for zeroth-order optimization in settings where gradient information is unavailable or expensive to compute. These procedures mimick first-order strategies by approximating gradients through function evaluations along a set of random directions. In this talk, I will illustrate recent theoretical results under various assumptions on the objective function showing convergence rates for the case where a structure — such as orthogonality — is imposed on the random set of directions. I will also review review and extend several strategies for constructing structured direction matrices, and empirically compare them with unstructured approaches in terms of computational cost, gradient approximation quality, and convergence behavior.

Regularizers promoting low condition number

Peter Balazs (1), Daniel Haider (1), Rossen Nenov (1,2),

(1) Acoustic Research Institute, Austrian Academy of Sciences, Vienna, Austria; (2) Faculty of Mathematics, University of Vienna, Vienna, Austria;

In this work, we introduce two novel matrix regularizers specifically designed to promote low condition numbers in linear operators. While the condition number is a critical measure of numerical stability and plays a central role in various applications, it is also a highly discontinuous and non-convex function, making it challenging to optimize directly. To address this, we propose two structured regularization strategies: one formulated as a difference-of-convex (DC) function and the other as a fractional regularizer. Both approaches provide tractable alternatives that implicitly enforce well-conditioned solutions. We discuss their regularization properties and simulate their effects in numerical experiments. Ensuring low condition numbers is particularly valuable in numerous domains, including inverse problems, machine learning, and numerical optimization, where stability and robustness are paramount.

101. Feedback control and stabilization of nonlinear PDEs 11:30 – 12:45

Chair: S. S. Rodrigues FH HS 6

Exponential Decay for a Wave Equation with a Nonlinear Delayed Term

Waled Al-Khulaifi (1,2), Manal Alotibi (1,3), Nasser-Eddine Tatar (1,4),

(1) King Fahd University of Petroleum & Minerals, Department of Mathematics, Dhahran, Saudi Arabia;
(2) Interdisciplinary Research Center for Construction and Building Materials, KFUPM, Saudi Arabia;
(3) Center for Integrative Petroleum Research, KFUPM, Saudi Arabia;
(4) Interdisciplinary Research Center for Intelligent Manufacturing and Robotics, KFUPM, Saudi Arabia;

In this talk, we consider a wave equation that incorporates strong damping and a nonlinear delayed term. Previous studies, such as [1], have established exponential stability under the condition that the weight of the damping (or strong damping) dominates the weight of the delay term. This idea has also been extended to cases involving nonlinear terms, as demonstrated by [2, 3, 4]. Our study introduces a new perspective: we achieve exponential stability and, remarkably, identify scenarios where the delayed term does not need to be dominated by the damping term, yet the system remains exponentially stable.

[1] S. Nicaise, C. Pignotti, Stability and instability results of the wave equation with a delay term in the boundary or internal feedbacks, *SIAM Journal on Control and Optimization* **45**(5), 1561–1585.131, 2006. https://doi.org/10.1137/060648891

[2] A. Benaissa, N. Louhibi, Global existence and energy decay of solutions to a nonlinear wave equation with a delay term, *Georgian Mathematical Journal*, **20**(1) 1–24, 2013. https://doi.org/10.1515/gmj-2013-0006

[3] L. Ihaddadene, A. Khemmoudj, eneral decay for a wave equation with Wentzell boundary conditions and nonlinear delay terms, *International Journal of Control*, **95**(9) 2565-2580, 2022. https://doi.org/10.1080/002071

[4] A. B. Laid Djilali, A. Benaissa, lobal existence and energy decay of solutions to a viscoelastic timoshenko beam system with a nonlinear delay term, *Applicable Analysis*, **95**(12) 2637-2660, 2016. https://doi.org/10.1080/00036811.2015.1105961

On H²-conforming finite elements for phase-field control with point-source stabilization

Herbert Egger (1,2), Marvin Fritz (2), Karl Kunisch (2,3), Sergio Rodrigues (2),

(1) Institute for Numerical Mathematics, Johannes Kepler University Linz, Austria; (2) Radon Institute for Computational and Applied Mathematics, Linz, Austria; (3) Institute for Mathematics and Scientific Computing, Karl-Franzens University Graz, Austria;

We analyze the numerical stabilization of the Cahn–Hilliard equation using H^2-conforming finite elements and point-source actuators. The controlled phase-field equation

$$\partial_t y + \nu\Delta^2 y - \Delta f(y) = h - \mathcal{F}(y - y_r)$$

is stabilized around a reference trajectory y_r with the feedback $\mathcal{F}z = \sum_{j=1}^{M_\sigma} z(x_j)\delta_{x_j}$. A backward Euler discretization with H^2-conforming Bell elements yields solutions $y_h^n \in V_h$ satisfying

$$\langle \partial_\tau y_h^n, v_h\rangle + \langle \nu\Delta y_h^n, \Delta v_h\rangle = \langle f(y_h^n), \Delta v_h\rangle - \langle \mathcal{F}(y_h^n - y_r^n), v_h\rangle \quad \forall v_h \in V_h,$$

where $\partial_\tau u^n := \tau^{-1}(u^n - u^{n-1})$ and $V_h \subset H^2_N(\Omega)$. We prove the existence and uniqueness of discrete solutions using a fixed point approach and inverse estimates. Further, we show error bound and an energy estimate that proves uniform decay under spectral gap conditions, preserved discretely. Numerical implementations in Firedrake validate the theory. The quintic Bell elements enable sparse C^1-discretizations, outperforming traditional DG/penalty methods.

SCD semismooth* Newton method for solving Stokes problem with stick-slip boundary

Vladimír Arzt (1), Petr Beremlijski (1),
(1) Department of Applied Mathematics, FEECS, VSB-Technical University of Ostrava, Czechia;

This contribution presents Stokes problem with nonlinear stick-slip boundary conditions, solved using the SCD semismooth* Newton method. This method, initially developed by J. Outrata, H. Gfrerer, and J. Valdman was designed for the numerical solution of generalized equations where the set-valued part is an SCD (subspace containing derivative) mapping. It was initially applied to contact problems involving Coulomb friction (see [1] for more details). Due to the nonlinearity introduced by the stick-slip boundary conditions, the Stokes problem is well-suited for this type of solver. The method involves the approximation or linearization of the set-valued term in the generalized equation based on the graph of the limiting coderivative. A linear system is solved in each Newton step via the Schur complement approach.

The method exhibits local superlinear convergence and outperforms other variants of semismooth Newton methods (see [2]) in terms of the number of iterations and overall computational time.

[1] H. Gfrerer, J.V. Outrata, and J. Valdman. On the Application of the SCD Semismooth* Newton Method to Variational Inequalities of the Second Kind. *Set-Valued and Variational Analysis*, **30**:1453–1484, 2022.

[2] J. Haslinger, R. Kučera, T. Sassi and V. Šátek. Dual strategies for solving the Stokes problem with stick-slip boundary conditions in 3D. In *Mathematics and Computers in Simulation*, **189**:191-206, 2021.

102. Topology optimization: theory, numerics and applications 11:30 – 12:20

Chair: **K. Sturm, P. Gangl** **FH HS 4**

A Newton-type scheme for topology optimisation

Phillip Baumann (1), Peter Gangl (1), Kevin Sturm (2),

(1) RICAM, Austrian Academy of Sciences, Linz, Austria; (2) ASC, TU Wien, Vienna, Austria;

In this talk, we discuss a second order algorithm to address PDE-constrained topology optimisation problems. Based on the notion of the topological state derivative we introduce a Newton-type scheme to identify descent directions. These are then further utilised in a level-set approach to obtain smooth changes of the topology.
We apply this approach to a variety of model problems, which necessitate slight modifications of the algorithm. Finally, we validate our findings in numerical tests.

Shape optimization in Stokes problems with stick-slip boundary conditions

Vladimír Arzt (1), Petr Beremlijski (1),

(1) Department of Applied Mathematics, FEECS, VSB-Technical University of Ostrava, Czechia;

This contribution deals with the numerical solution of shape optimization problems in fluid mechanics described by the Stokes problem with stick-slip boundary conditions. It is a problem of minimization of a composite function. To evaluate it for a specific boundary shape, we have to solve the so-called state problem, which in our case is described as a Stokes problem with stick-slip boundary conditions. The mentioned composite function is generally nondifferentiable due to the nonlinear boundary condition. The considered problem is thus described as a nonsmooth optimization problem. Therefore, an algorithm suitable for nonsmooth optimization should be used to solve it. We have used the bundle trust method. This method needs a function value and an arbitrary subgradient from the whole generalized gradient in each iteration. In our case, we only approximate this subgradient. The aim of the contribution is to present methods for solving shape optimization problems for Stokes problems and to present some numerical experiments.

103. Stochastic Control and Applications — 11:30 – 11:55

Chair: **A. Calvia, E. Bandini** — **FH HS 5**

The convergence problem for linear quadratic Mean Field Games in Hilbert Spaces and applications

Salvatore Federico (1), Daria Ghilli (2), Fausto Gozzi (3), Michele Ricciardi (3),

(1) Department of Mathematics, Bologna, Italy; (2) Department of Economics and Management, Pavia, Italy; (3) Department of Economics and Finance, LUISS, Rome, Italy;

We study a class of linear quadratic Mean Field Games (MFG) in infinite dimension, where the state variable lives in a Hilbert space. Our motivations are problems where the state equation is a PDE or a delay equation which can be written as an ODE in a suitable Hilbert space. As a starting point, we study the case, considered in most finite dimensional contributions on the topic, where the dependence on the distribution enters just in the objective functional through the mean. This feature allows, similarly to the finite dimensional case, to reduce the usual mean field game system to a Riccati equation and a forward-backward coupled system of abstract evolution equations. Such system is completely new in infinite dimension and no results have been proved on it so far. In the talk, we study the Nash system and the Master equation (ME) associated. Note that the solutions of the MFG are just the trajectories of the ME, which is an infinite dimensional PDE written in the space of measures. We prove existence and uniqueness of solutions to such system and to the ME, a verification theorem and the convergence of the solution to the Nash system to the solution of the ME. Finally we apply the results to a production output planning problem with delay in the control variable and to a vintage capital model where the production function depends on the mean of vintage capital.

104. Networks — 11:30 – 12:15

Chair: **S. Wrzaczek, M. Freiberger, M. Kuhn** — **FH HS 7**

Bayesian persuasion in networks: divisibility and network irrelevance

Toygar T. Kerman (1), Anastas P. Tenev (1), Yevgeny Tsodikovich (2),

(1) Institute of Economics, Corvinus University of Budapest, Budapest, Hungary; (2) Department of Economics, Ben-Gurion University of the Negev, Israel;

We study a multiple-receiver Bayesian persuasion model in which the sender wants to persuade a critical mass of receivers. Receivers are connected in a network and can perfectly observe their immediate neighbors' signals, which complicates the problem of the sender. We simplify the problem by considering signaling schemes ("experiments") in which certain receivers are never targeted, effectively dividing the network and preventing some of the information spillovers. Using this divided network approach, we derive lower bounds on the value of the sender, find experiments that obtain them, and provide sufficient conditions for the lower bounds to coincide with the optimal value. Our approach is robust to the addition and severance of some connections without affecting the results, sometimes even when adding more connections than existed in the original network. Finally, we demonstrate how our approach can be implemented if the network is nondivisible (e.g., a star) and compute the lower bound for such cases. This allows us to better interpret cultural phenomena such as echo chambers and opinion polarization.

Endogenous criticality, universality, and evolutionary dynamics in strategically formed random networks

Florian Herold (1),

(1) Institute of Economics, University of Bamberg, Bamberg, Germany;

Many large systems exhibit a phase transition, i.e. there is a critical threshold p_c of some order parameter p such that below p_c the system is in a very different phase than above p_c. Over the last decades in physics, mathematics, and other areas a large and beautiful set of theories, techniques and mathematical tools (renormalization, power laws, scaling relations, self-organized criticality) have been developed to analyze behavior at and around such critical thresholds. Here we are interested in large networks of agents who strategically influence the connection probabilities in a network. For concreteness, consider a model of agents located on a transitive two-dimensional lattice, say on a 2D triangular lattice. Agents strategically choose the probability of protecting themselves against a virus, e.g. by vaccinating. Vaccinated agents are protected and we assume that they can not spread the virus to their neighbours. The resulting networks corresponds to a (site) percolation model exhibiting a phase transition: Above a critical probability p_c a large fraction of vertices (here agents) are almost surely connected by a giant component. Below the critical probability all components are almost surely of finite size. Close to the critical probability different characteristics have a power law distribution. The corresponding critical exponents are expected to be universal, i.e. they depend only on the dimension of the lattice, but not on most other details of the lattice. Our leading example will be independent site percolation on the 2D-triangular lattice (or equivalently bond percolation on the 2D-hexagonal lattice), for which Smirnov [2001 a,b] proved rigorously conformal invariance and thus the size of the critical exponents. We show that there is a broad class of economic settings in which (sufficiently) below the critical threshold agents have an incentive to increase their probability of connecting to their neighbours, while (sufficiently) above the critical threshold, agents have an incentive to lower their connection probability. In Nash-equilibrium agents will then choose a probability close to the critical probability p_c. Assuming the universality conjecture is true, several policy implications will then be robust to different modeling assumptions about the underlying lattice structure.

Finally, by adding a time component and considering a game theoretic evolutionary dynamics, interesting dynamic effects arise.

105. Economics of climate change — 12:15 – 12:45

Chair: S. Wrzaczek, M. Freiberger, M. Kuhn — FH HS 7

International climate policy and risk sharing

Markus Epp (1), Marten Hillebrand (2),

(1) Department of Economic Theory, University of Freiburg, Freiburg, Germany;
(2) Department of Economic Theory, University of Freiburg, Freiburg, Germany;

The paper studies the strategic interaction of finitely many heterogeneous countries in a stochastic growth model of climate change. Production in each country is based on capital and fossil energy and subjected to both exogenous fundamental shocks and endogenous climate risk. Climate risk depends on global temperature reflecting aggregate emissions from fossil energy. We study the scope for international risk sharing in this model under different market arrangements and climate policies. In terms of markets, we consider the three scenarios of autarky where regions do not trade, incomplete markets with commodity trade and a riskless international bond market, and complete markets with trade on international markets for capital, fossil fuels, and output and a full set of state-contingent Arrow securities in each period. Climate policy takes the form of carbon taxation in each country. We distinguish the two policy scenarios of full cooperation where all countries coordinate on a globally optimal carbon tax and the non-cooperative case where each region sets the domestically optimal tax. Our main result provides analytical characterizations of optimal carbon taxation in closed form depending on the degree of cooperation and the market structure. Specifically, we show that globally optimal carbon taxes incorporate the expected discounted future global climate damages while locally optimal tax policies incorporate only domestic damages. The market structure determines the stochastic discount factors (SDF) in these formulas. In the autarky case, SDFs are unrelated across regions while their expected values align in the incomplete markets case. Complete markets lead to a unique global SDF applied by all regions. In terms of risk sharing, countries are fully exposed to domestic risk under autarky which can partially be diversified in the incomplete markets case. In the complete markets scenario, countries fully diversify their idiosyncratic production risk leading to a constant world consumption distribution and consumption levels proportional to global consumption in each region. Numerical simulations based on calibrated parameter values are used to illustrate the results and quantify the welfare effects of cooperation and international risk sharing.

106. Continuous Optimization: Theory and Applications — 13:45 – 15:00

Chair: R. Bot, Y. Malitsky, A. Daniilidis — FH Nöbauer 8

Bayesian transfer operators in reproducing kernel hilbert spaces

Septimus Boshoff (1), Sebastian Peitz (2),

(1) Data Science for Engineering, Paderborn University, Paderborn, Germany; (2) Safe Autonomous Systems, TU Dortmund, Dortmund, Germany;

The Koopman operator, as a linear representation of a nonlinear dynamical system, has been attracting attention in many fields of science. Recently, Koopman operator theory has been combined with another concept that is popular in data science: reproducing kernel Hilbert spaces (RKHSs). Kernels provide a flexible and nonparametric machinery to implicitly construct basis functions. We follow this thread into Gaussian process methods, and illustrate how these methods can alleviate *two* pervasive problems with kernel-based Koopman algorithms. The first being sparsity: 1) most kernel methods don't scale well and require a compression of sorts to become practical. The computational demands and memory requirements typically grow as the cube and square (respectively) of the number of training points, limiting the direct implementation to problems with at most a few thousand data points. We show that not only can the computational demands be significantly reduced, but the resilience against sensor noise can be improved. The second problem 2) involves hyperparameter optimization and dictionary learning to adapt the model to the dynamical system in order to achieve competitive results.

Nonconvex nonsmooth multicomposite optimization and its applications to recurrent neural networks

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(1) Department of Applied Mathematics, Hong Kong Polytechnic University, Kowloon, Hong Kong, China;
(2) Pengcheng Laboratory, Shenzhen, Guangdong, China;

We consider a class of nonconvex nonsmooth multicomposite optimization problems where the objective function consists of a Tikhonov regularizer and a composition of multiple nonconvex nonsmooth component functions. Such optimization problems arise from tangible applications in machine learning and beyond. To define and compute its first-order and second-order d(irectional)-stationary points effectively, we first derive the closed-form expression of the tangent cone for the feasible region of its constrained reformulation. Building on this, we establish its equivalence with the corresponding constrained and ℓ_1-penalty reformulations in terms of global optimality and d-stationarity. The equivalence offers indirect methods to attain the first-order and second-order d-stationary points of the original problem in certain cases. We apply our results to the training process of recurrent neural networks (RNNs).

Poster Session, Thursday, July 17th

107. Poster session 15:10 – 16:00

Freihaus, 2nd*Floor*

Pricing Equity Swaps using the Vasicek Model

Ghada Alobaidi (1), (1) American University of Sharjh, AE;

Optimal Control of the Navier-Stokes Equations with Mixed Boundary Conditions

Tabea Hüfken (1), (1) Martin-Luther Universität Halle-Wittenberg, DE;

Parallel-in-time preconditioning for time-dependent variational mean field games

Heidi Wolles (1), (1) University of Edinburgh, UK;

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